Export Conditions in Small Countries and their Effects on Domestic Markets

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Abstract

In a small country's industries, it is common that both small and large firms export a significant share of their total production. How does better export access affect the domestic market when this occurs? Incorporating investments in quality that require fixed outlays and increase a variety's appeal in all countries, we show that an export shock entails two opposing mechanisms. On the one hand, it induces quality upgrades that raise the domestic market share of large firms. On the other hand, it fosters entry of small firms, making large firms lose domestic market share and downgrade quality. Using Danish data, we show that small firms in some industries are so heavily export-oriented that better export opportunities reallocate domestic market share towards the least productive domestic firms. And while competition by small firms reduces some large firms' domestic markups, it also leads some to downgrade quality and suffer a substantial fall in profits.

Keywords: large firms, small firms, quality, export access, small country, Denmark.
JEL codes: F12, F14, L11.

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1 Introduction

Examples of industries where both small firms (SFs) and large firms (LFs) coexist abound. For instance, multinational pharmaceuticals tend to compete with a myriad of SFs producing generic brands; Coca-Cola and PepsiCo operate simultaneously with numerous small companies that produce private-label brands; and sport-apparel firms such as Nike and Adidas serve markets along with countless small businesses. Such a feature is particularly relevant when export opportunities in small countries are analyzed. These economies are characterized by both SFs and LFs exporting a large portion of their production, given the constraints imposed by a small home market. Consequently, better export conditions affect firms of different sizes and business models.[1]

When SFs and LFs coexist and export intensively, how does better export access differentially affect each firm's decisions? And what are the consequences that this has over the home market? In this paper, we address these questions using a framework with oligopolistic firms embedded into a monopolistic-competition setup, as in Shimomura and Thisse (2012). In particular, we study the effects of an industry-wide export shock, based on how better export opportunities to only SFs or LFs activate different mechanisms that impact the home market.

Relative to standard models with firm heterogeneity, partitioning firms enriches the analysis by reflecting the distinctive features of firms more appropriately according to their size. This becomes especially pertinent when there are stark differences in firm size within the same industry, as is the case when small businesses compete with leading companies.

Specifically, we describe SFs as in Melitz (2003), thereby capturing several stylized facts about small companies that have been identified in the literature since at least Dunne et al. (1988). This characterizes SFs as entrepreneurs that explore their possibilities in the industry and make entry decisions with uncertainty about their profitability. Eventually, they either do not succeed and exit the market, or survive and operate as negligible firms, with the most productive ones also exporting.

In contrast, LFs are regarded as well-established businesses that know their efficiency, are the most productive firms at home, and earn positive profits. Moreover, we treat them as granular entities exhibiting idiosyncratic features, without imposing a specific model relation between productivity and exporting. This determines that an export shock might actually benefit or hurt LFs in equilibrium, entailing quite different consequences for the home market.

[1] In addition to Denmark, which is the country we use for the empirical application, other studies have shown the widespread coexistence of SFs and LFs within an industry. See, for instance, Hottman et al. (2016) for consumer-goods industries in the US and Gaubert and Itskhoki (2021) for French manufacturing. Regarding the fact that both SFs and LFs tend to export in small countries, see Mayer and Ottaviano (2008) for various examples based on European countries.

Better export opportunities in our framework impact the domestic market through two mechanisms. First, they induce entry of SFs by increasing a SF's expected profits, which results in tougher domestic competition. Second, they stimulate investments in quality of all firms, and in particular of LFs. Quality in our model is interpreted in a broad sense: it reflects overhauls in a product's physical features, but also improvements in perceived features, after-sales services, and brand image (Sutton, 1991). Formally, we define quality as requiring fixed outlays, and enhancing both demand and the consumers' willingness to pay in *all* countries served. By these properties, better export opportunities increase a firm's scale and hence reduce the average cost of investing, which ends up fostering investments. This makes domestic varieties more appealing, thus increasing in particular the revenue and markup at home of each LF.[2]

After formalizing the setup in Section 2 and deriving the equilibrium in Section 3, we study how an export shock affects the domestic industry in Section 4. We start by considering a scenario where all firms are modeled as in Melitz and choose quality, with the goal of establishing some benchmark results relative to our setup. In this setting, an export shock reduces the profits of firms exclusively serving home and makes them downgrade quality. On the contrary, firms that start or continue exporting, which correspond to the most productive firms in Melitz, always benefit and upgrade quality. The result is akin to Bustos (2011), who obtains it in a setting where firms only decide whether to invest or not.

An identical description of outcomes holds for SFs in our framework. However, LFs, which are the most productive firms, could be either negatively or positively impacted by an industry-wide export shock. This occurs due to the existence of two opposing channels, which we identify by considering export shocks to each type of firm in isolation.

On the one hand, an export shock to only SFs increases their expected profits, thus fostering entry of SFs and strengthening competition at home. This raises a LF's cost per unit of quality by reducing its total sales, and also decreases a LF's market power at home due to tougher domestic competition. As a corollary, LFs reduce their domestic prices, downgrade quality, have a fall in profits, and domestic market share is reallocated towards SFs.

On the other hand, an export shock to only LFs benefits LFs by increasing their sales volume, thereby reducing their average costs in quality and hence providing incentives to raise quality. Since a LF's quality upgrade increases its variety's appeal at home, its position in the domestic market is affected: it charges higher domestic markups, earns greater profits, and

[2] We could have alternatively considered cost-reducing investments. The key difference is that investments in quality increase prices, which is consistent with recent evidence documenting that LFs tend to charge higher prices (e.g., Hottman et al. 2016 and Foster et al. 2016). There is also indirect empirical evidence indicating that exporters, and hence larger firms, charge higher prices on average (see, for instance, Manova and Zhang 2012, Eckel et al. 2015, and Gervais 2015).

increases its domestic market share at the expense of SFs.

Whether a LF is ultimately positively or negatively impacted depends on the magnitude of these opposing channels. Our results indicate that the strength of each channel can be inferred in terms of observables through the export intensities of SFs and LFs, defined as the share of exports in revenues. Specifically, the export intensity of SFs captures the extent to which competition becomes tougher at home. This follows because, when SFs have greater export intensity, an export shock entails higher increases in a SF's expected profit, implying relatively more entry. Additionally, a LF with greater export intensity benefits more from better export opportunities, and is simultaneously more shielded from tougher domestic competition.

To illustrate how the export intensities of SFs and LFs can lead to starkly different effects over the home market, we conduct several calibration exercises. With this aim, after laying out the procedure to quantify the model in Section 5, we analyze various Danish sectors in Section 6. Denmark is a particularly suitable choice, since it constitutes a small highly open economy where exporting is pervasive even among SFs: about half of the SFs in manufacturing are exporters and have an average export intensity of around 25%. Also, a market structure with coexistence of SFs and LFs is ubiquitous, with industries displaying this feature accounting for more than 80% of the total manufacturing revenue.

We first consider a representative industry in Danish manufacturing. The results indicate that an export shock makes each LF upgrade quality and earn greater profits. Moreover, LFs as a group gain presence at home, although this masks a heterogeneous impact on each LF, since they are differentially affected by better export access and tougher domestic competition. The top LF, which has a higher export intensity, gains domestic market share and increases prices at home. On the contrary, the rest of LFs, which exhibit a greater home bias, end up with lower domestic market shares and reduce their markups at home, despite their quality upgrades.

Additionally, we analyze outcomes for Denmark's top sectors by revenues, expenditures, and exports. First, we consider Food & Beverages. The calibration for this industry features SFs with high export intensity, thereby implying a pronounced increase in domestic competition following an export shock. Furthermore, LFs exhibit a significant home bias, so that they do not benefit substantially from increases in sales volume, but are severely affected by tougher domestic competition. These features determine that, even though LFs upgrade quality, they all charge lower markups at home and lose domestic market share. Moreover, the top two LFs have a substantial fall in profits.

We also provide results for Chemicals. SFs in this sector are even more export-oriented than the other cases analyzed, leading to significant entry of SFs and hence marked increases in domestic competition. Moreover, LFs exhibit a high degree of heterogeneity in their export intensities, precluding a general characterization of how they are impacted. This feature can be clearly demonstrated by comparing the impact of an export shock on the top two firms. Regarding the top LF, its export revenue considerably surpasses its domestic sales, determining that it substantially upgrades quality. Consequently, this firm gains domestic market share, charges higher markups at home, and garners greater profit. On the contrary, the second top LF is mainly oriented to the local market, and so an export shock represents primarily tougher domestic competition for it. Thus, it downgrades quality, reduces its domestic markups, loses domestic market share, and has a fall in profit.

Related Literature and Contributions. Our paper talks to a vast literature studying the effects of export shocks on R&D investments, and particularly on product innovation. It is consistent with mounting evidence on a positive relation between firm size and R&D expenditures,[3] with better export opportunities in particular increasing the scale of production and hence stimulating innovation (see, for instance, the survey by Shu and Steinwender 2019). It is also consistent with other literature, which emphasizes that better export opportunities can result in negative effects for firms by increasing competition (Baldwin and Gu 2009; Aghion et al. 2018). Indeed, consistent with these opposing effects, studies like Lileeva and Trefler (2010) and Bustos (2011) show that firms tend to be heterogeneously impacted by better export access.

Our paper contributes to this literature by setting a model that simultaneously incorporates these positive and negative effects of an export shock. Furthermore, we focus on industries with three features: i) coexistence of SFs and LFs, ii) large differences in firm size, so that LFs are better described as oligopolistic firms and SFs as monopolistic firms, and iii) SFs significantly engaged in exporting, as is common in small countries.

The model follows Shimomura and Thisse (2012) and Alfaro and Warzynski (2020), by partitioning firms according to their size. The approach has the advantage of accounting for a handful of leading firms, while retaining the benefits of modeling SFs as a continuum. Unlike these papers, we study the impact of better export opportunities in an industry in isolation, and extend their frameworks to account for choices on quality that affect all the countries served.[4] Moreover, we model LFs as companies that could be either domestic- or export-oriented, and hence either benefit or be hurt by an export shock. This implies that a LF could ultimately

[3] See Cohen (2010) for a survey of the literature. More recently, Knott and Vieregger (2020) corroborate this relation by using the most comprehensive survey of innovation available in the US by the US Census Bureau. They show that all types of innovation are increasing in scale, including product and service innovation. Moreover, the mechanism is a reduction in average costs through larger sales, consistent with the seminal studies by Cohen and Klepper (1996a; 1996b).

[4] For other papers using a similar approach, see Parenti (2018) and Anderson et al. (2020).

upgrade or downgrade quality, with different consequences over the domestic market regarding markups and market share. The outcome starkly contrasts with a setting à la Melitz, where the most productive firms always benefit and upgrade quality.

2 Model Setup

We consider a world economy with a set of countries $\mathcal{C}$. We utilize the convention that any variable subscript ij refers to i as the origin country and j as the destination country. Throughout this section, we describe the model through countries indices $i, j \in \mathcal{C}$. All the derivations and proofs of this paper are relegated to Appendix A.

2.1 Generalities

In each country i, there is a unitary mass of identical agents that are immobile across countries. Moreover, labor is the only production factor, and each agent offers a unit of labor inelastically. We suppose the existence of two sectors. One consists of a differentiated good, while the other is a homogeneous good produced and sold in each country under perfect competition. We take the latter as the numéraire and suppose that its technology of production determines wages w_i in each country i.

The differentiated industry in i comprises a set of single-product firms $\overline{\Omega}_i$, where each can potentially serve any country j with a unique variety. The coexistence of different types of firms is introduced by partitioning each $\overline{\Omega}_i$ into a finite set $\overline{\mathscr{L}}_i$ and a real interval $\overline{\mathcal{N}}_i$, whose letters are respectively mnemonics for "large" and "negligible". We refer to any firm $\omega \in \overline{\mathscr{L}}_i$ as a LF from i, and a firm $\omega \in \overline{\mathcal{N}}_i$ as a SF from i. Morever, firm $\omega \in \overline{\mathscr{L}}_i$ affects the price index of a country, while any firm $\omega \in \overline{\mathcal{N}}_i$ is negligible for industry conditions.

In terms of notation, we denote by Ω_{ji} the subset of varieties from j sold in i, with $\Omega_i := \cup_{k \in \mathcal{C}} \Omega_{ki}$ being the set of total varieties available in i. Likewise, $\Omega_{ji}^{\mathcal{N}} := \overline{\mathcal{N}}_j \cap \Omega_{ji}$ and $\Omega_{ji}^{\mathscr{L}} := \overline{\mathscr{L}}_j \cap \Omega_{ji}$ are the subsets of varieties available in i that are produced by SFs and LFs from j, respectively.

2.2 Supply Side

SFs from i are ex-ante identical and do not know their productivity. Each can receive a productivity draw φ and get a unique variety assigned by paying a sunk entry cost F_i. We suppose that productivity is a continuous random variable, with non-negative support $\left[\underline{\varphi}_i, \overline{\varphi}_i\right]$ and cumulative distribution function G_i. The mass of SFs that pay the entry cost is denoted by M_i^E.

The number of LFs from i is exogenous, with each having assigned a unique variety $\omega \in \overline{\mathscr{L}}_i$ and productivity φ_ω that is common knowledge across the world. We suppose that $\varphi_\omega > \overline{\varphi}_i$ for any $\omega \in \overline{\mathscr{L}}_i$, so that any LF from i is more productive than the most productive SFs from i.

A SF or LF ω that serves j from i produces with constant marginal costs c_{ij}^ω, given by $c\left(\varphi_\omega, \tau_{ij}^\omega\right) := \frac{w_i}{\varphi_\omega}\tau_{ij}^\omega$. The term τ_{ij}^ω represents trade costs, and satisfy $\tau_{ii}^\omega := 1$ and $\tau_{ij}^\omega := \tau^\omega \tau_{ij}$ if $j \neq i$. For any SF ω from i, we also suppose that τ^ω is symmetric and denote it by $\tau^{\mathcal{N}_i}$, with τ_{ij}^ω denoted by $\tau_{ij}^{\mathcal{N}}$.

The fact that trade costs can be decomposed into a firm-specific component (τ^ω) and a common component (τ_{ij}) serves two purposes. First, it enables us to investigate the effects of export shocks that are specific to a group of firms: the impact of better export access for SFs only (by varying $\tau^{\mathcal{N}_i}$), for LFs only (by varying τ^ω for each $\omega \in \overline{\mathscr{L}}_i$), or for all firms (by varying τ_{ij}). Second, firm-specific trade costs for LFs allow for scenarios where a LF has greater domestic sales relative to other LFs, without implying that its exports are greater too (or that it exports at all). Consequently, we do not impose any restrictions on the export intensity of LFs.

As for decisions, LFs from i and the mass M_i^E of SFs decide whether to pay an overhead fixed cost f_{ij} and serve country j. If firm ω does so, it chooses a price p_{ij}^ω. Additionally, any firm ω that becomes active in at least one market decides on the quality of its variety, z_i^ω. This variable simultaneously affects every market served, and entails sunk costs $I_i^\omega := f^z z_i^\omega$. Throughout the paper, we refer to z_i^ω as *quality* and I_i^ω as *investments*.

2.3 Demand Side

Preferences in each country i are represented by a two-tier utility function. The upper tier is quasilinear between the homogeneous and differentiated good. Formally, $U_i := E_i \ln\left(\mathbb{Q}_i\right) + \mathbb{Q}_i^0$, where $E_i > 0$, $\mathbb{Q}_i^0$ is the quantity consumed of the homogeneous good, and $\mathbb{Q}_i$ is the quantity index of the differentiated good. Assuming that income is high enough that there is consumption of both goods, the optimal expenditure on the differentiated good is $\mathbb{P}_i\mathbb{Q}_i = E_i$.

Preferences for the differentiated good are given by an augmented CES sub-utility function, so that the demand of a firm ω from j in i is given by

$$Q_{ji}^\omega := E_i \left(\mathbb{P}_i\right)^{\sigma-1} \left(p_{ji}^\omega\right)^{-\sigma} \left(z_j^\omega\right)^\delta, \tag{1}$$

where $\sigma > 1$, $\delta \in (0,1)$, and $\mathbb{P}_j$ is j's price index given by

$$\mathbb{P}_i := \left\{ \sum_{k\in\mathcal{C}} \left[\int_{\omega\in\Omega_{ki}^{\mathcal{N}}} \left(p_{ki}^{\omega}\right)^{1-\sigma} \left(z_k^{\mathcal{N}}\right)^{\delta} \mathrm{d}\omega + \sum_{\omega\in\Omega_{ki}^{\mathcal{L}}} \left(p_{ki}^{\omega}\right)^{1-\sigma} \left(z_k^{\omega}\right)^{\delta} \right] \right\}^{\frac{1}{1-\sigma}}.$$

Moreover, letting $R_{ij}^{\omega} := p_{ij}^{\omega} Q_{ij}^{\omega}$, ω's market share in j is defined by $s_{ij}^{\omega} := \frac{R_{ij}^{\omega}}{E_j}$ and can be expressed as

$$s\left(p_{ij}^{\omega}, z_i^{\omega}, \mathbb{P}_j\right) := \frac{\left(p_{ij}^{\omega}\right)^{1-\sigma} \left(z_i^{\omega}\right)^{\delta}}{\mathbb{P}_j^{1-\sigma}}. \tag{2}$$

The parameter δ measures the impact of quality on demand. It represents the quality elasticity of demand for SFs, with $\delta\left(1 - s_{ij}^{\omega}\right)$ as the quality elasticity for a LF ω. Likewise, ω's price elasticity of demand in j is given by $\varepsilon_{ij}^{\omega} := \left|\frac{\mathrm{d}\ln Q_{ij}^{\omega}}{\mathrm{d}\ln p_{ij}^{\omega}}\right|$, where $\varepsilon_{ij}^{\omega} = \sigma$ if ω is a SF and $\varepsilon\left(s_{ij}^{\omega}\right) = \sigma + s_{ij}^{\omega}\left(1-\sigma\right)$ if ω is a LF.

Throughout the paper, we assume that $\varepsilon_{ij}^{\omega}\left(1 - s_{ij}^{\omega}\right) - s_{ij}^{\omega} > 0$ for any $i, j \in \mathcal{C}$. This holds as long as s_{ij}^{ω} is not disproportionately large, as is the case in the Danish data for domestic firms.[5] The assumption allows us to obtain some definite results when we perform comparative statics. More generally, it rules out some counter-intuitive results that arise in models with LFs under a CES demand.

2.4 Small-Economy Assumption

Our focus is on a country H that is small in the sense of Demidova and Rodríguez-Clare (2009; 2013). This definition establishes that changes in H's domestic conditions and its domestic firms do not affect the aggregate conditions of any foreign country. It formally determines that $\left(\mathbb{P}_j, M_j^E\right)_{j\in\mathcal{C}\setminus\{H\}}$ is not impacted by a trade shock in H.[6] Notice that this does not rule out extensive margin adjustments, since the survival productivity cutoffs of foreign firms in H are still endogenous.

Incorporating the small-economy assumption, we directly simplify the set of countries under consideration and take $\mathcal{C} := \{H, F\}$, where F constitutes a composite country that represents the rest of the world.

[5] For instance, given $\sigma := 3.53$, which is the value for our representative Danish manufacturing industry, it is satisfied as long as no firm has a market share greater than 70%.

[6] The small-country assumption can be rationalized through a framework where each country has a continuum of trading partners and H is part of it (see Alfaro 2019).

3 Equilibrium

We proceed to outline the equilibrium conditions. Since the derivations are algebraically intensive and contain several unwieldy expressions, we concentrate on the main elements that are necessary to explain the results. A formal treatment is relegated to the appendix.

Throughout the paper, we consider that there is always a positive mass of active SFs in equilibrium. This implies that each LF is always active and serves its domestic market. Furthermore, we suppose that there is selection into exporting among SFs, and that any SF which exports also finds it profitable to serve its domestic market.

3.1 Optimal Decisions by Small Firms

As in the previous section, we keep supposing that $i, j \in \mathcal{C}$. A SF ω from i that pays the entry cost makes choices by maximizing π_i^ω, which is given by

$$\pi_i^\omega := \sum_{k \in \mathcal{C}} \pi_{ik}^\omega - f^z z_i^\omega,$$

where $\pi_{ik}^\omega := Q_{ik}^\omega \left(p_{ik}^\omega - c_{ik}^\omega\right) - f_{ik}$.

Optimal prices of a SF with productivity φ that is active in j is

$$p^{\mathcal{N}} \left(\varphi; \tau_{ij}^{\mathcal{N}}\right) := \frac{\sigma}{\sigma - 1} \frac{\tau_{ij}^{\mathcal{N}} w_i}{\varphi}. \tag{3}$$

Any SF ω that serves a market also makes a choice on quality. This variable affects all the markets served by ω, making ω's decision interdependent across markets. Utilizing that $R_{ij}^\omega = 0$ when country $j \neq i$ is not served, the optimal level of quality is

$$z_i^\omega = \frac{\delta}{f^z} \sum_{k \in \mathcal{C}} \frac{R_{ik}^\omega}{\sigma}. \tag{4}$$

This determines that optimal investments satisfy $I_i^\omega = \delta \left(\sum_{k \in \mathcal{C}} \frac{R_{ik}^\omega}{\sigma}\right)$, where $\frac{R_{ik}^\omega}{\sigma}$ corresponds to ω's optimal variable profits in country k. Consequently, optimal investments are a fixed proportion $\delta \in (0, 1)$ of ω's total variable profits, and so our setup captures the **cost-spreading advantage of a larger firm size** (Cohen, 2010): increases in a firm's total revenue, $\sum_{k \in \mathcal{C}} R_{ik}^\omega$, spread out the cost f^z between more units, hence reducing the average cost of quality. Due to this, any shock increasing a firm's revenue (e.g., better export opportunities) provides incentives to upgrade quality.

Once we have identified optimal choices, we can determine the optimal profits of SFs. A SF from H only serving home has optimal revenues $r_H^d \left(\mathbb{P}_H, \varphi\right)$, and so its profit is

$$\pi_H^d \left(\mathbb{P}_H, \varphi\right) := \frac{(1 - \delta)}{\sigma} r_H^d \left(\mathbb{P}_H, \varphi\right) - f_{HH}.$$

In turn, the productivity cutoff that makes this SF indifferent between serving home or not, φ^*_{HH}, is pinned down by the condition $\pi^d_H(\mathbb{P}_H, \varphi^*_{HH}) = 0$. It determines a function $\varphi^*_{HH}(\mathbb{P}_H)$.

Regarding an exporting SF from H, the sum of its revenue in each market is denoted $r^x_H\left(\mathbb{P}_H, \varphi, \tau^{\mathcal{N}}_{HF}\right)$. Unlike a setting without choices in quality, the fact that markets are interdependent determines that r^x_H is not separable in terms of revenue per country.[7] This implies that its total profit is

$$\pi^x_H\left(\mathbb{P}_H, \varphi, \tau^{\mathcal{N}}_{HF}\right) := \frac{(1-\delta)}{\sigma} r^x_H\left(\mathbb{P}_H, \varphi, \tau^{\mathcal{N}}_{HF}\right) - f_{HH} - f_{HF}.$$

Moreover, the productivity cutoff to export, φ^*_{HF}, takes into account the profits obtained in all countries served. Formally, φ^*_{HF} is the solution to $\pi^d_H(\mathbb{P}_H, \varphi^*_{HF}) = \pi^x_H\left(\mathbb{P}_H, \varphi^*_{HF}, \tau^{\mathcal{N}}_{HF}\right)$, and defines a function depending on the domestic price index, $\varphi^*_{HF}\left(\mathbb{P}_H, \tau^{\mathcal{N}}_{HF}\right)$.

Finally, the expression for expected profits also takes into account that an exporting SF's revenues are not separable. Formally, evaluating expected profits at the optimal productivity cutoffs, the free-entry condition is given by

$$\pi^{\mathbb{E}}_H\left(\mathbb{P}_H, \tau^{\mathcal{N}}_{HF}\right) := \int_{\varphi^*_{HH}(\mathbb{P}_H)}^{\varphi^*_{HF}\left(\mathbb{P}_H, \tau^{\mathcal{N}}_{HF}\right)} \pi^d_H(\mathbb{P}_H, \varphi)\, \mathrm{d}G_H(\varphi) + \int_{\varphi^*_{HF}\left(\mathbb{P}_H, \tau^{\mathcal{N}}_{HF}\right)}^{\overline{\varphi}_H} \pi^x_H\left(\mathbb{P}_H, \varphi, \tau^{\mathcal{N}}_{HF}\right) \mathrm{d}G_H(\varphi) = F_H. \tag{FE}$$

3.2 Optimal Decisions by Large Firms and Equilibrium

Next, we focus on a LF ω from H. Since the decisions of the foreign LFs do not play an essential role in our results, we relegate their description to the appendix.

LF ω's optimal decisions can be expressed as functions of its market share, which in turn is a function of the price index. Thus, its optimal price in j is

$$p^\omega_{Hj}\left(s^\omega_{Hj}\right) := m^\omega_{Hj} c^\omega_{Hj}, \tag{5}$$

where m^ω_{Hj} are ω's markups in j, with m^ω_{HH} given by $m\left(s^\omega_{HH}\right) := \frac{\varepsilon\left(s^\omega_{HH}\right)}{\varepsilon\left(s^\omega_{HH}\right)-1}$, and $m^\omega_{HF} := \frac{\sigma}{\sigma-1}$. Markups abroad reflect that no firm from H can affect the foreign country's industry conditions, given that H is assumed small.

Furthermore, using that $R^\omega_{ij} = E_j s^\omega_{ij}$, LF ω's optimal quality is

$$z^\omega_H\left(s^\omega_{HH}, s^\omega_{HF}\right) := \frac{\delta}{f^z}\left[\frac{R^\omega_{HH}\left(1 - s^\omega_{HH}\right)}{\varepsilon\left(s^\omega_{HH}\right)} + \frac{R^\omega_{HF}}{\sigma}\right], \tag{6}$$

yielding optimal investments $I^\omega_H\left(s^\omega_{HH}, s^\omega_{HF}\right) := f^z z^\omega_H\left(s^\omega_{HH}, s^\omega_{HF}\right)$. The description of the LFs' choices is akin to those by SFs, with the only difference that LFs influence industry conditions at home. The interpretations of optimal choices are similar too, and hence reflect the cost-

[7] Formally, $r^x_H\left(\mathbb{P}_H, \varphi, \tau^{\mathcal{N}}_{HF}\right) = \kappa\left[E_H\left(\mathbb{P}_H\varphi\right)^{\sigma-1} + E_F\left(\frac{\mathbb{P}_F\varphi}{\tau^{\mathcal{N}}_{HF}}\right)^{\sigma-1}\right]^{\frac{1}{1-\delta}}$ for some constant κ.

spreading advantage of a larger firm size.

Finally, for future reference, we obtain an expression for $\overline{\pi}_H^\omega$. We refer to it as ω's gross profits, which correspond to ω's total profits net of quality costs, but gross of market fixed costs. It is given by

$$\overline{\pi}_H^\omega \left(s_{HH}^\omega, s_{HF}^\omega\right) := \frac{R_{HH}^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]}{\varepsilon\left(s_{HH}^\omega\right)} + \frac{R_{HF}^\omega \left(1-\delta\right)}{\sigma}. \tag{7}$$

We additionally define the industry gross profits by $\overline{\Pi}_H^{\mathscr{L}} := \sum_{\omega\in\mathscr{L}_H} \overline{\pi}_H^\omega \left(s_{HH}^\omega, s_{HF}^\omega\right)$.[8]

4 Mechanisms: Results and Illustrations

In this section, we investigate the impact of better export access on the domestic market. We begin by presenting outcomes when all firms are characterized as in Melitz to establish some benchmark results. After this, we study export shocks to (i) only SFs, (ii) only LFs, and (iii) all firms.

Although (i) and (ii) could be deemed important on their own, we primarily utilize them to identify the mechanisms of adjustment.[9] These cases rationalize in particular why the outcomes for LFs under (iii) are ambiguous.

4.1 Benchmark Results

A framework with firms described as in Melitz is a special case of our setting. It arises when there are no LFs, so that all firms are treated as SFs. The following proposition identifies its effects.

Proposition 1: Melitz with Choices in Quality

Suppose that the set of LFs in H is empty, so that all firms are characterized as in Melitz. Moreover, consider a small reduction in H's export trade costs.

Then, a firm ω from H that exclusively serves home invests less in quality (I_H^ω) and earns lower profits (π_H^d). On the contrary, a firm ω from H that either becomes an exporter or continues exporting invests more in quality (I_H^ω) and earns greater total profits (π_H^x).

[8] The model is closed by an equilibrium condition for the market stage, which requires that the sum of market shares in each country equals one. We relegate its description to the appendix, since it is not necessary to identify the results we are interested in—we only need the equilibrium value of $\mathbb{P}_H$, and this can be directly identified through (FE).

[9] Even when it is not our primary goal, it is possible to conceive scenarios with reductions in export trade costs that are targeted at a specific group of firms. For instance, a government could implement policies that only favor multinationals, as is suggested by Freund and Pierola (2015) to boost exports. Alternatively, a government could establish export subsidies for a specific product in an industry. If one LF mainly produces this product, the policy is basically equivalent to targeting this firm (see Gaubert et al. 2021).

The proposition shows that a firm's decision on quality is unambiguous in this setting. In particular, firms that start or continue exporting always benefit from better export opportunities, and hence are stimulated to upgrade quality. These firms correspond to the most productive firms from H, thus encompassing those that we subsequently take as LFs.

The result is based on an industry where firms are heterogeneous, but negligible. On the contrary, we consider industries with significant differences in firm size. This implies that the most productive firms could be companies such as Adidas and Nike in sports apparel, Ikea in furniture, or Apple and Samsung in cell phones. Once we model them as oligopolistic firms with idiosyncratic features, we will show that the results in Proposition 1 could be reversed: the most productive firms in a country could downgrade quality and have a fall in profits following an export shock.

4.2 Export Shock to Small Firms

We begin by studying export shocks affecting SFs or LFs in isolation. The goal is twofold. First, to identify the opposing mechanisms that explain the ambiguous impact on LFs after an industry-wide export shock. Second, to identify observables capturing the magnitude of each mechanism, getting a grasp of the industry features that lead to specific outcomes.

The following proposition deals in particular with an export shock to SFs. It highlights how a reduction in export trade costs induces entry of SFs, thus strengthening domestic competition and impacting domestic LFs negatively.

Proposition 2: Export Shock to SFs

Suppose a small reduction in $\tau^{\mathcal{N}_H}$, which is the firm-specific component of trade costs only affecting SFs. Then, H's price index decreases. Moreover,

- *M_H^E becomes greater, and SFs from H increase their domestic market share as a group.*
- *Each LF ω from H invests less in quality (I_H^ω), decreases its domestic markup (m_{HH}^ω), loses domestic market share (s_{HH}^ω), and garners lower total profits ($\overline{\pi}_H^\omega$).*

The proposition captures effects on the domestic economy caused by an increase in the SFs' expected profits. This induces a larger mass of SFs to enter the industry, with a subset of them surviving and serving the domestic market. Consequently, competition increases at home, which is reflected in a lower price index in H.

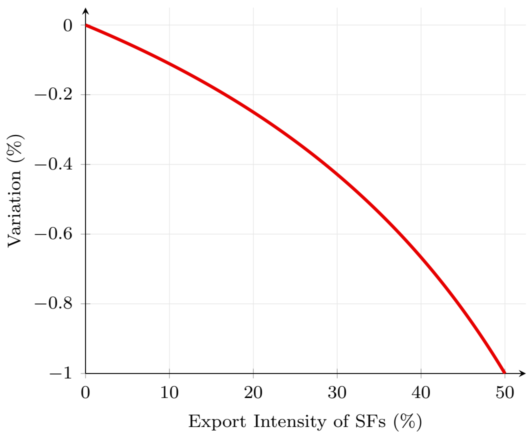
Greater domestic competition negatively impacts H's LFs in two ways. First, it reduces a LF's sales, thereby increasing the average cost of quality. Thus, a LF downgrades quality and sets lower domestic prices. Second, greater domestic competition decreases a LF's market power, which reinforces the reduction in prices at home through decreases in markups. Both effects determine that each LF earns a lower total profit and supplies a cheaper lower-quality variety domestically. Additionally, LFs reduce their presence at home, and the domestic market share is reallocated towards SFs.

While the proposition establishes that H's price index decreases and LFs' variables are negatively impacted, it is silent regarding magnitudes. Next, we show that the strength of these effects can be inferred through observables using the export intensity of firms.

First, greater export intensity of SFs is associated with more pronounced reductions in the domestic price index. This follows since $\mathrm{d}\ln \mathbb{P}_H = \frac{e_H^{\mathcal{N}}}{d_H^{\mathcal{N}}} \mathrm{d}\ln \tau^{\mathcal{N}_H}$, where $\mathrm{d}\ln \tau^{\mathcal{N}_H} < 0$, and $e_H^{\mathcal{N}}$ and $d_H^{\mathcal{N}} := 1 - e_H^{\mathcal{N}}$ are respectively the export and domestic intensity of SFs as a group.

Figure 1 illustrates this relation for a given reduction in export trade costs. The graph indicates that the variation in the price index is always negative, as established in the proposition, thus taking values in the negative vertical axis. Furthermore, the curve's negative slope shows that a greater $e_H^{\mathcal{N}}$ triggers a more pronounced decrease in $\mathbb{P}_H$. It reflects a larger positive impact of better export opportunities on a SF's expected profit. This acts by inducing a more pronounced entry of SFs, and therefore causing a more marked increase in domestic competition.

Figure 1. *1% Decrease in Export Trade Costs of SFs - Effect on the Domestic Price Index*

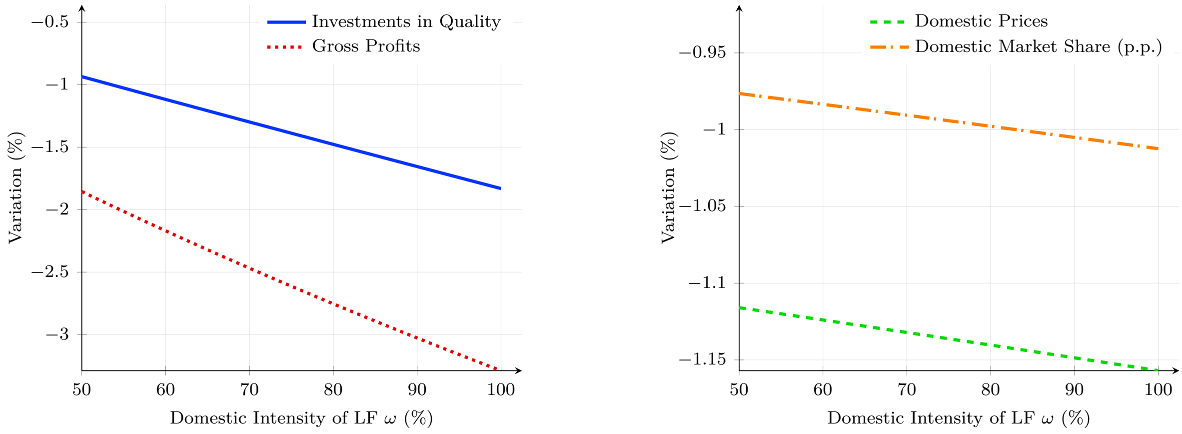


Likewise, the impact on each LF can be measured through a LF's export intensity, or equivalently a LF's domestic intensity. Intuitively, a greater LF's domestic intensity predicts a more pronounced impact from tougher domestic competition. This is demonstrated in Figure 2, where we depict the relation between a LF's variables and its domestic intensity, for a given reduction in the domestic price index.

The fact that a LF is hurt by this type of export shock is reflected through negative values in the vertical axis. Additionally, the negative slope of each curve captures that greater domestic intensity of a LF is associated with more pronounced decreases in its investments, domestic prices, domestic market share, and profits. It basically shows that the magnitude in which quality decreases depends on how important the domestic market is for a LF's total sales.

Figure 2. *1% Decrease in Export Trade Costs of SFs - Impact on LFs*

(a) LF's Investment and Profits **(b)** LF's Domestic Price and Market Share



4.3 Export Shock to Large Firms

The following proposition considers an export shock that affects LFs exclusively. Without loss of generality, we consider that each LF exports, with a LF only serving home arising as the limiting case with zero export intensity. The proposition establishes that each LF benefits from this type of shock and increases its domestic presence at the expense of domestic SFs.

Proposition 3: Export Shock to LFs

Suppose a small reduction in τ^ω for each $\omega \in \overline{\mathscr{L}}_H$. Then, H's price index remains the same. Moreover,

- *M_H^E decreases, and SFs from H lose domestic market share as a group.*
- *Each LF ω from H invests more in quality (I_H^ω), increases its domestic markup (m_{HH}^ω), gains domestic market share (s_{HH}^ω), and garners greater total profits ($\overline{\pi}_H^\omega$).*

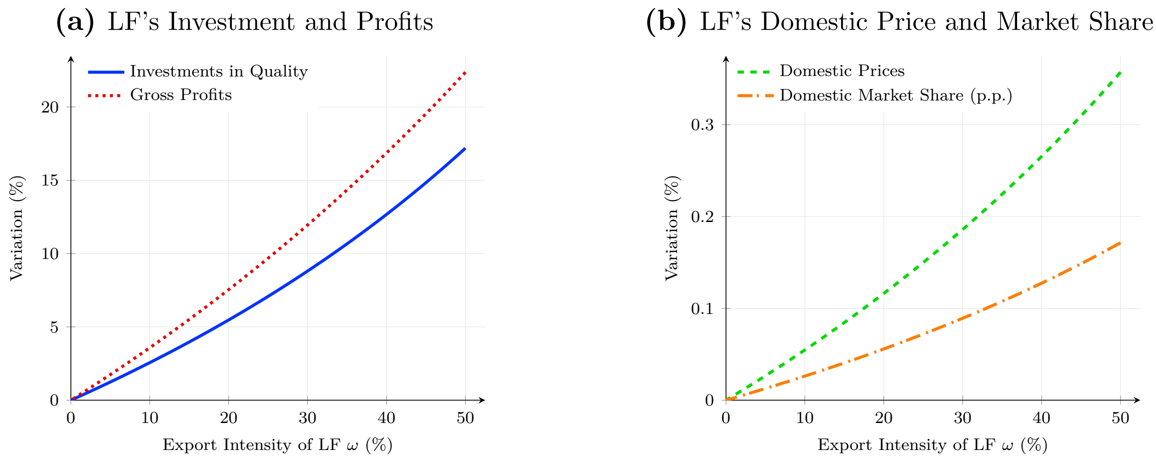
Relative to an export shock affecting SFs exclusively, this scenario affects the domestic market through a different mechanism: the expansion of effective market size for LFs. A greater scale of production allows a LF to spread out the fixed costs of quality across more units, which creates more favorable conditions to invest in quality. Since quality affects all markets simultaneously and increases a consumer's willingness to pay, each LF ends up selling more at home, charging higher domestic markups, and garnering greater total profits. Overall, this generates a reallocation of domestic market share towards LFs, reducing the mass of domestic SFs that pay the entry cost as its counterpart.

Furthermore, unlike an export shock to SFs, an export shock to LFs does not affect the domestic price index. The reason lies in the existence of two opposing effects on the competitive environment that are perfectly offset. On the one hand, greater investments by LFs initially create a tougher competitive environment at home. On the other hand, tougher competition reduces the expected profits of SFs, which crowds out SFs from the industry and softens competition. In equilibrium, both effects cancel out and leave the price index unaltered.[10]

While the results indicate that a LF is positively affected, it remains to study the magnitude in which its variables are impacted. This can be inferred in terms of observables through the export intensity of the LF. The intuition is that the increase in sales following an export shock is more significant when a LF has greater export intensity, translating into a more pronounced reduction in the average cost of quality.

The mechanism is illustrated in Figure 3, where we consider a given reduction in a LF's export trade cost. The graph indicates that all curves have values in the positive quadrant, reflecting that the variation in each variable is positive. Moreover, each curve has a positive slope, capturing that greater export intensity of a LF is associated with larger increases in total sales, therefore inducing heavier investments in quality. The graph also illustrates how this entails greater increases in a LF's domestic market share, thus determining a more marked crowding out of SFs.

Figure 3. *1% Decrease in Export Trade Costs of LFs - Impact on LFs*



[10] This property can be noticed by inspecting (FE), which completely identifies H's price index. It reflects that expected profits are independent of the LFs' export trade costs, and only depend on the SF's export trade costs. Consequently, more aggressive behavior by LFs only generates a reallocation of domestic market share between types of firms, without affecting the competitive environment at home.

4.4 Export Shock to All Firms

An industry-wide export shock combines the mechanisms arising under export shocks to each type of firm. We first state the results, and then employ the intuitions of an export shock to SFs and LFs to explain them.

Proposition 4: Export Shock to All Firms

Suppose a small reduction in τ_{HF}, which is the common component of trade cost shared by all firms. Then, H's price index decreases. Moreover,

- *For SFs from H: their domestic productivity cutoff increases (φ^{*}_{HH}). Moreover, a SF ω that only serves home invests less in quality (I^{ω}_{H}) and earns lower profits (π^{d}_{H}). Instead, a SF ω that becomes an exporter or continues exporting invests more in quality (I^{ω}_{H}) and earns greater profits (π^{x}_{H}).*
- *For a LF ω from H: there is an ambiguous impact on its investments in quality (I^{ω}_{H}), domestic markup (m^{ω}_{HH}), domestic market share (s^{ω}_{HH}), and total profits ($\overline{\pi}^{\omega}_{H}$).*

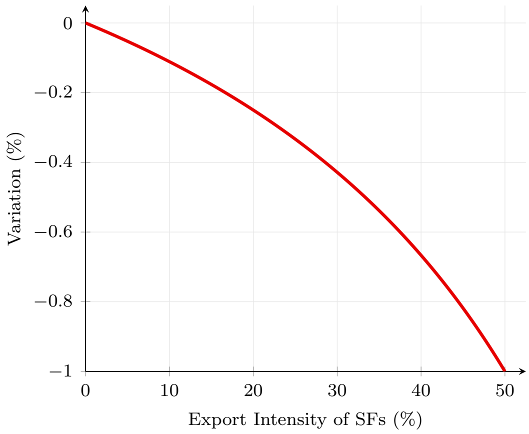
An industry-wide export shock affects each active firm through two mechanisms: tougher domestic competition and better export opportunities. The total impact on each differs by type of firm. As for the least-productive SFs from H, they serve home exclusively, both before and after the export shock. Thus, they do not benefit from better export conditions, but are negatively affected by the tougher competitive environment at home. This implies that their profits reduce, becoming harder for them to survive. Moreover, since they sell less, their average costs of quality increase and so they downgrade quality.

Regarding SFs from H that start or continue exporting after the export shock, they are also impacted by tougher domestic competition, but additionally benefit from better conditions to export. The proposition establishes that, overall, these firms are positively affected and always earn greater total profits. Furthermore, their total revenues increase, which reduces the average cost of quality and leads them to upgrade quality.

Finally, LFs from H are modeled as entities with idiosyncratic features, so that they could be domestic- or export-oriented. Both options are plausible, depending on a LF's features and model business. An example of domestic-oriented LF is given by a foreign firm that serves the market by setting operations within the country, rather than doing it through exports. Likewise, an export-oriented LF emerges if a foreign firm installs operations in the country to use it as an export platform. A corollary of this is that a LF could be benefited or hurt in equilibrium. Whether one or the other occurs depends ultimately on two aspects: i) the strength in which domestic competition becomes tougher, and ii) the relative importance that exports and domestic sales have for a LF.

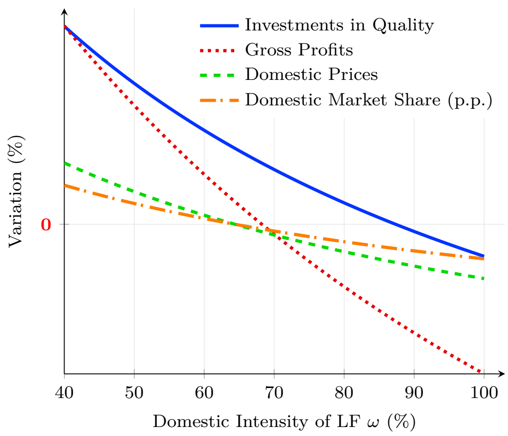
Regarding i), the export intensity of SFs identifies the magnitude in which the price index decreases. As in the case of an export shock to SFs, this follows because the SFs' export intensity reflects the magnitude in which a SF's expected profits increase, and hence the magnitude in which entry of SFs occurs. Formally, the variation in H's price index is given by $\mathrm{d}\ln \mathbb{P}_H = \frac{e_H^{\mathcal{N}}}{d_H^{\mathcal{N}}} \mathrm{d}\ln \tau_{HF}$. Thus, the price index always decreases given $\mathrm{d}\ln \tau_{HF} < 0$, and greater export intensity of SFs entails more pronounced decreases in $\mathbb{P}_H$. This is demonstrated in Figure 4.

Figure 4. *1% Decrease in Export Trade Costs of All Firms - Domestic Price Index*



As for ii), the relative importance of the domestic and foreign markets for a LF is captured through its domestic intensity. Figure 5 illustrates this feature by plotting the impact on each variable of a LF.

Figure 5. *1% Decrease in Export Trade Costs of All Firms - LF's Variables*



The graph exhibits curves with negative slopes, reflecting that a LF with greater domestic intensity is more impacted by tougher domestic competition and benefits less by better export access. Furthermore, each curve takes positive values when a LF's domestic intensity is low, and negative ones when it is high. Based on this, we can distinguish between two scenarios.

First, for low values of a LF's domestic intensity, the impact on a LF is akin to an export shock to LFs exclusively: a LF mainly benefits from scale effects due to better export opportunities. This determines that a LF upgrades quality, charges a higher domestic markup, and increases its domestic market share and profits. Second, the impact on a LF with high domestic intensity resembles the case of an export shock to SFs: it is primarily affected by tougher domestic competition, and so it downgrades quality, charges lower domestic prices, loses domestic market share, and has a fall in profits.

5 Numerical Approach and Data Description

We have established that an industry-wide export shock has an ambiguous impact on LFs. Due to this, it is an empirical matter whether LFs benefit or are hurt in equilibrium by better export opportunities. To analyze this, we present numerical results based on calibrations for several Danish industries. They demonstrate how an export shock affects LFs differently, depending on the export intensities of SFs and LFs.

The exercises consider finite but relatively small variations in export trade costs, with a range of changes between 0% and 10%. This is necessary since our results assume that SFs and LFs are active, precluding variations in export trade costs large enough that make all SFs or LFs stop operating. Furthermore, small changes enable us to interpret results through the propositions we derived, which are only valid under small changes in export trade costs.

We consider a baseline approach where the productivity distribution of SFs is discrete. This allows us to obtain results by calibrating just a few variables and parameters, thereby highlighting the role of export intensities to identify outcomes. The case with a continuous productivity distribution (specifically, a bounded Pareto) is presented in Appendix D. It generates the same qualitative results, with quite similar magnitudes.[11]

5.1 Computation

We consider a scenario where export trade costs in H are initially given by $\left(\tau_{HF}^{\mathcal{N}}\right)'$ for SFs and by $(\tau_{HF}^{\omega})'$ for each LF ω, with common component τ_{HF}'. The outcomes of this case are compared against a counterfactual scenario where export trade costs become $\left(\tau_{HF}^{\mathcal{N}}\right)''$ for SFs and $(\tau_{HF}^{\omega})''$ for each LF ω, with common component τ_{HF}''.

[11] Results under a bounded Pareto require solving a more complex system of equations and calibrating several additional parameters. Consequently, it obscures the identification of the primary factors determining results. Nonetheless, the choice of productivity distribution only has second-order effects on our outcomes, since changes in export trade costs are small. This explains why the results presented in Appendix D are almost the same. The property follows because the productivity distribution only affects how survival productivity cutoffs are impacted, which in turn only affects expected profits directly. However, since marginal entrants have zero profits, changes in the productivity cutoffs have a minor impact on the price index. A similar property arises for trade liberalization in the Melitz model with symmetric countries, as shown by Melitz and Redding (2015) and Arkolakis et al. (2019).

For the computation of results, we utilize the "hat-algebra" procedure, as in Dekle et al. (2008). This implies that, for any variable x, we respectively denote its equilibrium value under each set of export trade costs by x' and x'', and its proportional change by $\widehat{x} := \frac{x''}{x'}$. The approach enables us to compute proportional changes of each variable of interest, given proportional changes in export trade costs $\widehat{\tau}^{\mathcal{N}}_{HF}$ and $\widehat{\tau}^{\omega}_{HF}$ for each LF ω.

The procedure needs values of $\widehat{\tau}^{\mathcal{N}}_{HF}$ and $\widehat{\tau}^{\omega}_{HF}$ for each LF ω. They are defined depending on the export shock we consider. For instance, considering a 10% decrease, an export shock that only applies to SFs means $\widehat{\tau}^{\mathcal{N}}_{HF} = 0.9$ and $\widehat{\tau}^{\omega}_{HF} = 1$ for each $\omega \in \overline{\mathscr{L}}_H$; when it only applies to LFs, it is $\widehat{\tau}^{\mathcal{N}}_{HF} = 1$ and $\widehat{\tau}^{\omega}_{HF} = 0.9$ for each $\omega \in \overline{\mathscr{L}}_H$; and, finally, if it applies to all firms, it is $\widehat{\tau}^{\mathcal{N}}_{HF} = 0.9$ and $\widehat{\tau}^{\omega}_{HF} = 0.9$ for each $\omega \in \overline{\mathscr{L}}_H$.

We suppose that the productivity of SFs is a discrete random variable with support $\{\varphi^I, \varphi^D, \varphi^X\}$, where $\varphi^I < \varphi^D < \varphi^X$. The superscripts are mnemonics for respectively "inactive", "domestic", and "exporters", due to the role that we ascribe to them. Specifically, in equilibrium, a SF from H that obtains φ^I does not serve any country; if it gets φ^D, it is efficient enough to serve home, but not to serve the foreign country; and the draw φ^X is obtained by the most productive SFs, which serve both the domestic and foreign market.

To express the system for computing results in terms of observables, we begin by defining some variables. Let R^d_H be the total revenue of SFs from H serving home exclusively, and R^x_H the total revenue of exporting SFs. Moreover, R^d_{ij} and R^x_{ij} denote the SFs' revenue of each type of firm from specific markets. Let $e^g_H := \frac{R^g_{HF}}{R^g_H}$ be the export intensity of group g and $d^g_H := 1 - e^g_H$ its domestic intensity, where $g \in \{d, x, \mathcal{N}\}$ depending on the group of SFs that we focus on. Finally, define $\lambda^d_H := \frac{R^d_H}{R^d_H + R^x_H}$ and $\lambda^x_H := 1 - \lambda^d_H$, which are respectively the revenue share of SFs only serving home and of exporting SFs, relative to all SFs' revenue.

As we show in Appendix A.5, given $\widehat{\tau}^{\mathcal{N}}_{HF}$ and $\widehat{\tau}^{\omega}_{HF}$ for each LF ω, the computation of effects can be obtained by solving the following system for each $\omega \in \overline{\mathscr{L}}_H$:

$$1 - \left(\widehat{\mathbb{P}}_H\right)^{\frac{\sigma-1}{1-\delta}} \left(\lambda^d_H\right)' = \left(\left(\widehat{\mathbb{P}}_H\right)^{\sigma-1} \left(d^x_H\right)' + \left(e^x_H\right)' \left(\widehat{\tau}^{\mathcal{N}}_{HF}\right)^{1-\sigma}\right)^{\frac{1}{1-\delta}} \left(\lambda^x_H\right)', \tag{8a}$$

$$\widehat{p}^{\omega}_{HH} = \widehat{m}^{\omega}_{HH} = \widehat{\varepsilon}^{\omega}_{HH} \frac{\left(\varepsilon^{\omega}_{HH}\right)' - 1}{\widehat{\varepsilon}^{\omega}_{HH} \left(\varepsilon^{\omega}_{HH}\right)' - 1}, \tag{8b}$$

$$\widehat{I}^{\omega}_H = \widehat{z}^{\omega}_H = 1 + \left(\rho^{\omega}_{HH}\right)' \left[\frac{\widehat{s}^{\omega}_{HH}}{\widehat{\varepsilon}^{\omega}_{HH}} \frac{1 - \widehat{s}^{\omega}_{HH} \left(s^{\omega}_{HH}\right)'}{1 - \left(s^{\omega}_{HH}\right)'} - 1\right] + \left(\rho^{\omega}_{HF}\right)' \left[\widehat{s}^{\omega}_{HF} - 1\right], \tag{8c}$$

$$\widehat{s}^{\omega}_{HH} = \frac{\left(\widehat{m}^{\omega}_{HH}\right)^{1-\sigma} \left(\widehat{z}^{\omega}_H\right)^{\delta}}{\left(\widehat{\mathbb{P}}_H\right)^{1-\sigma}}, \tag{8d}$$

$$\widehat{s}^{\omega}_{HF} = \widehat{R}^{\omega}_{HF} = \left(\widehat{\tau}^{\omega}_{HF}\right)^{1-\sigma} \left(\widehat{z}^{\omega}_H\right)^{\delta}, \tag{8e}$$

where $\widehat{\varepsilon}^{\omega}_{HH} = 1 + (1 - \widehat{s}^{\omega}_{HH}) \frac{(s^{\omega}_{HH})'(\sigma-1)}{\sigma - (s^{\omega}_{HH})'(\sigma-1)}$ and

$$\rho^{\omega}_{HH} := \frac{d^{\omega}_{H}\left(1 - s^{\omega}_{HH}\right)/\varepsilon^{\omega}_{HH}}{d^{\omega}_{H}\left(1 - s^{\omega}_{HH}\right)/\varepsilon^{\omega}_{HH} + e^{\omega}_{H}/\sigma}, \tag{8f}$$

$\rho^{\omega}_{FH} := 1 - \rho^{\omega}_{HH}$, $d^{\mathcal{N}}_{H} := \frac{R^{\mathcal{N}}_{HH}}{R^{\mathcal{N}}_{HH} + R^{\mathcal{N}}_{HF}}$ and $e^{\mathcal{N}}_{H} := 1 - d^{\mathcal{N}}_{H}$.

In addition, results for gross profits can be obtained by computing

$$\widehat{\pi}^{\omega}_{H} = 1 + (\phi^{\omega}_{HH})' \left\{ \frac{\widehat{s}^{\omega}_{HH}}{\widehat{\varepsilon}^{\omega}_{HH}} \frac{1 - \delta\left(1 - \widehat{s}^{\omega}_{HH}\left(s^{\omega}_{HH}\right)'\right)}{1 - \delta\left(1 - \left(s^{\omega}_{HH}\right)'\right)} - 1 \right\} + (\phi^{\omega}_{HF})'\left(\widehat{s}^{\omega}_{HF} - 1\right), \tag{8g}$$

$$\widehat{\Pi}^{\mathscr{L}}_{H} = \sum_{\omega \in \mathscr{L}_H} \psi^{\omega}_{H} \widehat{\pi}^{\omega}_{H}, \tag{8h}$$

with

$$\phi^{\omega}_{HH} := \frac{d^{\omega}_{H}\left[1 - \delta\left(1 - s^{\omega}_{HH}\right)\right]/\varepsilon^{\omega}_{HH}}{d^{\omega}_{H}\left[1 - \delta\left(1 - s^{\omega}_{HH}\right)\right]/\varepsilon^{\omega}_{HH} + e^{\omega}_{H}\left(1 - \delta\right)/\sigma}, \tag{8i}$$

$\phi^{\omega}_{HF} := 1 - \phi^{\omega}_{HH}$, and

$$\psi^{\omega}_{H} := \frac{\widetilde{s}^{\omega}_{HH}\left[1 - \delta\left(1 - s^{\omega}_{HH}\right)\right]/\varepsilon^{\omega}_{HH} + \widetilde{s}^{\omega}_{HF}\left(1 - \delta\right)/\sigma}{\sum_{\omega \in \mathscr{L}_H}\left[\widetilde{s}^{\omega}_{HH}\left[1 - \delta\left(1 - s^{\omega}_{HH}\right)\right]/\varepsilon^{\omega}_{HH} + \widetilde{s}^{\omega}_{HF}\left(1 - \delta\right)/\sigma\right]}, \tag{8j}$$

where $\widetilde{s}^{\omega}_{Hj} := \frac{R^{\omega}_{Hj}}{Y^{\text{ind}}_{H}}$, with Y^{ind}_{H} defined as the industry's income in H (i.e., the sum of domestic and exports sales by all firms from H).

Given estimates of σ and δ, the computation of effects requires knowledge of $e^{\mathcal{N}}_{H}$ and e^{x}_{H} for SFs, and s^{ω}_{HH}, $\widetilde{s}^{\omega}_{HH}$ and $\widetilde{s}^{\omega}_{HF}$ for each LF ω. Given values for these variables, we can recover any other variable that is necessary for calculating results (see Appendix A.4 for further details). Due to this, next we only describe how to identify these terms.

5.2 Data Description

We utilize two datasets compiled by Statistics Denmark, which provide information on Danish manufacturing for the year 2005. Both datasets are presented at the firm-product level and disaggregated at the 8-digit level according to the Combined Nomenclature (CN). Throughout the analysis, we refer to a sector as a 2-digit industry and reserve the term industry to a 4-digit industry, according to the NACE rev. 1.1 classification.

The first dataset is the Prodcom survey, from which we obtain information on total turnover for each firm. This survey covers any production unit with at least ten employees that has manufacturing as its main activity. Moreover, its coverage is quite high, since the information is collected to ensure that at least 90% of the total production value in each industry is covered. We consider any firm that is included in this dataset as domestic.

Additionally, we draw on a dataset collected by Danish customs that contains trade data, covering transactions by both manufacturing and non-manufacturing firms. It has a coverage of 95% for imports and 97% for exports for trading partners in the EU, while the universe of transactions is covered for non-EU countries.

5.3 Construction of Variables and Calibration

We assemble the data in a way consistent with how our model is specified. This requires expressing variables at the industry level. To accomplish it, we gather the data such that turnover, exports, and imports at the 8-digit CN level are aggregated at the 4-digit NACE level. Next, we describe how to calculate $e_H^{\mathcal{N}}$ and e_H^x for SFs, and s_{HH}^{ω}, $\widetilde{s}_{HH}^{\omega}$ and $\widetilde{s}_{HF}^{\omega}$ for each LF ω.

First, we need to distinguish between LFs and SFs in each industry. We take a firm as large if it is non-trivial for the expenditure and revenue of its industry. Formally, we define a LF as having at least a 3% of domestic market share and 5% of industry revenue share. In terms of the data, this leads us to take the top 4 firms as LFs. Utilizing this definition, we end up with a sample of industries having an average of 57 firms each. The outcomes are robust to the specific cutoff chosen to define LFs, as shown in the sensitivity analysis of Appendix D. This occurs because firms below a certain threshold have virtually no impact on the industry's aggregate outcomes.

The domestic market share of LF ω (i.e. s_{HH}^{ω}) is expressed relative to industry expenditures, which are the sum of domestic sales and imports. A firm's domestic sales are computed as the difference between its total turnover and its export revenues. Likewise, imports comprise industry goods that are either acquired by non-manufacturing firms (i.e., firms not belonging to the Prodcom dataset, such as retailers) or manufacturing firms from other industries. This allows us to allocate each good imported to a specific industry, and hence reflect the import competition in an industry accurately.

To obtain domestic intensities and revenue shares, we take turnover as income, and split it into domestic and export sales for each firm. Based on this, we compute the export intensity of SFs (i.e. $e_H^{\mathcal{N}}$) as the total exports of the group relative to the SFs' income. Furthermore, we calculate the domestic and export shares of LF ω (i.e. $\widetilde{s}_{HH}^{\omega}$ and $\widetilde{s}_{HF}^{\omega}$) as domestic and export sales relative to the industry revenue. Identifying the difference in average export intensity between SFs and LFs also allows us to compute e_H^x.

Concerning the model parameters, only two are necessary to quantify the effects of export shocks: σ and δ. Regarding the former, we make use of the estimates by Soderbery (2015). They are based on the methodology by Broda and Weinstein (2006), but accounting for small-sample biases. The average of these estimates using industry-revenue weights is $\sigma := 3.53$, which we use throughout the paper.

As for δ, we calibrate its value by fitting, as close as possible to the model, each LF's domestic market share variation not explained by prices. Basically, the approach is based on the definition of quality by Khandelwal (2010). We first net out the effect of prices on domestic market shares, whose residuals measure quality in a way consistent with our broad definition of it (i.e., any non-price choice that affects a variety's appeal). Then, we fit these residuals to the investments in quality predicted by the model, determining that $\delta := 0.68$. A detailed description of the procedure is included in Appendix B.

5.4 Sample of Industries

We utilize a sample of industries that are consistent with our model. To do this, we discard industries without a pool of SFs or not having at least one LF. We define LFs through domestic market shares, thus avoiding some issues related to a definition based on revenue shares. This determines a somewhat more stringent condition empirically, relative to utilizing revenue shares.[12,13]

In Figure 6a, we indicate how representative the final sample of industries is relative to the original dataset. This is done in terms of income, expenditure, and exports for manufacturing and three specific sectors we analyze: Food & Beverages, Chemicals, and Machinery. The results reveal that a market structure with SFs and LFs coexisting explains a substantial share of each variable, especially in terms of income and exports. The share in expenditure is somewhat lower, which reflects that some industries are served exclusively through imports.

Figure 6b characterizes the industries not covered. This has the goal of determining whether they are excluded due to an absence of a pool of SFs or for not having LFs. The graph depicts the percentage of industries in our final sample relative to industries that contain a pool of SFs. The coverage is almost complete (i.e., almost 100% in each dimension, and never less than 80%), revealing that most industries not included are due to the absence of a pool of SFs, rather than the lack of at least one LF. As a corollary, industries with a set of negligible firms operating are better described by a coexistence of SFs and LFs, rather than a pure monopolistic

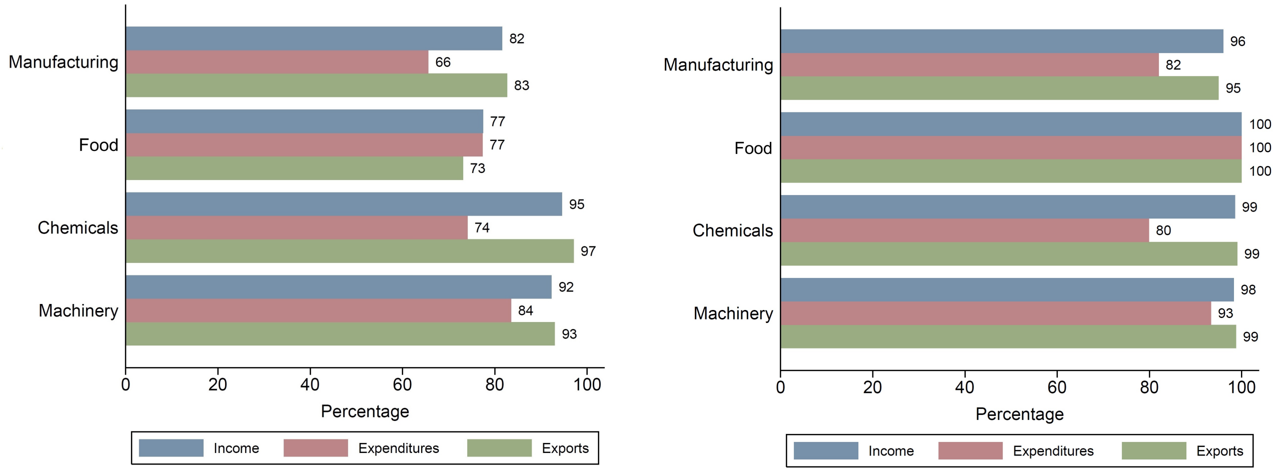
[12]The use of domestic market share is to avoid scenarios where firms accumulate high shares of revenue in the industry, but the total revenue relative to expenditures is negligible. In these cases, a firm having a large fraction of the total industry's revenues is not equivalent to having market power or being relevant to the whole sector; rather, it reflects the low level of operation by domestic firms in the industry.

[13]Specifically, our criterion for incorporating an industry into the final sample is that there is at least one firm with a domestic market share greater than 3%, and that there is a pool of SFs operating. For the latter, we ensure that SFs are actually negligible by checking that in each industry there are at least 10 firms, and removing any industry where the 10 firms or 20% of the firms with the lowest domestic market share accumulate more than 6% of total domestic market share.

competition market structure.

Figure 6. *Final Sample of Industries*

(a) Final Sample relative to Total **(b)** Final Sample relative to Industries With SFs



Finally, in Table 1 we describe the features of our final sample of industries. The information is aggregated at the sector level and sorted by the contribution to total exports in manufacturing. From this, we infer that exporting is a widespread activity, as is usual in small countries. In particular, the percentage of exporters among SFs is on average almost 50%. This provides evidence that the conditions to access foreign markets are of relevance for all firms, and not only for the largest ones.[14]

Table 1. *Final Sample of Industries - Information in %*

	Exports	**Income**	**Expenditure**	**Exporters**	**SFs Exporters**	**LFs Exporters**
Chemicals	28.3	17.1	11.9	73	70	90
Machinery	16.6	12.7	13.0	53	51	85
Food & Beverages	16.0	18.4	16.8	63	60	84
Medical Equipment	7.1	4.8	4.1	67	66	83
Electrical/Machinery	7.0	7.3	6.7	55	52	85
Other Manufactures	6.2	5.8	4.8	59	57	96
Rubber & Plastic	5.8	5.5	5.8	62	61	65
Metal Products	3.0	8.6	8.4	31	30	82
Glass & Cement	2.2	3.1	2.2	40	40	44
Media Equipment	1.9	1.7	3.7	59	56	88
Wood	1.7	4.0	5.1	32	29	70
Basic Metals	1.4	1.5	5.7	44	40	69
Paper	1.1	2.7	3.9	38	35	75
Textiles	0.8	0.8	1.7	60	57	88
Printing	0.6	5.1	4.6	26	25	63
Motor Vehicles	0.3	0.8	1.4	37	34	75
Average of Sectors	6.2	6.2	6.2	50	48	78

Note: Exports, income, and expenditures calculated as a percentage relative to the total. Exporters, SFs exporters, and LFs exporters are the number of firms that export relative to the total firms in each sector. All values are calculated based on the final sample of industries that we utilize.

[14] For a comparison of features between LFs and SFs, see Alfaro (2020). Using the same data, the article shows that LFs have greater revenue productivity, pay higher wages, are more capital intensive, and are more likely to export and import.

In addition, the table reveals that Chemicals, Machinery, and Food & Beverages rank among the top three sectors by total exports, income, and expenditures. This constitutes our basis for selecting them in the numerical exercises.

6 Quantitative Exercises

We begin by performing numerical simulations in a representative Danish manufacturing industry. After that, we present results calibrating the model for specific sectors. The results reveal that the model entails different predictions for LFs, depending on the export intensity of SFs and LFs.

6.1 Manufacturing

The calibration for Danish manufacturing matches average features using industry-revenue weights. Its main characteristics are presented in Table 2.

Table 2. *A Representative Manufacturing Industry - Information in %*

Firm	Domestic Market Share	Domestic Intensity	Export Intensity	Domestic Revenue as % of Industry Income	Export Revenue as % of Industry Income
Top 1	16.31	53.41	46.59	17.46	15.23
Top 2	7.28	64.42	35.58	8.11	4.48
Top 3	4.89	67.76	32.24	5.38	2.56
Top 4	3.38	64.45	35.55	3.68	2.03
SFs		75.82	24.18		

The calibration captures the pervasiveness of international transactions in small countries. Imports accrue almost 40% of the total expenditure, revealing their relevance to obtaining domestic market shares that reflect each firm's market power.[15] Additionally, the importance of exporting can be appreciated through the export intensity of SFs, which is almost 25%.

By solving and computing the system (8), we quantify the effects of export shocks. Table 3 illustrates Proposition 2, which indicates that an export shock to SFs in isolation impacts LFs negatively by increasing domestic competition. Likewise, it illustrates Proposition 3, which establishes that an export shock to LFs benefits LFs by increasing their total sales and hence reducing the average costs of quality.

[15] After some algebraic manipulation, the share of imports in expenditure equals $1 - \left(\sum_{\omega \in \overline{\mathscr{L}}_H} s^{\omega}_{HH}\right)\left[1 + \frac{1 - \sum_{\omega \in \overline{\mathscr{L}}_H} (\widetilde{s}^{\omega}_{HH} + \widetilde{s}^{\omega}_{HF})}{\sum_{\omega \in \overline{\mathscr{L}}_H} \widetilde{s}^{\omega}_{HH}} d^{\mathcal{N}}_{H}\right]$.

Table 3. *Impact of a 10% Reduction in Export Trade Costs - Manufacturing*

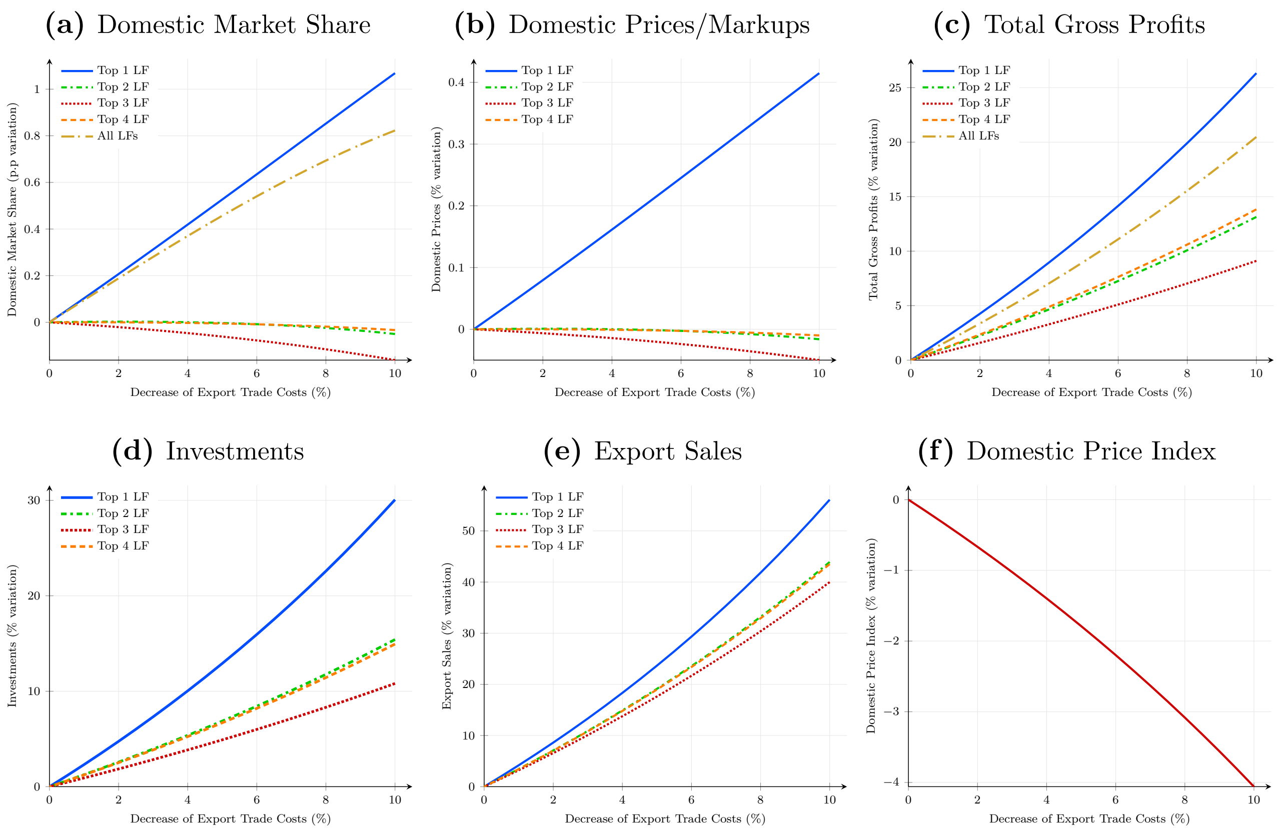
(a) Impact on each LF

Better Export Access	**Firm**	**Domestic Market Share** Change (p.p.)	**Domestic Prices/Markups** Change (%)	**Quality Investments** Change (%)	**Export Revenues** Change (%)	**Total Gross Profits** Change (%)
For All Firms	Top 1	1.07	0.41	30.07	56.10	26.33
	Top 2	-0.05	-0.02	15.42	43.92	13.14
	Top 3	-0.16	-0.05	10.81	39.99	9.11
	Top 4	-0.03	-0.01	14.94	43.51	13.83
Only For LFs	Top 1	3.69	1.48	42.54	66.13	45.54
	Top 2	1.51	0.49	34.39	59.60	36.16
	Top 3	0.96	0.30	31.79	57.50	33.02
	Top 4	0.77	0.23	36.25	61.11	37.24
Only For SFs	Top 1	-2.43	-0.90	-10.99	-7.61	-15.31
	Top 2	-1.40	-0.44	-16.07	-11.23	-18.68
	Top 3	-1.00	-0.31	-17.72	-12.42	-19.64
	Top 4	-0.70	-0.21	-17.49	-12.26	-18.79

(b) Impact on LFs as Group

Better Export Access	**Domestic Market Share** Change (p.p.)	**Total Gross Profits** Change (%)
For All Firms	0.82	20.48
Only For LFs	6.93	41.39
Only For SFs	-5.52	-16.82

Figure 7. *Manufacturing - Decrease in Export Trade Costs for all Firms*



An industry-wide export shock represents the most interesting case, given the ambiguity of outcomes for LFs. Table 3b indicates the impact of this shock on aggregate variables, whereas

Figure 7 establishes the impact on each LF's variables. The results point out that the total profits and domestic market share of LFs as a group increase. Nonetheless, aggregate results of this sort mask the heterogeneous impact across LFs. In particular, although all LFs upgrade quality and garner higher profits, differences in their export intensity entail a dissimilar impact on each LF's domestic prices and domestic market shares.[16]

Specifically, the top LF has higher export intensity than the rest of LFs. Thus, it benefits more from better export access and is simultaneously more shielded from tougher domestic competition. Due to this, its increases in investments are more pronounced than the rest of LFs, rationalizing its gains in domestic market share and the higher domestic price. On the contrary, the rest of LFs have lower export intensity, making them be more impacted by tougher domestic competition and less benefited from better export opportunities. Consequently, even when they upgrade quality and their varieties become more appealing domestically, each loses presence in the domestic market and charges lower markups at home.

6.2 The Role of Investments in Quality

In addition to the partition of firms into small and large, one key feature that distinguishes our model is the incorporation of investments in quality. To clearly show the role of quality for results, next we replicate the exercise assuming that firms only decide on prices.[17] The outcomes are presented in Table 4 and Figure 8.

The comparison of each LF's profit in Tables 3 and 4 indicates that the qualitative impact is the same. Nonetheless, its magnitude is magnified when quality is incorporated, so that both losses and gains become more pronounced. This feature can be graphically observed by comparing Figures 7b and 8b.

Furthermore, without LFs deciding on quality, an industry-wide export shock only affects the domestic market by inducing entry of SFs. This represents an increase in competition for LFs, making a LF always lose domestic market share and decrease its domestic markup. The result can be appreciated in Figures 8a and 8b. The fact that an export shock only induces entry of SFs also explains why export shocks to all firms and SFs have an identical quantitative impact on a LF's domestic markup and market share, while an export shock to LFs does not affect any of these variables (see Table 4).

Instead, better export access incorporating investments in quality allows a LF to additionally gain domestic market share and charge higher markups, since quality affects a variety's appeal

[16] While LFs also differ in their domestic market share, this only plays a secondary role in the qualitative outcomes. For instance, if we consider that each LF has the export intensity of the top LF but allow them to have different domestic market shares, the signs of all effects are the same as those of the top firm.

[17] In terms of our model, absence of investments in quality is equivalent to assuming $\delta \to 0$.

in all countries served. Overall, a LF can increase or decrease its domestic market share and markups, as it can be appreciated in Figures 7a and 7b.

Table 4. *Impact of a 10% Reduction in Export Trade Costs - Manufacturing (No Investments)*

(a) Impact on each LF

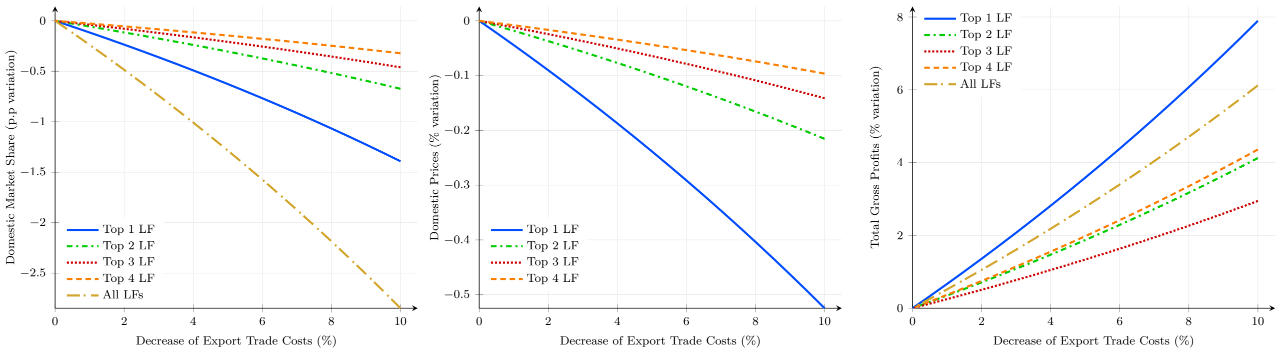
Better Export Access	**Firm**	**Domestic Market Share** Change (p.p.)	**Domestic Prices/Markups** Change (%)	**Quality Investments** Change (%)	**Export Revenues** Change (%)	**Total Gross Profits** Change (%)
For All Firms	Top 1	-1.39	-0.52	0	30.55	7.90
	Top 2	-0.67	-0.22	0	30.55	4.13
	Top 3	-0.46	-0.14	0	30.55	2.95
	Top 4	-0.32	-0.10	0	30.55	4.36
Only For LFs	Top 1	0	0	0	30.55	13.29
	Top 2	0	0	0	30.55	10.50
	Top 3	0	0	0	30.55	9.61
	Top 4	0	0	0	30.55	10.69
Only For SFs	Top 1	-1.39	-0.52	0	0	-5.40
	Top 2	-0.67	-0.22	0	0	-6.37
	Top 3	-0.46	-0.14	0	0	-6.67
	Top 4	-0.32	-0.10	0	0	-6.33

(b) Impact on LFs as Group

Better Export Access	**Domestic Market Share** Change (p.p.)	**Total Gross Profits** Change (%)
For All Firms	-2.85	6.12
Only For LFs	0	11.98
Only For SFs	-2.85	-5.86

Figure 8. *Manufacturing - Decrease in Export Trade Costs for all Firms (No Investments)*

(a) Domestic Market Share **(b)** Domestic Prices/Markups **(c)** Total Gross Profits



6.3 Specific Sectors

In the following, we provide results for some specific sectors. The analysis considers the top three sectors in terms of contribution to exports, income, and expenditures: Food & Beverages, Chemicals, and Machinery.

As we did for manufacturing, we calibrate the model to match average values obtained through industry-revenue weights. Due to potential confidentiality issues, we only describe the calibration features verbally. Furthermore, since the qualitative results for Machinery are similar to manufacturing, we relegate this case to Appendix C. Instead, we focus on Food & Beverages and Chemicals, whose results underscore different patterns in outcomes due to the features of their SFs and LFs.

6.3.1 Food & Beverages

The case of Food & Beverages serves as an example of the detrimental effects that an export shock can have on LFs. This occurs when the shock mainly represents tougher domestic competition for a LF, without generating substantial benefits from better export access. Scenarios like this can take place due to different motives. For instance, it can occur when LFs are primarily local leaders, rather than global ones. Alternatively, it can arise if LFs are foreign-owned firms that set operations in the country to serve the local market, rather than using it as an export platform.

There are two characteristics in particular that distinguish the calibration of Food & Beverages from manufacturing. First, SFs have higher export intensity, so that an export shock induces more entry of SFs and hence a more marked decrease in the domestic price index. Second, LFs have a more pronounced home bias in sales. Thus, LFs benefit less from better export access, and are simultaneously more exposed to tougher competition at home. Both facts explain the results presented in Table 5.

Specifically, an industry-wide export shock determines that, albeit LFs invest more, each LF loses presence in the domestic market. Moreover, the top two LFs, which are the firms with the greatest home bias among LFs, are so heavily impacted by tougher domestic competition that they end up with lower profits. Overall, the high average domestic intensity of LFs translates into an almost null variation in industry profits.

Table 5. *Impact of a 10% Reduction in Export Trade Costs - Food and Beverages*

(a) Impact on each LF

Better Export Access	Firm	Domestic Market Share Change (p.p.)	Domestic Prices/Markups Change (%)	Quality Investments Change (%)	Export Revenues Change (%)	Total Gross Profits Change (%)
For All Firms	Top 1	-1.59	-0.69	6.78	36.50	-1.66
	Top 2	-0.93	-0.30	0.68	31.15	-3.14
	Top 3	-0.44	-0.14	7.20	36.87	4.72
	Top 4	-0.03	-0.01	19.44	47.31	18.22
Only For LFs	Top 1	2.54	1.18	21.41	48.96	23.04
	Top 2	1.26	0.42	25.02	51.95	26.53
	Top 3	1.12	0.35	32.31	57.92	33.70
	Top 4	0.88	0.27	44.13	67.39	45.22
Only For SFs	Top 1	-4.10	-1.74	-14.04	-9.78	-22.09
	Top 2	-2.02	-0.65	-21.51	-15.18	-25.28
	Top 3	-1.39	-0.43	-21.20	-14.96	-23.70
	Top 4	-0.77	-0.23	-19.50	-13.72	-20.81

(b) Impact on LFs as Group

Better Export Access	Domestic Market Share Change (p.p.)	Total Gross Profits Change (%)
For All Firms	-3.00	0.33
Only For LFs	5.81	26.61
Only For SFs	-8.28	-22.73

6.3.2 Chemicals

One of the distinctive features of Chemicals is the high export intensity of its SFs. This is around 40%, which is higher than in both Manufacturing and Food & Beverages. Consequently, an export shock to SFs triggers an even more marked increase in domestic competition relative to these sectors. Additionally, LFs feature substantial heterogeneity in terms of their export intensity, entailing starkly different responses following an export shock. This can be observed in Table 6.

The table shows that an export shock to all firms decreases the domestic market share of LFs as a group, although their total profits increase. These outcomes combine the idiosyncratic responses by quite dissimilar LFs, which can be illustrated by focusing on the first two top LFs. The features of these firms are markedly different. The top firm is heavily oriented to foreign markets, with its exports even greater than its sales at home. Thus, this firm substantially benefits from a reduction in export trade costs, making it invest significantly more, with its domestic market share, domestic markups, and total profit becoming greater.

On the contrary, the second top firm features the opposite characteristics: its revenue comes mainly from domestic sales, and its exports are even lower than the third top firm in the sector. Consequently, the negative impact of tougher domestic competition is pronounced enough to make it reduce its investments. This leads to a decrease in its domestic market share, domestic markups, and total profit.

Table 6. *Impact of a 10% Reduction in Export Trade Costs - Chemicals*

(a) Impact on each LF

Better Export Access	Firm	Domestic Market Share Change (p.p.)	Domestic Prices/Markups Change (%)	Quality Investments Change (%)	Export Revenues Change (%)	Total Gross Profits Change (%)
For All Firms	Top 1	2.19	0.98	58.37	78.46	51.05
	Top 2	-2.27	-0.77	-7.35	23.94	-14.02
	Top 3	-0.37	-0.11	19.80	47.61	16.98
	Top 4	-0.80	-0.24	-5.11	25.97	-7.64
Only For LFs	Top 1	7.13	3.41	70.11	87.36	73.20
	Top 2	1.60	0.57	24.35	51.40	26.12
	Top 3	1.66	0.53	50.26	72.20	52.08
	Top 4	0.79	0.24	31.92	57.61	32.96
Only For SFs	Top 1	-4.48	-1.84	-10.51	-7.27	-16.24
	Top 2	-3.62	-1.21	-28.31	-20.25	-34.29
	Top 3	-1.69	-0.52	-23.72	-16.82	-26.18
	Top 4	-1.40	-0.42	-30.90	-22.22	-33.12

(b) Impact on LFs as Group

Better Export Access	Domestic Market Share Change (p.p.)	Total Gross Profits Change (%)
For All Firms	-1.25	35.45
Only For LFs	11.19	62.33
Only For SFs	-11.18	-20.64

7 Conclusion

We studied the impact of better export conditions on the domestic markets of small economies, where both small and large firms tend to be export-oriented due to the limited size of the home market. With this goal, we built a model where firms choose prices and quality, and the latter affects a variety's appeal in each country served. Moreover, firms were partitioned according to their size, with small firms characterized as in Melitz and large firms taken as oligopolistic firms with an idiosyncratic export intensity.

We began by considering the impact of better export opportunities when all firms are described as in Melitz. Our results indicated that an export shock increases the profits of exporters and induces them to upgrade quality. As a corollary, the effects on the most productive firms are unambiguous and positive.

On the contrary, the impact on large firms in our setting is ambiguous. This is explained because better export opportunities trigger two opposing mechanisms. The first one occurs through increases in a small firm's expected profitability, which induces entry and strengthens domestic competition. This reduces a large firm's profit, investment, and domestic markup, and reallocates domestic market share towards small firms. The second mechanism acts through the impact of better export conditions on a large firm's sales volume. This decreases a large firm's average cost of quality, making them garner greater profits and upgrade quality. The latter increases its domestic market share and allows it to charge higher domestic markups.

To identify conditions under which one or the other mechanism dominates, we identified observables that capture the magnitude of each channel. This led us to highlight the role of export intensities. Specifically, greater export intensity of small firms translates into more domestic competition, since an export shock impacts expected profits more and hence more small firms are induced to enter. Additionally, a large firm's export intensity reflects the degree to which a large firm benefits from scale effects and is hurt by tougher domestic competition.

Given the ambiguity of results, we performed several calibration exercises for Danish manufacturing. The goal was to show how an export shock entails different outcomes depending on the export intensities of firms in an industry. The results revealed that large firms are far from being uniformly affected, even within industries. More precisely, an export shock induces some large firms to upgrade quality and charge higher domestic markups, with positive effects on their profits and domestic market share. However, it is common to observe the exact opposite pattern for other large firms.

Our results highlight how incorporating large firms into an analysis can revert results arising under monopolistic competition. The fact becomes particularly relevant considering the recent evidence on the rise of superstar firms (Autor et al. 2017, De Loecker et al. 2020): in a context where industries are becoming increasingly dominated by large firms, accounting for their idiosyncratic features is of first-order relevance for the impact on an industry.

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Online Appendix - not for publication

A Proofs and Derivations

To make this appendix self-contained, some equations included in the main body are restated here. Also, we streamline notation in several ways. First, we only express the arguments of equilibrium functions in terms of either endogenous variables or parameters that vary in equilibrium. Furthermore, we denote

$$\alpha := \left(\frac{\sigma}{\sigma-1}w_H\right)^{1-\sigma},$$
$$\beta := \left(\frac{\delta}{\sigma f^z}\right),$$

and for $i \in \mathcal{C}$ and $j \neq i$,

$$\mathcal{D}_i := E_i\left(\mathbb{P}_i\right)^{\sigma-1}, \tag{A1}$$
$$\mathcal{X}_i := E_j\left(\mathbb{P}_j\right)^{\sigma-1}\left(\tau_{ij}^{\mathcal{N}}\right)^{1-\sigma}. \tag{A2}$$

A.1 Equilibrium Derivation

In this section, we derive the optimal decisions of each type of firm. Moreover, we derive the equilibrium conditions, including the one regarding the market stage, which was omitted in the main part of the paper.

A.1.1 Optimal Variables of Large Firms

Optimal prices for LFs from $i \in \mathcal{C}$ in $j \in \mathcal{C}$ are given by the well-known expression

$$p_{ij}^{\omega} = m\left(s_{ij}^{\omega}\right)c_{ij}^{\omega}. \tag{A3}$$

where $p_{HF}^{\omega} = \frac{\sigma}{\sigma-1}c_{HF}^{\omega}$, and $m\left(s_{ij}^{\omega}\right) := \frac{\varepsilon\left(s_{ij}^{\omega}\right)}{\varepsilon\left(s_{ij}^{\omega}\right)-1}$ otherwise.

Regarding the solution for quality, the first-order condition determines that for firm ω from $i \in \mathcal{C}$

$$\frac{\partial \pi_i^{\omega}}{\partial z_i^{\omega}} = \sum_{k\in\mathcal{C}} Q_{ik}^{\omega}\left(p_{ik}^{\omega} - c_{ik}^{\omega}\right)\left(\frac{\mathrm{d}\ln Q_{ik}^{\omega}}{\mathrm{d}\ln z_i^{\omega}}\right)\frac{1}{z_i^{\omega}} - f^z = 0, \tag{A4}$$

where $Q_{ik}^{\omega}\left(p_{ik}^{\omega} - c_{ik}^{\omega}\right) = \frac{R_{ik}^{\omega}}{\varepsilon_{ik}^{\omega}}$ by using optimal prices.

Suppose LF ω from F. Since it has market power in each $k \in \mathcal{C}$, then $\frac{\mathrm{d}\ln Q_{Fk}^{\omega}}{\mathrm{d}\ln z_F^{\omega}} = \delta\left(1 - s_{Fk}^{\omega}\right)$. Therefore, its solution to (A4) determines

$$z_F\left(s_{FH}^{\omega}\right) := \delta\left[\frac{R_{FH}^{\omega}}{\varepsilon_{FH}^{\omega}}\frac{\left(1-s_{FH}^{\omega}\right)}{f^z} + \frac{R_{FF}^{\omega}}{\varepsilon_{FF}^{\omega}}\frac{\left(1-s_{FF}^{\omega}\right)}{f^z}\right], \tag{A5}$$

with optimal investments in quality $I_F\left(s_{FH}^{\omega}\right) := f^z z_F\left(s_{FH}^{\omega}\right)$.

Furthermore, if $i = H$, then $\frac{\mathrm{d}\ln Q_{HH}^{\omega}}{\mathrm{d}\ln z_H^{\omega}} = \delta\left(1 - s_{HH}^{\omega}\right)$ and $\frac{\mathrm{d}\ln Q_{HF}^{\omega}}{\mathrm{d}\ln z_H^{\omega}} = \delta$, which establishes (6).

A.1.2 Optimal Variables of Small Firms

We can derive optimal quality for SF ω from i by using (A4). This requires using $\frac{\mathrm{d}\ln Q_{ik}^{\omega}}{\mathrm{d}\ln z_i^{\omega}} = \delta$ for any $k \in \mathcal{C}$, so that (4) is obtained. Thus, consider a SF from H with productivity φ that exclusively serves the domestic market. Given (4) evaluated at optimal prices, we obtain

$$z_H^d(\mathbb{P}_H, \varphi) := (\alpha\beta)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} (\mathcal{D}_H)^{\frac{1}{1-\delta}}, \tag{A6}$$

with investments $I_H^d(\mathbb{P}_H, \varphi) := f^z z_H^d(\mathbb{P}_H, \varphi)$. Moreover, optimal revenues are

$$r_H^d(\mathbb{P}_H, \varphi) := \left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} (\mathcal{D}_H)^{\frac{1}{1-\delta}}. \tag{A7}$$

This determines that optimal profits can be expressed as

$$\pi_H^d(\mathbb{P}_H, \varphi) := \frac{1-\delta}{\sigma} r_H^d(\mathbb{P}_H, \varphi) - f_{HH}. \tag{A8}$$

Moreover, the survival productivity cutoff at home is given by $\pi_H^d(\mathbb{P}_H, \varphi_{HH}^*) = 0$, which yields

$$\varphi_{HH}^*(\mathbb{P}_H) := \left[\frac{\left(\frac{\sigma f_{HH}}{1-\delta}\right)^{1-\delta}}{(\alpha\beta^{\delta}) E_H}\right]^{\sigma-1} \frac{1}{\mathbb{P}_H}. \tag{A9}$$

Consider now a SF from $i \in \mathcal{C}$ with productivity φ that exports. (4) evaluated at optimal prices determines

$$z_i^x\left(\mathbb{P}_H, \varphi, \tau_{ij}^{\mathcal{N}}\right) := (\alpha\beta)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} (\mathcal{D}_i + \mathcal{X}_i)^{\frac{1}{1-\delta}}, \tag{A10}$$

with investments for $i = H$ given by $I_H^x\left(\mathbb{P}_H, \varphi, \tau_{HF}^{\mathcal{N}}\right) := f^z z_H^x\left(\mathbb{P}_H, \varphi, \tau_{HF}^{\mathcal{N}}\right)$. Besides, optimal revenues at home and in country $j \neq i$, and total optimal revenues are respectively

$$r_{HH}^x\left(\mathbb{P}_H, \varphi, \tau_{HF}^{\mathcal{N}}\right) := \left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} \mathcal{D}_H (\mathcal{D}_H + \mathcal{X}_H)^{\frac{\delta}{1-\delta}}, \tag{A11}$$

$$r_{ij}^x\left(\mathbb{P}_H, \varphi, \tau_{ij}^{\mathcal{N}}\right) := \left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} \mathcal{X}_i (\mathcal{D}_i + \mathcal{X}_i)^{\frac{\delta}{1-\delta}}, \tag{A12}$$

$$r_i^x\left(\mathbb{P}_H, \varphi, \tau_{ij}^{\mathcal{N}}\right) := \left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} (\mathcal{D}_i + \mathcal{X}_i)^{\frac{1}{1-\delta}}. \tag{A13}$$

Thus, optimal profits are

$$\pi_i^x\left(\mathbb{P}_i, \varphi, \tau_{ij}^{\mathcal{N}}\right) := \frac{(1-\delta)}{\sigma} r_i^x - f_{ii} - f_{ij}, \tag{A14}$$

where the productivity cutoff for exporters is determined by $\pi_i^d\left(\mathbb{P}_i, \varphi_{ij}^*\right) = \pi_i^x\left(\mathbb{P}_i, \varphi_{ij}^*, \tau_{ij}^{\mathcal{N}}\right)$, so that

$$\varphi_{ij}^*\left(\mathbb{P}_i, \tau_{ij}^{\mathcal{N}}\right) := \left[\frac{f_{ij}}{\frac{1-\delta}{\sigma}(\alpha\beta^{\delta})^{\frac{1}{1-\delta}}\left\{(\mathcal{D}_i + \mathcal{X}_i)^{\frac{1}{1-\delta}} - (\mathcal{D}_i)^{\frac{1}{1-\delta}}\right\}}\right]^{\frac{1-\delta}{\sigma-1}}. \tag{A15}$$

A.1.3 Equilibrium Conditions

In the working-paper version of our article, we have derived the equilibrium conditions without imposing that H is a small country. After that, we have shown which conditions are relevant for our results. Based on that analysis, next we directly state the conditions that are relevant when H is a small country. Recall that the small-economy assumption determines that $\left(\mathbb{P}_F, M_F^E\right)$ is unaffected by changes in H's market conditions or its firms.

The equilibrium conditions exploit the existence of a single sufficient statistic. Thus, a firm's optimal choices in H are functions of its market shares, which in turn are determined by $\mathbb{P}_H$.

Specifically, we need to state the free-entry condition in H and the equilibrium condition at the market stage. The free-entry condition has already been derived and is given by (FE). So, next we focus on the equilibrium condition at the market stage. This requires that the sum of optimal market shares of firms serving H sums to one.

We begin by stating the optimal market shares in H for each type of firm. Regarding SFs, we need to distinguish between SFs from H that exclusively serve home, and SFs from any country that export. For the former, we use (A7), (A9), and (A15) to determine that the optimal market share is a function given by

$$s^d_{HH}\left(\mathbb{P}_H, M^E_H; \tau^{\mathcal{N}}_{HF}\right) := M^E_H \int_{\varphi^*_{HH}(\mathbb{P}_H)}^{\varphi^*_{HF}\left(\mathbb{P}_H, \tau^{\mathcal{N}}_{HF}\right)} \frac{r^d_H\left(\mathbb{P}_H, \varphi\right)}{E_H} \mathrm{d}G_H\left(\varphi\right).$$

As for SFs from H that export, the procedure is the same but using (A11) instead of (A7), so that

$$s^x_{HH}\left(\mathbb{P}_H, M^E_H; \tau^{\mathcal{N}}_{HF}\right) := M^E_H \int_{\varphi^*_{HF}\left(\mathbb{P}_H, \tau^{\mathcal{N}}_{HF}\right)}^{\overline{\varphi}_H} \frac{r^x_{HH}\left(\mathbb{P}_H, \varphi, \tau^{\mathcal{N}}_{HF}\right)}{E_H} \mathrm{d}G_H\left(\varphi\right),$$

whereas for SFs from F that export to H we use (A12) instead of (A7), so that

$$s^x_{FH}\left(\mathbb{P}_H\right) := M^E_F \int_{\varphi^*_{FH}(\mathbb{P}_H)}^{\overline{\varphi}_F} \frac{r^x_{FH}\left(\mathbb{P}_H, \varphi\right)}{E_H} \mathrm{d}G_F\left(\varphi\right).$$

Therefore, we can define the optimal market share in H of SFs serving that country by a function

$$S^{\mathcal{N}}_H\left(\mathbb{P}_H, M^E_H; \tau^{\mathcal{N}}_{HF}\right) := s^d_{HH}\left(\mathbb{P}_H, M^E_H; \tau^{\mathcal{N}}_{HF}\right) + s^x_{HH}\left(\mathbb{P}_H, M^E_H; \tau^{\mathcal{N}}_{HF}\right) + s^x_{FH}\left(\mathbb{P}_H\right). \tag{A16}$$

As for LF ω from H, the simultaneous solution to (2), (5), and (6) determines that its market share in H can be expressed as a function $s^{\omega}_{HH}\left(\mathbb{P}_H, \tau^{\omega}_{HF}\right)$. Analogously, the solution to (2), (A3), (A5) determines a function $s^{\omega}_{FH}\left(\mathbb{P}_H\right)$ for a LF ω from F.

Using these results, the market-clearing condition in H is

$$S^{\mathcal{N}}_H\left(\mathbb{P}_H, M^E_H; \tau^{\mathcal{N}}_{HF}\right) + \sum_{\omega \in \Omega^{\mathcal{L}}_{HH}} s^{\omega}_{HH}\left(\mathbb{P}_H, \tau^{\omega}_{HF}\right) + \sum_{\omega \in \Omega^{\mathcal{L}}_{FH}} s^{\omega}_{FH}\left(\mathbb{P}_H\right) = 1. \tag{A17}$$

In summary, the equilibrium conditions have been expressed in a way that we can exploit separability properties and the existence of sufficient statistics. Specifically, all the equilibrium values can be obtained by pinning down $\left(\mathbb{P}^*_H, M^{E*}_H\right)$ through the system comprising (FE) and (A17). In particular, $\mathbb{P}^*_H$ can be identified through (FE) with independence of M^{E*}_H. Likewise, once that $\mathbb{P}^*_H$ is obtained, it is possible to obtain solutions for each optimal variable. For LF ω from H, this implies we can determine $s^{\omega}_{Hj}\left(\mathbb{P}_H, \tau^{\omega}_{HF}\right)$ for $j \in \mathcal{C}$, and then identify its optimal prices and investments in quality through (5) and (6), respectively.

A.2 Intermediate Results

We start by establishing some intermediate results that allow us to perform subsequent calculations more easily. In particular, we characterize how the optimal decisions by LFs (i.e., prices and investment) are impacted by variations in market shares and export trade costs. After this, we solve for the system of equations consisting of each firm's market shares, and characterize the relation between market shares with the domestic price index and export trade costs. Finally, we outline the impact on gross profits, and then describe how the price index is impacted by export trade costs.

A.2.1 Partial Effects on Optimal Choices

Consider LF ω from $i \in \mathcal{C}$ in $j \in \mathcal{C}$. We begin by characterizing partial effects on prices. Conditional on ω's market share, (A3) determines

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln \tau_{ij}^{\omega}} = 1. \tag{A18}$$

As for the effect of market share on prices, (A3) determines that $\ln p_{ij}^{\omega} = \ln m_{ij}^{\omega} + \ln c_{ij}^{\omega}$ and therefore,

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{\partial \ln m_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{\partial \ln m_{ij}^{\omega}}{\partial \ln \varepsilon_{ij}^{\omega}} \frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}}.$$

In turn, $\ln m_{ij}^{\omega} = \ln \varepsilon_{ij}^{\omega} - \ln \left(\varepsilon_{ij}^{\omega} - 1\right)$ and $\varepsilon_{ij}^{\omega} = \sigma + s_{ij}^{\omega}(1-\sigma)$. Using these results,

$$\frac{\partial \ln m_{ij}^{\omega}}{\partial \ln \varepsilon_{ij}^{\omega}} = 1 - \frac{\varepsilon_{ij}^{\omega}}{\varepsilon_{ij}^{\omega} - 1} = 1 - m_{ij}^{\omega},$$

and, since $\frac{\partial \varepsilon_{ij}^{\omega}}{\partial s_{ij}^{\omega}} = 1 - \sigma$,

$$\frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{s_{ij}^{\omega}(1-\sigma)}{\varepsilon_{ij}^{\omega}}. \tag{A19}$$

This establishes that $\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \left(1 - m_{ij}^{\omega}\right) \frac{s_{ij}^{\omega}(1-\sigma)}{\varepsilon_{ij}^{\omega}}$ which, by using that $1 - m_{ij}^{\omega} = \frac{-1}{\varepsilon_{ij}^{\omega}-1}$ and $\varepsilon_{ij}^{\omega} - 1 = (\sigma - 1)\left(1 - s_{ij}^{\omega}\right)$, becomes

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{s_{ij}^{\omega}}{\left(1 - s_{ij}^{\omega}\right)\varepsilon_{ij}^{\omega}}. \tag{A20}$$

Since H is a small economy, the expression for $\frac{\partial \ln p_{HF}^{\omega}}{\partial \ln \tau_{HF}^{\omega}}$ is still given by (A18). Nonetheless, since any LF from H is negligible for any foreign country, its markups are constant in F, determining that $\frac{\partial \ln p_{HF}^{\omega}}{\partial \ln s_{HF}^{\omega}} = 0$.

For future references, we summarize the results for a LF from H that we use in subsequent derivations:

$$\frac{\partial \ln p_{HH}^{\omega}}{\partial \ln s_{HH}^{\omega}} = \frac{\partial \ln m_{HH}^{\omega}}{\partial \ln s_{HH}^{\omega}} = \frac{s_{HH}^{\omega}}{\left(1 - s_{HH}^{\omega}\right)\varepsilon_{HH}^{\omega}}, \tag{A21a}$$

$$\frac{\partial \ln p_{HF}^{\omega}}{\partial \ln \tau_{HF}^{\omega}} = 1. \tag{A21b}$$

Now we obtain some partial effects for LF ω from H regarding investments. Define $\rho_{HH}^{\omega} := \frac{R_{HH}^{\omega}\left(1-s_{HH}^{\omega}\right)/\varepsilon_{HH}^{\omega}}{R_{HH}^{\omega}\left(1-s_{HH}^{\omega}\right)/\varepsilon_{HH}^{\omega}+R_{HF}^{\omega}/\sigma}$ and $\rho_{HF}^{\omega} := \frac{R_{HF}^{\omega}/\sigma}{R_{HH}^{\omega}\left(1-s_{HH}^{\omega}\right)/\varepsilon_{HH}^{\omega}+R_{HF}^{\omega}/\sigma}$. The terms ρ_{HH}^{ω} and ρ_{HF}^{ω} satisfy $\rho_{HH}^{\omega} + \rho_{HF}^{\omega} = 1$, and represent the relative importance of market H and F in ω's total investments, respectively. They can be reexpressed in terms of observables by

$$\rho_{HH}^{\omega} = \frac{d_{H}^{\omega}\left(1 - s_{HH}^{\omega}\right)/\varepsilon_{HH}^{\omega}}{d_{H}^{\omega}\left(1 - s_{HH}^{\omega}\right)/\varepsilon_{HH}^{\omega} + e_{H}^{\omega}/\sigma}, \tag{A22}$$

and $\rho^{\omega}_{HF} = 1 - \rho^{\omega}_{HH}$, which gives $\rho^{\omega}_{HF} = \frac{e^{\omega}_H/\sigma}{d^{\omega}_H\left(1-s^{\omega}_{HH}\right)/\varepsilon^{\omega}_{HH}+e^{\omega}_H/\sigma}$.

As for the impact of trade costs on z^{ω}_H, we can use that $R^{\omega}_{Hj} = E_j s^{\omega}_{Hj}$ for $j \in \{H, F\}$, so that optimal quality is a function $z^{\omega}_H\left(s^{\omega}_{HH}, s^{\omega}_{HF}\right)$. This implies in particular that z^{ω}_H does not depend directly on τ^{ω}_{HF}.

Next, we characterize how s^{ω}_{HH} and s^{ω}_{HF} impact investments. Concerning s^{ω}_{HH},

$$\begin{aligned}\frac{\partial \ln z^{\omega}_H}{\partial \ln s^{\omega}_{HH}} &= \frac{\partial \ln\left[R^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)/\varepsilon^{\omega}_{HH} + R^{\omega}_{HF}/\sigma\right]}{\partial \ln s^{\omega}_{HH}}, \\ &= \rho^{\omega}_{HH}\frac{\partial \ln\left(\frac{R^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}{\varepsilon^{\omega}_{HH}}\right)}{\partial \ln s^{\omega}_{HH}}.\end{aligned}$$

Besides, since $R_{HH} = E_H s^{\omega}_{HH}$, then

$$\frac{\partial \ln\left(\frac{R^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}{\varepsilon^{\omega}_{HH}}\right)}{\partial \ln s^{\omega}_{HH}} = \frac{\partial \ln\left(E_H s_{HH}\right)}{\partial \ln s^{\omega}_{HH}} - \frac{s^{\omega}_{HH}}{1-s^{\omega}_{HH}} - \frac{\partial \ln \varepsilon^{\omega}_{HH}}{\partial \ln s^{\omega}_{HH}},$$

where we have used that $\frac{\partial \ln\left(1-s^{\omega}_{HH}\right)}{\partial \ln s^{\omega}_{HH}} = \frac{-s^{\omega}_{HH}}{1-s^{\omega}_{HH}}$. Using that $\frac{\partial \ln \varepsilon^{\omega}_{HH}}{\partial \ln s^{\omega}_{HH}} = (1-\sigma)\frac{s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}}$ and $(1-\sigma)\, s^{\omega}_{HH} = \varepsilon^{\omega}_{HH} - \sigma$ due to the definition of $\varepsilon^{\omega}_{HH}$,

$$\frac{\partial \ln\left(\frac{R^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}{\varepsilon^{\omega}_{HH}}\right)}{\partial \ln s^{\omega}_{HH}} = 1 - \frac{\varepsilon^{\omega}_{HH}-\sigma}{\varepsilon^{\omega}_{HH}} - \frac{s^{\omega}_{HH}}{1-s^{\omega}_{HH}}.$$

Gathering terms and using that $\sigma\left(1-s^{\omega}_{HH}\right) = \varepsilon^{\omega}_{HH} - s^{\omega}_{HH}$,

$$\frac{\partial \ln\left(\frac{R^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}{\varepsilon^{\omega}_{HH}}\right)}{\partial \ln s^{\omega}_{HH}} = \frac{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right) - s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)},$$

which allows us to conclude that

$$\frac{\partial \ln z^{\omega}_H}{\partial \ln s^{\omega}_{HH}} = \rho^{\omega}_{HH}\frac{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right) - s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}.$$

Regarding the impact of s^{ω}_{HF} on z^{ω}_H, we can proceed in the same fashion as above, so that

$$\frac{\partial \ln z^{\omega}_H}{\partial \ln s^{\omega}_{HF}} = \rho^{\omega}_{HF}\frac{\partial \ln\left(R^{\omega}_{HF}/\sigma\right)}{\partial \ln s^{\omega}_{HF}}.$$

Therefore, since $R^{\omega}_{HF} = E_F s^{\omega}_{HF}$,

$$\frac{\partial \ln z^{\omega}_H}{\partial \ln s^{\omega}_{HF}} = \rho^{\omega}_{HF}.$$

Gathering all the results and using that $\frac{\partial \ln I^{\omega}_H}{\partial \ln s^{\omega}_{HH}} = \frac{\partial \ln z^{\omega}_H}{\partial \ln s^{\omega}_{HH}}$, we end up with

$$\frac{\partial \ln I^{\omega}_H}{\partial \ln s^{\omega}_{HH}} = \frac{\partial \ln z^{\omega}_H}{\partial \ln s^{\omega}_{HH}} = \rho^{\omega}_{HH}\frac{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right) - s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}, \tag{A23a}$$

$$\frac{\partial \ln I^{\omega}_H}{\partial \ln s^{\omega}_{HF}} = \frac{\partial \ln z^{\omega}_H}{\partial \ln s^{\omega}_{HF}} = \rho^{\omega}_{HF}. \tag{A23b}$$

Since we rule out extremely large market shares such that $\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right) - s^{\omega}_{HH} > 0$, it follows that $\frac{\partial \ln I^{\omega}_H}{\partial \ln s^{\omega}_{HH}} = \frac{\partial \ln z^{\omega}_H}{\partial \ln s^{\omega}_{HH}} > 0$.

A.2.2 Partial Effects on Market Shares

Given the characterization of optimal prices and investments for H, summarized by (A21) and (A23), we proceed to study how the market shares of a LF ω from H are impacted by H's price index and export trade costs. For the latter, we present results only for τ^{ω}_{HF} since $\frac{\partial \ln \tau^{\omega}_{HF}}{\partial \ln \tau^{\omega}} = \frac{\partial \ln \tau^{\omega}_{HF}}{\partial \ln \tau_{HF}} = 1$, which implies that changes in its components τ^{ω} or τ_{HF} have the same logarithmic impact on market shares.

In logarithms, the system of market-shares equations for LF ω is

$$\begin{aligned} \ln s^{\omega}_{HH} &= (1-\sigma)\ln p^{\omega}_{HH}\left(s^{\omega}_{HH}\right) + \delta \ln z^{\omega}_{H}\left(s^{\omega}_{HH}, s^{\omega}_{HF}\right) - (1-\sigma)\ln \mathbb{P}_{H}, \\ \ln s^{\omega}_{HF} &= (1-\sigma)\ln p^{\omega}_{HF}\left(\tau^{\omega}_{HF}\right) + \delta \ln z^{\omega}_{H}\left(s^{\omega}_{HH}, s^{\omega}_{HF}\right) - (1-\sigma)\ln \mathbb{P}_{F}. \end{aligned} \tag{A24}$$

Before differentiating the system, we obtain an intermediate result to express $1 - \frac{\partial \ln s^{\omega}_{HH}}{\partial \ln p^{\omega}_{HH}} \frac{\partial \ln p^{\omega}_{HH}}{\partial \ln s^{\omega}_{HH}} - \frac{\partial \ln s^{\omega}_{HH}}{\partial \ln z^{\omega}_{H}} \frac{\partial \ln z^{\omega}_{H}}{\partial \ln s^{\omega}_{HH}}$. By using (A21) and (A23),

$$1-(\sigma-1)\frac{\partial \ln p^{\omega}_{HH}}{\partial \ln s^{\omega}_{HH}} - \delta \frac{\partial \ln z^{\omega}_{H}}{\partial \ln s^{\omega}_{HH}} = 1-(\sigma-1)\frac{s^{\omega}_{HH}}{\left(1-s^{\omega}_{HH}\right)\varepsilon^{\omega}_{HH}} - \delta\rho^{\omega}_{HH}\frac{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}.$$

Working out the expression and, in particular, using that $(\sigma-1)\,s^{\omega}_{HH} = \sigma - \varepsilon^{\omega}_{HH}$, we establish that

$$1 - \frac{\partial \ln s^{\omega}_{HH}}{\partial \ln p^{\omega}_{HH}}\frac{\partial \ln p^{\omega}_{HH}}{\partial \ln s^{\omega}_{HH}} - \frac{\partial \ln s^{\omega}_{HH}}{\partial \ln z^{\omega}_{H}}\frac{\partial \ln z^{\omega}_{H}}{\partial \ln s^{\omega}_{HH}} = \frac{\left(\sigma-\varepsilon^{\omega}_{HH}s^{\omega}_{HH}\right) - \delta\rho^{\omega}_{HH}\left[\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}\right]}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)},$$

which can be shown that it is always positive, since $\delta\rho^{\omega}_{HH} \in (0,1)$.

Using this result, along with (A21) and (A23), we differentiate the system (A24) and express it in a matrix way:

$$\begin{pmatrix} \frac{\left(\sigma-\varepsilon^{\omega}_{HH}s^{\omega}_{HH}\right)-\delta\rho^{\omega}_{HH}\left[\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}\right]}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)} & -\delta\rho^{\omega}_{HF} \\ -\delta\rho^{\omega}_{HH}\left[\frac{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}\right] & 1-\delta\rho^{\omega}_{HF} \end{pmatrix} \begin{pmatrix} \mathrm{d}\ln s^{\omega}_{HH} \\ \mathrm{d}\ln s^{\omega}_{HF} \end{pmatrix} = \begin{pmatrix} 0 & \sigma-1 \\ 1-\sigma & 0 \end{pmatrix} \begin{pmatrix} \mathrm{d}\ln \tau^{\omega}_{HF} \\ \mathrm{d}\ln \mathbb{P}_{H} \end{pmatrix}, \tag{A25}$$

where we define the matrix on the left-hand side (LHS) as J^{ω}_{H}. This determines that

$$\frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \tau^{\omega}_{HF}} = \frac{\delta\rho^{\omega}_{HF}(1-\sigma)}{\det J^{\omega}_{H}}, \tag{A26a}$$

$$\frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \mathbb{P}_{H}} = \frac{(\sigma-1)\left(1-\delta\rho^{\omega}_{HF}\right)}{\det J^{\omega}_{H}}, \tag{A26b}$$

$$\frac{\partial \ln s^{\omega}_{HF}}{\partial \ln \tau^{\omega}_{HF}} = \frac{(1-\sigma)}{\det J^{\omega}_{H}}\frac{\left(\sigma-\varepsilon^{\omega}_{HH}s^{\omega}_{HH}\right)-\delta\rho^{\omega}_{HH}\left[\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}\right]}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}, \tag{A26c}$$

$$\frac{\partial \ln s^{\omega}_{HF}}{\partial \ln \mathbb{P}_{H}} = \frac{(\sigma-1)\,\delta\rho^{\omega}_{HH}}{\det J^{\omega}_{H}}\left[\frac{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}\right], \tag{A26d}$$

where it can be shown that and (A26b) and (A26d) are positive, and (A26a) and (A26c) are negative.

A.2.3 Partial Effects on Profits

Next, we obtain expressions for partial effects on the optimal gross profits of LF ω. The optimal gross profits of a LF ω from H are given by (7). Each of the partial effects is given by

$$\frac{\partial \bar{\pi}^{\omega}_{H}}{\partial \ln \mathbb{P}_{H}} = \sum_{k\in\{H,F\}} \frac{\partial \bar{\pi}^{\omega}_{H}}{\partial \ln s^{\omega}_{Hk}}\frac{\partial \ln s^{\omega}_{Hk}}{\partial \ln \mathbb{P}_{H}},$$

$$\frac{\partial \overline{\pi}_H^\omega}{\partial \ln \tau_{HF}^\omega} = \sum_{k\in\{H,F\}} \frac{\partial \overline{\pi}_H^\omega}{\partial \ln s_{Hk}^\omega} \frac{\partial \ln s_{Hk}^\omega}{\partial \ln \tau_{HF}^\omega}.$$

Next, we begin by obtaining an expression for $\frac{\partial \overline{\pi}_H^\omega}{\partial \ln s_{HH}^\omega}$. This is given by

$$\frac{\partial \overline{\pi}_H^\omega}{\partial \ln s_{HH}^\omega} = \frac{R_{HH}^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]}{\varepsilon_{HH}^\omega} \left(\frac{\partial \ln \left(\frac{R_{HH}^\omega}{\varepsilon_{HH}^\omega} \left[1-\delta\left(1-s_{HH}^\omega\right)\right]\right)}{\partial \ln s_{HH}^\omega} \right) = \frac{R_{HH}^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]}{\varepsilon_{HH}^\omega} \left(1 - \frac{\partial \ln \varepsilon_{HH}^\omega}{\partial \ln s_{HH}^\omega} + \frac{\delta s_{HH}^\omega}{1-\delta\left(1-s_{HH}^\omega\right)} \right),$$

Proceeding in the same fashion with $\frac{\partial \overline{\pi}_H^\omega}{\partial \ln s_{HF}^\omega}$, it is determined that

$$\frac{\partial \overline{\pi}_H^\omega}{\partial \ln \mathbb{P}_H} = \frac{R_{HH}^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]}{\varepsilon_{HH}^\omega} \left(1 - \frac{\partial \ln \varepsilon_{HH}^\omega}{\partial \ln s_{HH}^\omega} + \frac{\delta s_{HH}^\omega}{1-\delta\left(1-s_{HH}^\omega\right)} \right) \frac{\partial \ln s_{HH}^\omega}{\partial \ln \mathbb{P}_H} + \frac{R_{HF}^\omega (1-\delta)}{\sigma} \frac{\partial \ln s_{HF}^\omega}{\partial \ln \mathbb{P}_H},$$

$$\frac{\partial \overline{\pi}_H^\omega}{\partial \ln \tau_{HF}^\omega} = \frac{R_{HH}^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]}{\varepsilon_{HH}^\omega} \left(1 - \frac{\partial \ln \varepsilon_{HH}^\omega}{\partial \ln s_{HH}^\omega} + \frac{\delta s_{HH}^\omega}{1-\delta\left(1-s_{HH}^\omega\right)} \right) \frac{\partial \ln s_{HH}^\omega}{\partial \ln \tau_{HF}^\omega} + \frac{R_{HF}^\omega (1-\delta)}{\sigma} \left(\frac{\partial \ln s_{HF}^\omega}{\partial \ln \tau_{HF}^\omega} \right).$$

Using (A19) and the definition of elasticity, we get that $\frac{\partial \ln \varepsilon_{HH}^\omega}{\partial \ln s_{HH}^\omega} = \frac{s_{HH}^\omega(1-\sigma)}{\varepsilon_{HH}^\omega} = \frac{\varepsilon_{HH}^\omega - \sigma}{\varepsilon_{HH}^\omega}$. Working out the expression, this becomes $1 - \frac{\partial \ln \varepsilon_{HH}^\omega}{\partial \ln s_{HH}^\omega} + \frac{\delta s_{HH}^\omega}{1-\delta\left(1-s_{HH}^\omega\right)} = \frac{\sigma - \delta\left[\varepsilon_{HH}^\omega\left(1-s_{HH}^\omega\right) - s_{HH}^\omega\right]}{\varepsilon_{HH}^\omega\left[1-\delta\left(1-s_{HH}^\omega\right)\right]}$. Therefore,

$$\frac{\partial \overline{\pi}_H^\omega}{\partial \ln \mathbb{P}_H} = \frac{R_{HH}^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]}{\varepsilon_{HH}^\omega} \frac{\partial \ln s_{HH}^\omega}{\partial \ln \mathbb{P}_H} \left(\frac{\sigma - \delta\left[\varepsilon_{HH}^\omega\left(1-s_{HH}^\omega\right) - s_{HH}^\omega\right]}{\varepsilon_{HH}^\omega\left[1-\delta\left(1-s_{HH}^\omega\right)\right]} \right) + \frac{R_{HF}^\omega (1-\delta)}{\sigma} \left(\frac{\partial \ln s_{HF}^\omega}{\partial \ln \mathbb{P}_H} \right), \tag{A27}$$

$$\frac{\partial \overline{\pi}_H^\omega}{\partial \ln \tau_{HF}^\omega} = \frac{R_{HH}^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]}{\varepsilon_{HH}^\omega} \frac{\partial \ln s_{HH}^\omega}{\partial \ln \tau_{HF}^\omega} \left(\frac{\sigma - \delta\left[\varepsilon_{HH}^\omega\left(1-s_{HH}^\omega\right) - s_{HH}^\omega\right]}{\varepsilon_{HH}^\omega\left[1-\delta\left(1-s_{HH}^\omega\right)\right]} \right) + \frac{R_{HF}^\omega (1-\delta)}{\sigma} \frac{\partial \ln s_{HF}^\omega}{\partial \ln \tau_{HF}^\omega}. \tag{A28}$$

Define

$$\phi_{HH}^\omega := \frac{R_{HH}^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]}{\overline{\pi}_H^\omega \varepsilon_{HH}^\omega}, \tag{A29}$$

$$\phi_{HF}^\omega := \frac{R_{HF}^\omega (1-\delta)}{\overline{\pi}_H^\omega \sigma}, \tag{A30}$$

which are equivalent to

$$\phi_{HH}^\omega = \frac{d_H^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]/\varepsilon_{HH}^\omega}{d_H^\omega \left[1-\delta\left(1-s_{HH}^\omega\right)\right]/\varepsilon_{HH}^\omega + e_H^\omega (1-\delta)/\sigma},$$

$$\phi_{HF}^\omega = 1 - \phi_{HH}^\omega,$$

where the relations follow by using the definition of $\overline{\pi}_H^\omega$ and by multiplying and dividing the RHS in (A29) and (A30) by $R_{HH}^\omega + R_{HF}^\omega$. The terms ϕ_{HH}^ω and ϕ_{HF}^ω represent, respectively, the relative importance of market H and F in ω's gross profits. Moreover, they satisfy that $\phi_{HH}^\omega + \phi_{HF}^\omega = 1$.

By using this result, we can divide the right-hand side (RHS) of (A27) and (A28) by $\overline{\pi}_H^\omega$, and obtain

$$\frac{\partial \ln \overline{\pi}_H^\omega}{\partial \ln \mathbb{P}_H} = \phi_{HH}^\omega \frac{\partial \ln s_{HH}^\omega}{\partial \ln \mathbb{P}_H} \left(\frac{\sigma - \delta\left[\varepsilon_{HH}^\omega\left(1-s_{HH}^\omega\right) - s_{HH}^\omega\right]}{\varepsilon_{HH}^\omega\left[1-\delta\left(1-s_{HH}^\omega\right)\right]} \right) + \phi_{HF}^\omega \frac{\partial \ln s_{HF}^\omega}{\partial \ln \mathbb{P}_H}, \tag{A31a}$$

$$\frac{\partial \ln \overline{\pi}_H^\omega}{\partial \ln \tau_{HF}^\omega} = \phi_{HH}^\omega \frac{\partial \ln s_{HH}^\omega}{\partial \ln \tau_{HF}^\omega} \left(\frac{\sigma - \delta\left[\varepsilon_{HH}^\omega\left(1-s_{HH}^\omega\right) - s_{HH}^\omega\right]}{\varepsilon_{HH}^\omega\left[1-\delta\left(1-s_{HH}^\omega\right)\right]} \right) + \phi_{HF}^\omega \frac{\partial \ln s_{HF}^\omega}{\partial \ln \tau_{HF}^\omega}, \tag{A31b}$$

where it can be shown that $\frac{\partial \ln \overline{\pi}_H^\omega}{\partial \ln \mathbb{P}_H} > 0$ and $\frac{\partial \ln \overline{\pi}_H^\omega}{\partial \ln \tau_{HF}^\omega} < 0$.

Next, we derive the partial effects of the gross profits of LFs as a group. First, notice that, by definition of $\overline{\Pi}_H^{\mathscr{L}}$,

$$\frac{\partial \overline{\Pi}_H^{\mathscr{L}}}{\partial \ln \mathbb{P}_H} = \sum_{\omega \in \overline{\mathscr{L}}_H} \frac{\partial \overline{\pi}_H^{\omega}}{\partial \ln \mathbb{P}_H},$$

$$\sum_{\omega \in \overline{\mathscr{L}}_H} \frac{\partial \overline{\Pi}_H^{\mathscr{L}}}{\partial \ln \tau_{HF}^{\omega}} = \sum_{\omega \in \overline{\mathscr{L}}_H} \frac{\partial \overline{\pi}_H^{\omega}}{\partial \ln \tau_{HF}^{\omega}}.$$

Multiplying and dividing the LHS by $\overline{\Pi}_H^{\mathscr{L}}$ and each sum in the RHS by $\overline{\pi}_H^{\omega}$, these equations can be equivalently restated

$$\frac{\partial \ln \overline{\Pi}_H^{\mathscr{L}}}{\partial \ln \mathbb{P}_H} = \sum_{\omega \in \overline{\mathscr{L}}_H} \psi_H^{\omega} \frac{\partial \ln \overline{\pi}_H^{\omega}}{\partial \ln \mathbb{P}_H}, \tag{A32a}$$

$$\sum_{\omega \in \overline{\mathscr{L}}_H} \frac{\partial \ln \overline{\Pi}_H^{\mathscr{L}}}{\partial \ln \tau_{HF}^{\omega}} = \sum_{\omega \in \overline{\mathscr{L}}_H} \psi_H^{\omega} \frac{\partial \ln \overline{\pi}_H^{\omega}}{\partial \ln \tau_{HF}}, \tag{A32b}$$

where $\psi_H^{\omega} := \frac{\overline{\pi}_H^{\omega}}{\overline{\Pi}_H^{\mathscr{L}}}$ and, so,

$$\psi_H^{\omega} := \frac{R_{HH}^{\omega} \left[1 - \delta \left(1 - s_{HH}^{\omega}\right)\right] / \varepsilon_{HH}^{\omega} + R_{HF}^{\omega} \left(1 - \delta\right) / \sigma}{\sum_{\omega \in \overline{\mathscr{L}}_H} \left[R_{HH}^{\omega} \left[1 - \delta \left(1 - s_{HH}^{\omega}\right)\right] / \varepsilon_{HH}^{\omega} + R_{HF}^{\omega} \left(1 - \delta\right) / \sigma\right]}.$$

For its computation, we divide numerator and denominator by the total income of the industry, Y_H^{ind}, so that

$$\psi_H^{\omega} = \frac{\widetilde{s}_{HH}^{\omega} \left[1 - \delta \left(1 - s_{HH}^{\omega}\right)\right] / \varepsilon_{HH}^{\omega} + \widetilde{s}_{HF}^{\omega} \left(1 - \delta\right) / \sigma}{\sum_{\omega \in \overline{\mathscr{L}}_H} \left[\widetilde{s}_{HH}^{\omega} \left[1 - \delta \left(1 - s_{HH}^{\omega}\right)\right] / \varepsilon_{HH}^{\omega} + \widetilde{s}_{HF}^{\omega} \left(1 - \delta\right) / \sigma\right]},$$

where $\widetilde{s}_{Hj}^{\omega} := \frac{R_{Hj}^{\omega}}{Y_H^{\text{ind}}}$ for $j \in \{H, F\}$, with Y_H^{ind} defined as the industry's income in H (i.e., the sum of domestic and exports sales by domestic firms from H). In words, the term $\widetilde{s}_{Hj}^{\omega}$ represents the industry-revenue share coming from sales by ω in country $j \in \{H, F\}$. Thus, through these terms for each j, it is possible to capture the importance of domestic and export sales of ω in terms of H's industry income.

A.2.4 Impact on the Price Index

We exploit that $\mathbb{P}_H$ is pinned down by (FE) for H, irrespective of any other equation or endogenous variable. Thus, we are able to directly obtain the total variation of the price index rather than a partial effect.

Inspection of (FE) for H reveals that $\mathbb{P}_H$ is not impacted by variations in τ_{HF}^{ω} for any LF ω. On the other hand, changes in $\tau_{HF}^{\mathcal{N}}$ affect $\mathbb{P}_H$, which can be determined by differentiating (FE) for H:

$$\frac{\mathrm{d} \ln \mathbb{P}_H^*}{\mathrm{d} \ln \tau_{HF}^{\mathcal{N}}} = -\left(\frac{\mathrm{d} \ln \pi_H^{\mathbb{E}}}{\mathrm{d} \ln \mathbb{P}_H}\right)^{-1} \frac{\mathrm{d} \ln \pi_H^{\mathbb{E}}}{\mathrm{d} \ln \tau_{HF}^{\mathcal{N}}}. \tag{A33}$$

We proceed to calculate each of the terms of expected profits. With this goal, we express expected profits in terms of how we have defined revenues:

$$\pi_H^{\mathbb{E}} = \int_{\varphi_{HH}^*}^{\varphi_{HF}^*} \left[\frac{1-\delta}{\sigma} \left(\alpha \beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H\right)^{\frac{1}{1-\delta}} - f_{HH}\right] \mathrm{d}G_H\left(\varphi\right) + \int_{\varphi_{HF}^*}^{\overline{\varphi}_H} \left[\frac{1-\delta}{\sigma} \left(\alpha \beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H + \mathcal{X}_H\right)^{\frac{1}{1-\delta}} - f_{HH} - f_{FF}\right] \mathrm{d}G_H\left(\varphi\right),$$

where $\frac{\partial \ln \mathcal{D}_H}{\partial \ln \mathbb{P}_H} = -\frac{\mathrm{d}\ln \mathcal{X}_H}{\mathrm{d}\ln \tau^{\mathcal{N}}_{HF}} = \sigma - 1$, $\frac{\partial \ln(\mathcal{D}_H+\mathcal{X}_H)}{\partial \ln \mathbb{P}_H} = \frac{\mathcal{D}_H(\sigma-1)}{\mathcal{D}_H+\mathcal{X}_H}$, and $\frac{\partial \ln(\mathcal{D}_H+\mathcal{X}_H)}{\partial \ln \tau^{\mathcal{N}}_{HF}} = \frac{\mathcal{X}_H(1-\sigma)}{\mathcal{D}_H+\mathcal{X}_H}$. Thus,

$$\frac{\partial \pi^{\mathbb{E}}_H}{\partial \ln \mathbb{P}_H} = \frac{\sigma-1}{\sigma}\left\{\int_{\varphi^*_{HH}}^{\varphi^*_{HF}} \left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H\right)^{\frac{1}{1-\delta}} \mathrm{d}G_H(\varphi) + \int_{\varphi^*_{HF}}^{\overline{\varphi}_H} \left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H+\mathcal{X}_H\right)^{\frac{1}{1-\delta}} \frac{\mathcal{D}_H}{\mathcal{D}_H+\mathcal{X}_H}\right] \mathrm{d}G_H(\varphi)\right\}.$$

Moreover, given that $\frac{(\mathcal{D}_H+\mathcal{X}_H)^{\frac{1}{1-\delta}}}{\mathcal{D}_H+\mathcal{X}_H} = (\mathcal{D}_H+\mathcal{X}_H)^{\frac{\delta}{1-\delta}}$, we can use (A7), (A11) and (A12), so that

$$\frac{\partial \pi^{\mathbb{E}}_H}{\partial \ln \mathbb{P}_H} = \frac{\sigma-1}{\sigma}\left\{\int_{\varphi^*_{HH}}^{\varphi^*_{HF}} r^d_H\left(\mathbb{P}^*_H,\varphi\right)\mathrm{d}G_H(\varphi) + \int_{\varphi^*_{HF}}^{\overline{\varphi}_H} r^x_{HH}\left(\mathbb{P}^*_H,\varphi,\tau^{\mathcal{N}}_{HF}\right)\mathrm{d}G_H(\varphi)\right\}$$

By the same token,

$$\begin{aligned}\frac{\partial \pi^{\mathbb{E}}_H}{\partial \ln \tau^{\mathcal{N}}_{HF}} &= \frac{1-\sigma}{\sigma}\int_{\varphi^*_{HF}}^{\overline{\varphi}_H}\left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \varphi^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H+\mathcal{X}_H\right)^{\frac{1}{1-\delta}} \frac{\mathcal{X}_H}{\mathcal{D}_H+\mathcal{X}_H}\right]\mathrm{d}G_H(\varphi), \\ &= \frac{1-\sigma}{\sigma}\int_{\varphi^*_{HF}}^{\overline{\varphi}_H} r^x_{HF}\left(\mathbb{P}^*_H,\varphi,\tau^{\mathcal{N}}_{HF}\right)\mathrm{d}G_H(\varphi).\end{aligned}$$

All this implies that

$$\frac{\mathrm{d}\ln \mathbb{P}^*_H}{\mathrm{d}\ln \tau^{\mathcal{N}}_{HF}} = \frac{\int_{\varphi^*_{HF}}^{\overline{\varphi}_H} r^x_{HF}\left(\mathbb{P}^*_H,\varphi,\tau^{\mathcal{N}}_{HF}\right)\mathrm{d}G_H(\varphi)}{\int_{\varphi^*_{HH}}^{\varphi^*_{HF}} r^d_H\left(\mathbb{P}^*_H,\varphi\right)\mathrm{d}G_H(\varphi) + \int_{\varphi^*_{HF}}^{\overline{\varphi}_H} r^x_{HH}\left(\mathbb{P}^*_H,\varphi,\tau^{\mathcal{N}}_{HF}\right)\mathrm{d}G_H(\varphi)},$$

and multiplying and dividing by M^{E*}_H and $R^{\mathcal{N}}_H := R^{\mathcal{N}}_{HH} + R^{\mathcal{N}}_{HF}$,

$$\frac{\mathrm{d}\ln \mathbb{P}^*_H}{\mathrm{d}\ln \tau^{\mathcal{N}}_{HF}} = \frac{e^{\mathcal{N}}_H}{d^{\mathcal{N}}_H}, \tag{A34}$$

where $d^{\mathcal{N}}_H := \frac{R^{\mathcal{N}}_{HH}}{R^{\mathcal{N}}_H}$ and $e^{\mathcal{N}}_H := 1 - d^{\mathcal{N}}_H$ are the domestic and export intensities of SFs from H, respectively.

A.3 Export Shocks

Next, we concentrate on the propositions and results included in Section 4. Recall that, throughout the paper, we have assumed that $\varepsilon^{\omega}_{ij}\left(1 - s^{\omega}_{ij}\right) - s^{\omega}_{ij} > 0$ for $i, j \in \mathcal{C}$. This holds when no firm ω has a disproportionately large market share, as is the case in our Danish data for domestic firms. The assumption is incorporated into the results we present subsequently.

We begin by stating some lemmas that are necessary to determine the signs of each effect.

Lemma 1. $\det J^{\omega}_H > 0$.

Proof of Lemma 1. Differentiating the system (A24) determines (A25) and, hence, J^{ω}_H. This is defined by

$$J^{\omega}_H := \begin{pmatrix} \frac{\left(\sigma-\varepsilon^{\omega}_{HH}s^{\omega}_{HH}\right)-\delta\rho^{\omega}_{HH}\left[\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}\right]}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)} & -\delta\rho^{\omega}_{HF} \\ -\delta\rho^{\omega}_{HH}\left[\frac{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}\right] & 1-\delta\rho^{\omega}_{HF}\end{pmatrix}.$$

Using that $\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right) - s^{\omega}_{HH} > 0$, it can be shown that $\arg\inf_{\delta}\left(\det J^{\omega}_H\right) = 1$. Thus, the proof requires that $\det J^{\omega}_H > 0$ when $\delta \to 1$, which ensures that the lemma holds for any $\delta \in (0,1)$. Incorporating that $\delta \to 1$, $\det J^{\omega}_H > 0$ when

$$\frac{\left(\sigma-\varepsilon^{\omega}_{HH}s^{\omega}_{HH}\right)-\rho^{\omega}_{HH}\left[\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}\right]}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}\left(1-\rho^{\omega}_{HF}\right) > \rho^{\omega}_{HF}\rho^{\omega}_{HH}\left[\frac{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}}{\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)}\right].$$

Taking into account that $1-\rho^{\omega}_{HF}=\rho^{\omega}_{HH}$, this inequality holds if

$$\left(\sigma-\varepsilon^{\omega}_{HH}s^{\omega}_{HH}\right)-\rho^{\omega}_{HH}\left[\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}\right]>\rho^{\omega}_{HF}\left[\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}\right]$$

or, by using that $1=\rho^{\omega}_{HH}+\rho^{\omega}_{HF}$, if

$$\sigma-\varepsilon^{\omega}_{HH}s^{\omega}_{HH}>\varepsilon^{\omega}_{HH}\left(1-s^{\omega}_{HH}\right)-s^{\omega}_{HH}.$$

This inequality can be reexpressed as $\sigma+s^{\omega}_{HH}>\varepsilon^{\omega}_{HH}$ and, since $\sigma>\varepsilon^{\omega}_{HH}$ for any $s^{\omega}_{HH}>0$, the result follows. ∎

Using that $\varepsilon^{\omega}_{ij}\left(1-s^{\omega}_{ij}\right)-s^{\omega}_{ij}>0$ for $i,j\in\mathcal{C}$, it can be easily shown that $\left(\sigma-\varepsilon^{\omega}_{ij}s^{\omega}_{ij}\right)-\delta\rho^{\omega}_{ij}\left[\varepsilon^{\omega}_{ij}\left(1-s^{\omega}_{ij}\right)-s^{\omega}_{ij}\right]>0$ by using that $\delta\rho^{\omega}_{ij}<1$ and $\sigma\geq\varepsilon^{\omega}_{ij}$ with strict inequality if the firm ω is non-negligible. By using these results and Lemma 1, we can determine the sign of all the partial effects. For future references, we incorporate them as a lemma.

Lemma 2. *The following signs hold:*

- *for* (A21): $\frac{\partial\ln p^{\omega}_{HH}}{\partial\ln s^{\omega}_{HH}}=\frac{\partial\ln m^{\omega}_{HH}}{\partial\ln s^{\omega}_{HH}}>0$,
- *for* (A23): $\frac{\partial\ln I^{\omega}_{H}}{\partial\ln s^{\omega}_{HH}}=\frac{\partial\ln z^{\omega}_{H}}{\partial\ln s^{\omega}_{HH}}>0$, $\frac{\partial\ln I^{\omega}_{H}}{\partial\ln s^{\omega}_{HF}}=\frac{\partial\ln z^{\omega}_{H}}{\partial\ln s^{\omega}_{HF}}>0$,
- *for* (A26): $\frac{\partial\ln s^{\omega}_{HH}}{\partial\ln\mathbb{P}_{H}}>0$, $\frac{\partial\ln s^{\omega}_{HF}}{\partial\ln\mathbb{P}_{H}}>0$, $\frac{\partial\ln s^{\omega}_{HH}}{\partial\ln\tau^{\omega}_{HF}}<0$, *and* $\frac{\partial\ln s^{\omega}_{HF}}{\partial\ln\tau^{\omega}_{HF}}<0$.

  Moreover, with those results, it can be shown as a corollary that:
- *for* (A31): $\frac{\partial\ln\overline{\pi}^{\omega}_{H}}{\partial\ln\mathbb{P}_{H}}>0$ *and* $\frac{\partial\ln\overline{\pi}^{\omega}_{H}}{\partial\ln\tau^{\omega}_{HF}}<0$,
- *for* (A32): $\frac{\partial\ln\overline{\Pi}^{\mathscr{L}}_{H}}{\partial\ln\mathbb{P}_{H}}>0$ *and* $\frac{\partial\ln\overline{\Pi}^{\mathscr{L}}_{H}}{\partial\ln\tau^{\omega}_{HF}}<0$.

Lemma 3. $\frac{\mathrm{d}\ln(\mathcal{D}_H+\mathcal{X}_H)}{\mathrm{d}\ln\tau^{\mathcal{N}}_{HF}}<0$.

Proof of Lemma 3. Given the definition of $\mathcal{D}_H$ and $\mathcal{X}_H$, given by (A1) and (A2) respectively, and that $\frac{\mathrm{d}\ln\mathbb{P}_H}{\mathrm{d}\ln\tau^{\mathcal{N}}_{HF}}=\frac{e^{\mathcal{N}}_H}{d^{\mathcal{N}}_H}$ due to (A34),

$$\frac{\mathrm{d}\ln\left(\mathcal{D}_H+\mathcal{X}_H\right)}{\mathrm{d}\ln\tau^{\mathcal{N}}_{HF}}=(\sigma-1)\left(\frac{\mathcal{D}_H}{\mathcal{D}_H+\mathcal{X}_H}\frac{e^{\mathcal{N}}_H}{d^{\mathcal{N}}_H}-\frac{\mathcal{X}_H}{\mathcal{D}_H+\mathcal{X}_H}\right).$$

Given (4), (A6), and (A10), we obtain $\frac{\mathcal{D}_H}{\mathcal{D}_H+\mathcal{X}_H}=\frac{r^{x}_{HH}}{r^{x}_{H}}$ and $\frac{\mathcal{X}_H}{\mathcal{D}_H+\mathcal{X}_H}=\frac{r^{x}_{HF}}{r^{x}_{H}}$. Thus, $\frac{\mathrm{d}\ln(\mathcal{D}_H+\mathcal{X}_H)}{\mathrm{d}\ln\tau^{\mathcal{N}}_{HF}}<0$ iff

$$\frac{r^{x}_{HH}}{r^{x}_{HF}}<\frac{d^{\mathcal{N}}_H}{e^{\mathcal{N}}_H},$$

or, what is same, iff

$$\frac{E_H\left(\mathbb{P}_H\right)^{\sigma-1}}{E_F\left(\mathbb{P}_F\tau^{\mathcal{N}}_{HF}\right)^{\sigma-1}}<\frac{E_H\left(\mathbb{P}_H\right)^{\sigma-1}}{E_F\left(\mathbb{P}_F\tau^{\mathcal{N}}_{HF}\right)^{\sigma-1}}\frac{\int_{\varphi^{*}_{HH}}^{\varphi^{*}_{HF}}\varphi^{\sigma-1}\mathrm{d}G_H\left(\varphi\right)+\int_{\varphi^{*}_{HF}}^{\overline{\varphi}_H}\varphi^{\sigma-1}\mathrm{d}G_H\left(\varphi\right)}{\int_{\varphi^{*}_{HF}}^{\overline{\varphi}_H}\varphi^{\sigma-1}\mathrm{d}G_H\left(\varphi\right)},$$

which always holds. ∎

A.3.1 Benchmark Results

Next, we prove the results in Section 4.1, where we consider a setting as in Melitz. This setting arises by assuming that the set of LFs in each country is empty.

Proof of Proposition 1. Following a change in export trade costs, the change in the price index is given by (A34), irrespective if there are LFs or not. Thus, $\frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} = \frac{e_H^{\mathcal{N}}}{d_H^{\mathcal{N}}}$. We begin by analyzing what occurs with the choices in quality, according to the type of firm.

Consider a SF from H with productivity φ that serves home exclusively before and after the export trade shock. This firm chooses optimal quality through (A6), which using $\frac{\partial \ln \mathcal{D}_H}{\partial \ln \mathbb{P}_H} = \sigma - 1$ implies that

$$\frac{\mathrm{d}\ln I_H^d}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} = \frac{\sigma-1}{1-\delta}\frac{e_H^{\mathcal{N}}}{d_H^{\mathcal{N}}} > 0.$$

Thus, this firm reduces its investment.

Consider a SF from H with productivity φ that exports before and after the export shock. Then, it chooses quality in both scenarios through (A10), and so

$$\frac{\mathrm{d}\ln I_H^x}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} = \frac{1}{1-\delta}\frac{\mathrm{d}\ln\left(\mathcal{D}_H + \mathcal{X}_H\right)}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} < 0,$$

where the sign follows by Lemma 3. Therefore, this firm increases its investment.

Finally, consider a SF from H that changes the total markets served following the export shock. First, we show that this only entails the case where a firm exclusively serving home before the export shock becomes an exporter. To show this, we prove that the productivity cutoff to export, given by (A15), decreases following the export shock. Defining $\mathcal{A}_H := \left(\mathcal{D}_H + \mathcal{X}_H\right)^{\frac{1}{1-\delta}} - \left(\mathcal{D}_H\right)^{\frac{1}{1-\delta}}$, we obtain that

$$\frac{\mathrm{d}\mathcal{A}_H}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} := \left(\mathcal{D}_H + \mathcal{X}_H\right)^{\frac{1}{1-\delta}}\frac{1}{1-\delta}\frac{\mathrm{d}\ln\left(\mathcal{D}_H + \mathcal{X}_H\right)}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} - \left(\mathcal{D}_H\right)^{\frac{1}{1-\delta}}\frac{\sigma-1}{1-\delta}\frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}}.$$

Then, $\frac{\mathrm{d}\mathcal{A}_H}{\mathrm{d}\tau_{HF}^{\mathcal{N}}} < 0$ by using Lemma 3 and (A34), which implies that $\frac{\mathrm{d}\varphi_{HF}^*}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} > 0$. Therefore, if the export shock changes the markets served by a firm, this occurs only when a domestic firm becomes an exporter. Second, we show that this firm increases its investment. To do this, denote the equilibrium value of a variable in the scenario before the shock with a prime, and after the shock with two primes. By definition of the productivity cutoff to export,

$$\frac{1-\delta}{\sigma}\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}}\left(\varphi_{HF}'\right)^{\frac{\sigma-1}{1-\delta}}\left(\mathcal{D}_H'\right)^{\frac{1}{1-\delta}} = \frac{1-\delta}{\sigma}\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}}\left(\varphi_{HF}'\right)^{\frac{\sigma-1}{1-\delta}}\left(\mathcal{D}_H' + \mathcal{X}_H'\right)^{\frac{1}{1-\delta}} - f_{HF}. \tag{A35}$$

Moreover, we have shown $\frac{\mathrm{d}\ln(\mathcal{D}_H + \mathcal{X}_H)}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} < 0$, so that following a small decrease in the export trade shock:

$$\frac{1-\delta}{\sigma}\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}}\left(\varphi_{HF}'\right)^{\frac{\sigma-1}{1-\delta}}\left(\mathcal{D}_H' + \mathcal{X}_H'\right)^{\frac{1}{1-\delta}} < \frac{1-\delta}{\sigma}\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}}\left(\varphi_{HF}'\right)^{\frac{\sigma-1}{1-\delta}}\left(\mathcal{D}_H'' + \mathcal{X}_H''\right)^{\frac{1}{1-\delta}}. \tag{A36}$$

Thus, $\left(\mathcal{D}_H'\right)^{\frac{1}{1-\delta}} < \left(\mathcal{D}_H'' + \mathcal{X}_H''\right)^{\frac{1}{1-\delta}}$ by combining (A35) and (A36). With this result, we use that a domestic firm only serving home decides quality by (A6), and that it decides quality by (A10) when it starts exporting. Consequently, $\left(\mathcal{D}_H'\right)^{\frac{1}{1-\delta}} < \left(\mathcal{D}_H'' + \mathcal{X}_H''\right)^{\frac{1}{1-\delta}}$ implies that the firm upgrades quality.

As for profits, we derive the result for each type of firm. A firm serving home before and after the shock has lower profits since it is impacted in equilibrium only by a reduction in $\mathbb{P}_H$. As for a firm with productivity φ that only serves home before the export shock but starts exporting after, the result follows by using $\left(\mathcal{D}_H'\right)^{\frac{1}{1-\delta}} < \left(\mathcal{D}_H'' + \mathcal{X}_H''\right)^{\frac{1}{1-\delta}}$ and (A35), which implies that

$$\frac{1-\delta}{\sigma}\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}}\left(\varphi\right)^{\frac{\sigma-1}{1-\delta}}\left(\mathcal{D}_H'\right)^{\frac{1}{1-\delta}} < \frac{1-\delta}{\sigma}\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}}\left(\varphi\right)^{\frac{\sigma-1}{1-\delta}}\left(\mathcal{D}_H'' + \mathcal{X}_H''\right)^{\frac{1}{1-\delta}} - f_{HF}.$$

Finally, a firm that exports before and after the export shock has greater profits by (A36). ■

A.3.2 Export Shock to SFs

Next, we consider the results in Section 4.2. We begin by presenting a lemma.

Lemma 4. $\frac{\partial \ln s^{\omega}_{FH}}{\partial \ln \mathbb{P}_H} > 0$

Proof of Lemma 4. Optimal market shares can be obtained by using equation (A24) and that optimal prices and quality are given, respectively, by (A3) and (A5). Proceeding in a similar fashion as in the derivation for (A26), we can differentiate the system (A24) for LFs from F and obtain that:

$$\begin{pmatrix} \frac{(\sigma-\varepsilon^{\omega}_{FF}s^{\omega}_{FF})-\delta\rho^{\omega}_{FF}\left[\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}\right]}{\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)} & -\delta\rho^{\omega}_{FH}\frac{\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)-s^{\omega}_{FH}}{\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)} \\ -\delta\rho^{\omega}_{FF}\left[\frac{\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}}{\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)}\right] & \frac{(\sigma-\varepsilon^{\omega}_{FH}s^{\omega}_{FH})-\delta\rho^{\omega}_{FH}\left[\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)-s^{\omega}_{FH}\right]}{\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)} \end{pmatrix} \begin{pmatrix} \frac{\partial \ln s^{\omega}_{FF}}{\partial \ln \mathbb{P}_H} \\ \frac{\partial \ln s^{\omega}_{FH}}{\partial \ln \mathbb{P}_H} \end{pmatrix} = \begin{pmatrix} 0 \\ \sigma - 1 \end{pmatrix}.$$

We have assumed in the main part of the paper that $\varepsilon^{\omega}_{Fk}\left(1-s^{\omega}_{Fk}\right)-s^{\omega}_{Fk} > 0$ for $k \in \mathcal{C}$. Given this, $\arg\inf_{\delta}\left(\det J^{\omega}_{F}\right) = 1$. Thus, if we show that $\det J^{\omega}_{F} > 0$ when $\delta \to 1$, the result follows for any $\delta \in (0,1)$. This holds when

$$\frac{(\sigma-\varepsilon^{\omega}_{FF}s^{\omega}_{FF})-\rho^{\omega}_{FF}\left[\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}\right]}{\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)}\frac{(\sigma-\varepsilon^{\omega}_{FH}s^{\omega}_{FH})-\rho^{\omega}_{FH}\left[\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)-s^{\omega}_{FH}\right]}{\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)} > \rho^{\omega}_{FH}\frac{\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)-s^{\omega}_{FH}}{\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)}\rho^{\omega}_{FF}\left[\frac{\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}}{\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)}\right],$$

and a sufficient condition for this to hold is that the following inequalities hold simultaneously:

$$\begin{aligned} (\sigma-\varepsilon^{\omega}_{FF}s^{\omega}_{FF})-\rho^{\omega}_{FF}\left[\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}\right] &\geq \rho^{\omega}_{FH}\left[\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}\right], \\ (\sigma-\varepsilon^{\omega}_{FH}s^{\omega}_{FH})-\rho^{\omega}_{FH}\left[\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)-s^{\omega}_{FH}\right] &\geq \rho^{\omega}_{FF}\left[\varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)-s^{\omega}_{FH}\right], \end{aligned}$$

with one of them holding with strict inequality. By using that $\rho^{\omega}_{FF}+\rho^{\omega}_{FH}=1$, this becomes

$$\begin{aligned} \sigma-\varepsilon^{\omega}_{FF}s^{\omega}_{FF} &\geq \varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}, \\ \sigma-\varepsilon^{\omega}_{FH}s^{\omega}_{FH} &\geq \varepsilon^{\omega}_{FH}\left(1-s^{\omega}_{FH}\right)-s^{\omega}_{FH}, \end{aligned}$$

where both are satisfied since $\sigma \geq \varepsilon^{\omega}_{Fk}$ for any $k \in \mathcal{C}$ and one of them has to be holding with strict inequality with non-negligible firms. Therefore, $\det J^{\omega}_{F} > 0$.

Finally, solving the system is determined that

$$\frac{\partial \ln s^{\omega}_{FH}}{\partial \ln \mathbb{P}_H} = \frac{(\sigma-1)}{\det J^{\omega}_{F}}\frac{(\sigma-\varepsilon^{\omega}_{FF}s^{\omega}_{FF})-\delta\rho^{\omega}_{FF}\left[\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}\right]}{\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)},$$

which, by using that $(\sigma-\varepsilon^{\omega}_{FF}s^{\omega}_{FF})-\delta\rho^{\omega}_{FF}\left[\varepsilon^{\omega}_{FF}\left(1-s^{\omega}_{FF}\right)-s^{\omega}_{FF}\right] > 0$, is positive. ■

Proof of Proposition 2. Since only SFs from H have a better export access, then $\mathrm{d}\ln\tau^{\mathcal{N}_H} < 0$. By (A34) we know that $\frac{\mathrm{d}\ln\mathbb{P}^{*}_{H}}{\mathrm{d}\ln\tau^{\mathcal{N}_H}} > 0$, which determines that $\mathbb{P}^{*}_{H}$ decreases.

As for LFs from H, any firm ω is impacted by $\mathbb{P}^{*}_{H}$ exclusively. Thus, the total impact on each variable is given by

$$\mathrm{d}\ln s^{\omega}_{HH} = \left(\frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \mathbb{P}_H}\frac{\mathrm{d}\ln\mathbb{P}^{*}_{H}}{\mathrm{d}\ln\tau^{\mathcal{N}}_{HF}}\right)\mathrm{d}\ln\tau^{\mathcal{N}_H} < 0, \tag{A37a}$$

$$\mathrm{d}\ln I^{\omega}_{H} = \left(\frac{\partial \ln z^{\omega}_{H}}{\partial \ln s^{\omega}_{HH}}\frac{\mathrm{d}\ln s^{\omega}_{HH}}{\mathrm{d}\ln\tau^{\mathcal{N}}_{HF}} + \frac{\partial \ln z^{\omega}_{H}}{\partial \ln s^{\omega}_{HF}}\frac{\partial \ln s^{\omega}_{HF}}{\partial \ln \mathbb{P}_H}\frac{\mathrm{d}\ln\mathbb{P}^{*}_{H}}{\mathrm{d}\ln\tau^{\mathcal{N}}_{HF}}\right)\mathrm{d}\ln\tau^{\mathcal{N}_H} < 0, \tag{A37b}$$

$$\mathrm{d}\ln p^{\omega}_{HH} = \left(\frac{\partial \ln p^{\omega}_{HH}}{\partial \ln s^{\omega}_{HH}} \frac{\mathrm{d}\ln s^{\omega}_{HH}}{\mathrm{d}\ln \tau^{\mathcal{N}}_{HF}}\right) \mathrm{d}\ln \tau^{\mathcal{N}_H} < 0, \tag{A37c}$$

$$\mathrm{d}\ln \overline{\pi}^{\omega}_{H} = \left(\frac{\partial \ln \overline{\pi}^{\omega}_{H}}{\partial \ln \mathbb{P}_H} \frac{\mathrm{d}\ln \mathbb{P}^{*}_{H}}{\mathrm{d}\ln \tau^{\mathcal{N}}_{HF}}\right) \mathrm{d}\ln \tau^{\mathcal{N}_H} < 0, \tag{A37d}$$

where we have used that $\frac{\partial \ln \tau^{\mathcal{N}}_{HF}}{\partial \ln \tau^{\mathcal{N}_H}} = 1$, and the signs follow by using Lemma 2 and (A34). This determines that each LF from H invests less in quality, decreases its domestic prices/markups, garners lower gross profits, and loses domestic market share. Moreover, since all LFs have lower gross profits and market fixed costs did not vary, the total profits are lower too. Consequently, the total profits of LFs from H as a group is lower too.

As for SFs from H, we begin by showing that the domestic survival productivity cutoff, φ^{*}_{HH}, increases. This is given by (A9), and noticing that $\frac{\mathrm{d}\ln \varphi^{*}_{HH}}{\mathrm{d}\ln \mathbb{P}_H} = -1$ and that $\mathbb{P}^{*}_{H}$ decreases, the result follows.

Moreover, to show that SFs from H gain domestic market share, we make use of (A17) for H. Notice that this equation is not affected by variations in $\tau^{\mathcal{N}_H}$. Moreover, given that H is a small economy, $\left(\mathbb{P}^{*}_{F}, M^{E*}_{F}\right)$ does not vary. Reexpressing it and stating it as a function of only the variables that change, (A17) for H becomes

$$s^{\mathcal{N}}_{HH}\left(\mathbb{P}^{*}_{H}, M^{E*}_{H}\right) + s^{\mathcal{N}}_{FH}\left(\mathbb{P}^{*}_{H}\right) + \sum_{k\in\{H,F\}} \sum_{\omega\in\Omega^{\mathcal{L}}_{kH}} s^{\omega}_{kH}\left(\mathbb{P}^{*}_{H}\right) = 1.$$

Besides, differentiating it,

$$\mathrm{d}s^{\mathcal{N}}_{HH} + \mathrm{d}s^{\mathcal{N}}_{FH} + \sum_{k\in\{H,F\}} \sum_{\omega\in\Omega^{\mathcal{L}}_{kH}} \mathrm{d}s^{\omega}_{kH} = 0.$$

Next, we show that $\mathrm{d}s^{\mathcal{N}}_{FH} < 0$, and $\mathrm{d}s^{\omega}_{kH} < 0$ for each $\omega \in \Omega^{\mathcal{L}}_{kH}$ and $k \in \{H, F\}$. First, notice that they are only impacted by changes in $\mathbb{P}^{*}_{H}$. By Lemma 2, we know that $\frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \mathbb{P}_H} > 0$ for any $\omega \in \Omega^{\mathcal{L}}_{HH}$. Moreover, by Lemma 4, $\frac{\partial \ln s^{\omega}_{FH}}{\partial \ln \mathbb{P}_H} > 0$ for $\omega \in \Omega^{\mathcal{L}}_{FH}$. Given that $\mathbb{P}^{*}_{H}$ decreases, this determines that $\sum_{k\in\{H,F\}} \sum_{\omega\in\Omega^{\mathcal{L}}_{kH}} \mathrm{d}s^{\omega}_{kH} < 0$. As for $\mathrm{d}s^{\mathcal{N}}_{FH}$,

$$s^{\mathcal{N}}_{FH} = M^{E*}_{F} \int_{\varphi^{*}_{FH}}^{\overline{\varphi}_F} \frac{\left[p^{\mathcal{N}}_{FH}(\varphi)\right]^{1-\sigma} \left(z^{\mathcal{N}}_{F}\right)^{\delta}}{\left(\mathbb{P}^{*}_{H}\right)^{1-\sigma}} \mathrm{d}G_F(\varphi),$$

and

$$\frac{\mathrm{d}s^{\mathcal{N}}_{FH}}{\mathrm{d}\mathbb{P}_H} = \underbrace{\frac{\partial s^{\mathcal{N}}_{FH}}{\partial \mathbb{P}_H}}_{+} + \underbrace{\frac{\partial s^{\mathcal{N}}_{FH}}{\partial \varphi^{*}_{FH}} \frac{\partial \varphi^{*}_{FH}}{\partial \mathbb{P}_H}}_{+},$$

where we have used that φ^{*}_{FH} is given by (A15). Since $\mathbb{P}^{*}_{H}$ decreases, this determines that $\mathrm{d}s^{\mathcal{N}}_{FH} < 0$. Therefore,

$$\mathrm{d}s^{\mathcal{N}}_{HH} = -\mathrm{d}s^{\mathcal{N}}_{FH} - \sum_{k\in\{H,F\}} \sum_{\omega\in\Omega^{\mathcal{L}}_{kH}} \mathrm{d}s^{\omega}_{kH} > 0,$$

and the result follows. ■

A.3.3 Export Shock to LFs

Proof of Proposition 3. Suppose that each LF ω from H has better export access, so that $\mathrm{d}\ln\tau^{\omega} < 0$. We exploit that $\frac{\partial \ln \tau^{\omega}_{HF}}{\partial \ln \tau^{\omega}} = 1$, which allows us to characterize the total impact through a variation in τ^{ω}_{HF}. The equilibrium price index of H is pinned down by (FE) for H. Thus, since τ^{ω}_{HF} does not affect that condition directly, then $\mathbb{P}^{*}_{H}$ does not vary. Moreover, this determines that φ^{*}_{HH} does not vary either by (A9).

Regarding LF ω from H, since $\mathbb{P}^{*}_{H}$ does not vary, it is only impacted by the variation in τ^{ω}. This determines that the total impact on each variable of ω is given by

$$\mathrm{d}\ln s^{\omega}_{HH} = \left(\frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \tau^{\omega}_{HF}}\right)\mathrm{d}\ln\tau^{\omega} > 0, \tag{A38a}$$

$$\mathrm{d}\ln I^{\omega}_{H} = \left(\frac{\partial \ln z^{\omega}_{H}}{\partial \ln s^{\omega}_{HH}}\frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \tau^{\omega}_{HF}} + \frac{\partial \ln z^{\omega}_{H}}{\partial \ln s^{\omega}_{HF}}\frac{\partial \ln s^{\omega}_{HF}}{\partial \ln \tau^{\omega}_{HF}}\right)\mathrm{d}\ln\tau^{\omega} > 0, \tag{A38b}$$

$$\mathrm{d}\ln p^{\omega}_{HH} = \left(\frac{\partial \ln p^{\omega}_{HH}}{\partial \ln s^{\omega}_{HH}}\frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \tau^{\omega}_{HF}}\right)\mathrm{d}\ln\tau^{\omega} > 0, \tag{A38c}$$

$$\mathrm{d}\ln \overline{\pi}^{\omega}_{H} = \left(\frac{\partial \ln \overline{\pi}^{\omega}_{H}}{\partial \ln \tau^{\omega}_{HF}}\right)\mathrm{d}\ln\tau^{\omega} > 0, \tag{A38d}$$

where we have used that $\frac{\partial \ln \tau^{\omega}_{HF}}{\partial \ln \tau^{\omega}} = 1$. The signs of each of these terms are determined by Lemma 2. Thus, each LF from H invests more in quality, increases its domestic prices/markups, and ends up with greater gross profits and domestic market share. Moreover, since all LFs have greater gross profits and market fixed costs do not vary, total profits increase. As a corollary, the total profits of LFs from H as a group increase too.

As for SFs from H, we need to show that M^{E*}_{H} decreases and that they lose domestic market share. Both can be shown by using (A17) for H. Given that H is a small economy, $\left(\mathbb{P}^{*}_{F}, M^{E*}_{F}\right)$ does not vary. Therefore, (A17) for H can be expressed as

$$s^{\mathcal{N}}_{HH}\left(\mathbb{P}^{*}_{H}, M^{E*}_{H}\right) + s^{\mathcal{N}}_{FH}\left(\mathbb{P}^{*}_{H}\right) + \sum_{\omega\in\Omega^{\mathscr{L}}_{HH}} s^{\omega}_{HH}\left(\mathbb{P}^{*}_{H}, \tau^{\omega}_{HF}\right) + \sum_{\omega\in\Omega^{\mathscr{L}}_{FH}} s^{\omega}_{FH}\left(\mathbb{P}^{*}_{H}\right) = 1.$$

Differentiating the expression,

$$\mathrm{d}s^{\mathcal{N}}_{HH} + \mathrm{d}s^{\mathcal{N}}_{FH} + \sum_{\omega\in\Omega^{\mathscr{L}}_{HH}} \mathrm{d}s^{\omega}_{HH} + \sum_{\omega\in\Omega^{\mathscr{L}}_{FH}} \mathrm{d}s^{\omega}_{FH} = 0.$$

We have already determined that $\mathbb{P}^{*}_{H}$ does not vary. Consequently, $\mathrm{d}s^{\mathcal{N}}_{FH} = \mathrm{d}s^{\omega}_{FH} = 0$ for each $\omega \in \Omega^{\mathscr{L}}_{FH}$. Moreover, By Lemma 2, we know that $\frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \tau^{\omega}_{HF}} < 0$ for each $\omega \in \Omega^{\mathscr{L}}_{HH}$. In addition, $\frac{\partial \ln s^{\mathcal{N}}_{HH}}{\partial \ln M^{E*}_{H}} > 0$. Thus,

$$\frac{\partial \ln s^{\mathcal{N}}_{HH}}{\partial \ln M^{E}_{H}}\mathrm{d}\ln M^{E*}_{H} + \sum_{\omega\in\Omega^{\mathscr{L}}_{HH}} \frac{\partial \ln s^{\omega}_{HH}}{\partial \ln \tau^{\omega}_{HF}}\mathrm{d}\ln\tau^{\omega} = 0,$$

and, since $\mathrm{d}\ln\tau^{\omega} < 0$ for each LF ω from H, it is determined that $\mathrm{d}\ln M^{E*}_{H} < 0$. Thus, M^{E*}_{H} decreases and SFs from H lose domestic market share. ■

A.3.4 Export Shock to All Firms

Proof of Proposition 4. The variation in the price index is given by (A34). Thus, all the results regarding LFs from H can be obtained by utilizing the results in Appendix A.2. Specifically,

$$\mathrm{d}\ln s_{HH}^{\omega} = \left(\frac{\partial \ln s_{HH}^{\omega}}{\partial \ln \mathbb{P}_H} \frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} + \frac{\partial \ln s_{HH}^{\omega}}{\partial \ln \tau_{HF}^{\omega}} \right) \mathrm{d}\ln \tau_{HF} \lesseqgtr 0, \tag{A39a}$$

$$\mathrm{d}\ln I_H^{\omega} = \left[\frac{\partial \ln z_H^{\omega}}{\partial \ln s_{HH}^{\omega}} \frac{\mathrm{d}\ln s_{HH}^{\omega}}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} + \frac{\partial \ln z_H^{\omega}}{\partial \ln s_{HF}^{\omega}} \left(\frac{\partial \ln s_{HF}^{\omega}}{\partial \ln \mathbb{P}_H} \frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} + \frac{\partial \ln s_{HF}^{\omega}}{\partial \ln \tau_{HF}^{\omega}} \right) \right] \mathrm{d}\ln \tau_{HF} \lesseqgtr 0, \tag{A39b}$$

$$\mathrm{d}\ln p_{HH}^{\omega} = \left(\frac{\partial \ln p_{HH}^{\omega}}{\partial \ln s_{HH}^{\omega}} \frac{\mathrm{d}\ln s_{HH}^{\omega}}{\mathrm{d}\ln \tau_{HF}^{\omega}} \right) \mathrm{d}\ln \tau_{HF} \lesseqgtr 0, \tag{A39c}$$

$$\mathrm{d}\ln \overline{\pi}_H^{\omega} = \left(\frac{\partial \ln \overline{\pi}_H^{\omega}}{\partial \ln \mathbb{P}_H} \frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}^{\mathcal{N}}} + \frac{\partial \ln \overline{\pi}_H^{\omega}}{\partial \ln \tau_{HF}^{\omega}} \right) \mathrm{d}\ln \tau_{HF} \lesseqgtr 0, \tag{A39d}$$

where we have used that $\frac{\partial \ln \tau_{HF}^{\mathcal{N}}}{\partial \ln \tau_{HF}} = \frac{\partial \ln \tau_{HF}^{\omega}}{\partial \ln \tau_{HF}} = 1$.As for SFs, all the proofs follow verbatim the proof of Proposition 1. ∎

A.4 Export Intensities in terms of Observables

We have utilized the export intensity of SFs and LFs for our graphical illustrations in Section 4. Here, we show that they are sufficient statistics when variations in export trade costs are infinitesimal. To express the models in terms of observables with infinitesimal variations, we need to compute (A21), (A23), (A26), (A31), (A32), and (A34). In terms of parameters, this requires values for σ and δ.

First, notice we have expressed (A34) in a way that it is completely determined by $e_H^{\mathcal{N}}$. As for (A21), (A23), (A26), and (A31), by inspection of the terms, it is determined that for its computation is necessary to have values for s_{HH}^{ω}, $\varepsilon_{HH}^{\omega}$, ρ_{HH}^{ω}, ρ_{HF}^{ω}, ϕ_{HH}^{ω}, and ϕ_{HF}^{ω} for each LF ω. All these expressions can be calculated by knowledge of the expenditure-based domestic market share and the domestic intensities of each LF ω. In terms of our notation, they correspond to s_{HH}^{ω} and d_H^{ω}. To see this, regarding $\varepsilon_{HH}^{\omega}$, its value is completely determined by s_{HH}^{ω}. As for ρ_{HH}^{ω} and ρ_{HF}^{ω}, by dividing numerator and denominator by $R_{HH}^{\omega} + R_{HF}^{\omega}$, we obtain $\rho_{HH}^{\omega} := \frac{d_H^{\omega}(1-s_{HH}^{\omega})/\varepsilon_{HH}^{\omega}}{d_H^{\omega}(1-s_{HH}^{\omega})/\varepsilon_{HH}^{\omega}+e_H^{\omega}/\sigma}$ and $\rho_{HF}^{\omega} := \frac{e_H^{\omega}/\sigma}{d_H^{\omega}(1-s_{HH}^{\omega})/\varepsilon_{HH}^{\omega}+e_H^{\omega}/\sigma}$, where e_H^{ω} can be computed since $e_H^{\omega} := 1 - d_H^{\omega}$. Similar procedure for ϕ_{HH}^{ω} and ϕ_{HF}^{ω}. Finally, for (A32), we need additionally information of $\widetilde{s}_{HH}^{\omega}$ and $\widetilde{s}_{HF}^{\omega}$ to compute ψ_H^{ω}.

When we compute results under finite changes in export trade costs, we need some additional information. Specifically, this is used to compute (8a), which plays a similar role to (A34) under infinitesimal changes. It represents the impact of an export trade shock on zero expected profits. Next, we show that all the terms in (8a) can be computed by knowledge of $e_H^{\mathcal{N}}$ and e_H^{x}.

To see this, notice that (8a) requires calibrations for $\left(\lambda_H^d\right)'$, $(\lambda_H^x)'$, $(d_H^x)'$ and $(e_H^x)'$. In Manufacturing, these terms can be identified given the information in Table 2, along with a given difference in export intensity between LFs and the SFs that export. As in Alfaro (2020), where the same Danish data is used, this average difference is 9.38%. Thus, we can infer from Table 2 that the LFs' export intensity is 37.49%, determining that $(e_H^x)' := 0.2811$ and so $(d_H^x)' := 1 - 0.2811$. Furthermore, we can use that $(\lambda_H^x)' = \frac{(e_H^{\mathcal{N}})'}{(e_H^x)'} = 0.86$ and so $\left(\lambda_H^d\right)' = 1 - 0.86$.

A.5 Discrete Changes in Export Trade Costs

Next, we derive the system of equations (8), which is used to compute effects under finite changes in export trade costs. We consider export trade costs in H at the initial situation given by $\left(\tau_{HF}^{\mathcal{N}}\right)'$ for SFs and $\left(\tau_{HF}^{\omega}\right)'$ for each LF ω, with common component of export trade costs τ_{HF}'. The results are compared relative to a counterfactual with export trade costs $\left(\tau_{HF}^{\mathcal{N}}\right)''$ for SFs and $\left(\tau_{HF}^{\omega}\right)''$ for each LF ω, with common component of export trade costs τ_{HF}''.

Depending on the experiment under analysis, we keep some of the export trade costs unaltered between both scenarios. Also, as in the main part of the paper, for any variable x, we denote its equilibrium under each set of export trade costs by x' and x'', and express the results by $\widehat{x} := \frac{x''}{x'}$.

We begin by establishing (8a). To do this, we reexpress (FE) for H incorporating the productivity distribution chosen for the numerical exercise:

$$\left[\frac{(1-\delta)}{\sigma} r_H^d \left(\mathbb{P}_H, \varphi^D\right) - f_{HH}\right] \Pr\left(\varphi^D\right) + \left[\frac{(1-\delta)}{\sigma} r_H^x \left(\mathbb{P}_H, \varphi^X, \tau_{HF}^{\mathcal{N}}\right) - f_{HF}\right] \Pr\!\left(\varphi^X\right) = F_H. \tag{A40}$$

Since (A40) holds under trade costs $\left(\tau_{HF}^{\mathcal{N}}\right)'$ and $\left(\tau_{HF}^{\mathcal{N}}\right)''$,

$$\left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \left(\varphi^D\right)^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H'\right)^{\frac{1}{1-\delta}} - f_{HH}\right] \Pr\left(\varphi^D\right) + \left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \left(\varphi^X\right)^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H' + \mathcal{X}_H'\right)^{\frac{1}{1-\delta}} - f_{HH} - f_{HF}\right] \Pr\left(\varphi^X\right) =$$
$$\left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \left(\varphi^D\right)^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H''\right)^{\frac{1}{1-\delta}} - f_{HH}\right] \Pr\left(\varphi^D\right) + \left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \left(\varphi^X\right)^{\frac{\sigma-1}{1-\delta}} \left(\mathcal{D}_H'' + \mathcal{X}_H''\right)^{\frac{1}{1-\delta}} - f_{HH} - f_{HF}\right] \Pr\left(\varphi^X\right).$$

Working out the expression, this becomes

$$1 - \left(\widehat{\mathbb{P}}_H\right)^{\frac{\sigma-1}{1-\delta}} \frac{\left(R_H^d\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'} = \left(\left(\widehat{\mathbb{P}}_H\right)^{\sigma-1} \frac{\left(R_{HH}^x\right)'}{\left(R_H^x\right)'} + \frac{\left(R_{HF}^x\right)'}{\left(R_H^x\right)'} \left(\widehat{\tau}_{HF}^{\mathcal{N}}\right)^{1-\sigma}\right)^{\frac{1}{1-\delta}} \frac{\left(R_H^x\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'}. \tag{A41}$$

Given the definitions included in the main part of the text, it is immediate to see that this determines (8a).

As for domestic prices and markups of LF ω, which are given by (8b), we begin by reexpressing the price elasticity of demand. Expressing $\varepsilon\left(s_{HH}^{\omega}\right) = \sigma + s_{HH}^{\omega}(1-\sigma)$ in terms of differences,

$$\left(\varepsilon_{HH}^{\omega}\right)'' - \left(\varepsilon_{HH}^{\omega}\right)' = \left[\left(s_{HH}^{\omega}\right)'' - \left(s_{HH}^{\omega}\right)'\right](1-\sigma),$$

and, by using that $x'' - x' = x'\left(\widehat{x} - 1\right)$ for any variable x, this can be reexpressed by

$$\widehat{\varepsilon}_{HH}^{\omega} = 1 + \left(1 - \widehat{s}_{HH}^{\omega}\right) \frac{\left(s_{HH}^{\omega}\right)'(\sigma-1)}{\sigma - \left(s_{HH}^{\omega}\right)'(\sigma-1)}.$$

Thus, given that $\widehat{m}_{HH}^{\omega} = \frac{\left(\varepsilon_{HH}^{\omega}\right)''}{\left(\varepsilon_{HH}^{\omega}\right)''-1} \frac{\left(\varepsilon_{HH}^{\omega}\right)'-1}{\left(\varepsilon_{HH}^{\omega}\right)'}$ for markups, then

$$\widehat{p}_{HH}^{\omega} = \widehat{m}_{HH}^{\omega} = \widehat{\varepsilon}_{HH}^{\omega} \frac{\left(\varepsilon_{HH}^{\omega}\right)' - 1}{\widehat{\varepsilon}_{HH}^{\omega}\left(\varepsilon_{HH}^{\omega}\right)' - 1}. \tag{A42}$$

Regarding investments from a LF ω, their variation is given by (8c). Making use of that $R\left(s_{Hk}^{\omega}\right) = E_k s_{Hk}^{\omega}$ for $k \in \mathcal{C}$, then

$$\left(I_H^{\omega}\right)'' - \left(I_H^{\omega}\right)' = \delta E_H \left[\frac{\left(s_{HH}^{\omega}\right)''}{\varepsilon\left[\left(s_{HH}^{\omega}\right)''\right]}\left[1 - \left(s_{HH}^{\omega}\right)''\right] - \frac{\left(s_{HH}^{\omega}\right)'}{\varepsilon\left[\left(s_{HH}^{\omega}\right)'\right]}\left[1 - \left(s_{HH}^{\omega}\right)'\right]\right] + \delta\frac{E_F}{\sigma}\left[\left(s_{HF}^{\omega}\right)'' - \left(s_{HF}^{\omega}\right)'\right]. \tag{A43}$$

The first term on the LHS can be reexpressed as $(I_H^\omega)'' - (I_H^\omega)' = \left(\widehat{I}_H^\omega - 1\right)(I_H^\omega)'$, while the second term on the RHS as $\delta\frac{E_F}{\sigma}\left[(s_{HF}^\omega)'' - (s_{HF}^\omega)'\right] = \delta\frac{R_{HF}^\omega}{\sigma}\left(\widehat{s}_{HF}^\omega - 1\right)$. Moreover, after working out the expression, the first term of the RHS becomes

$$E_H\delta\frac{(s_{HH}^\omega)'}{\left(\varepsilon_{HH}^\omega\right)'}\left[1-(s_{HH}^\omega)'\right]\left[\frac{\widehat{s_{HH}^\omega}}{\widehat{\varepsilon_{HH}^\omega}}\frac{1-(s_{HH}^\omega)'\,\widehat{s_{HH}^\omega}}{1-\left(s_{HH}^\omega\right)'}-1\right],$$

which determines that (A43) is

$$\left(\widehat{I}_H^\omega - 1\right)(I_H^\omega)' = \delta\frac{(R_{HH}^\omega)'}{\left(\varepsilon_{HH}^\omega\right)'}\left[1-(s_{HH}^\omega)'\right]\left[\frac{\widehat{s}_{HH}^\omega}{\widehat{\varepsilon}_{HH}^\omega}\frac{1-\widehat{s}_{HH}^\omega\,(s_{HH}^\omega)'}{1-\left(s_{HH}^\omega\right)'}-1\right]+\delta\frac{(R_{HF}^\omega)'}{\sigma}\left(\widehat{s}_{HF}^\omega-1\right).$$

Finally, dividing by $(I_H^\omega)'$, we can utilize ρ_{HH}^ω, as defined by for the case of infinitesimal variations in export trade costs, determining that

$$\widehat{I}_H^\omega = \widehat{z}_H^\omega = 1 + (\rho_{HH}^\omega)'\left[\frac{\widehat{s}_{HH}^\omega}{\widehat{\varepsilon}_{HH}^\omega}\frac{1-\widehat{s}_{HH}^\omega\,(s_{HH}^\omega)'}{1-\left(s_{HH}^\omega\right)'}-1\right]+(\rho_{HF}^\omega)'\left[\widehat{s}_{HF}^\omega-1\right]. \tag{A44}$$

Regarding market shares of LF ω, it is immediate to obtain (8d) and (8e):

$$\widehat{s}_{HH}^\omega = \frac{\left(\widehat{m}_{HH}^\omega\right)^{1-\sigma}\left(\widehat{z}_H^\omega\right)^\delta}{\left(\widehat{\mathbb{P}}_H\right)^{1-\sigma}},$$

$$\widehat{s}_{HF}^\omega = \left(\widehat{\tau}_{HF}^\omega\right)^{1-\sigma}\left(\widehat{z}_H^\omega\right)^\delta.$$

Regarding gross profits of LF ω, $\overline{\pi}_H^\omega$, their variations are given by (8g). To obtain this expression, we proceed in a similar fashion as for investments. Their difference is given by

$$(\overline{\pi}_H^\omega)'' - (\overline{\pi}_H^\omega)' = E_H\left\{\frac{(s_{HH}^\omega)''}{\left(\varepsilon_{HH}^\omega\right)''}\left[1-\delta\left(1-(s_{HH}^\omega)''\right)\right]-\frac{(s_{HH}^\omega)'}{\left(\varepsilon_{HH}^\omega\right)'}\left[1-\delta\left(1-(s_{HH}^\omega)'\right)\right]\right\}+\frac{E_F}{\sigma}(1-\delta)\left[(s_{HF}^\omega)''-(s_{HF}^\omega)'\right].$$

The first term of the RHS can be reexpressed as

$$\delta E\frac{(s_{HH}^\omega)'}{\left(\varepsilon_{HH}^\omega\right)'}\left[1-\delta\left(1-(s_{HH}^\omega)'\right)\right]\left\{\frac{\widehat{s}_{HH}^\omega}{\widehat{\varepsilon}_{HH}^\omega}\frac{1-\delta\left(1-\widehat{s}_{HH}^\omega\,(s_{HH}^\omega)'\right)}{1-\delta\left(1-\left(s_{HH}^\omega\right)'\right)}-1\right\},$$

and the second term of the RHS by $\frac{E_F}{\sigma}(1-\delta)\left[(s_{HF}^\omega)''-(s_{HF}^\omega)'\right] = \frac{E_F}{\sigma}(1-\delta)(s_{HF}^\omega)'\left(\widehat{s}_{HF}^\omega-1\right)$. This determines that

$$\left(\widehat{\overline{\pi}}_H^\omega - 1\right)(\overline{\pi}_H^\omega)' = \frac{E_H\,(s_{HH}^\omega)'}{\left(\varepsilon_{HH}^\omega\right)'}\left[1-\delta\left(1-(s_{HH}^\omega)'\right)\right]\left\{\frac{\widehat{s}_{HH}^\omega}{\widehat{\varepsilon}_{HH}^\omega}\frac{1-\delta\left(1-\widehat{s}_{HH}^\omega\,(s_{HH}^\omega)'\right)}{1-\delta\left(1-\left(s_{HH}^\omega\right)'\right)}-1\right\}+\frac{E_F\,(s_{HF}^\omega)'}{\sigma}(1-\delta)\left(\widehat{s}_{HF}^\omega-1\right).$$

Using the same definitions for ϕ_{HH}^ω and ϕ_{HF}^ω as in the case of infinitesimal changes, then

$$\widehat{\overline{\pi}}_H^\omega = 1 + (\phi_{HH}^\omega)'\left\{\frac{\widehat{s}_{HH}^\omega}{\widehat{\varepsilon}_{HH}^\omega}\frac{1-\delta\left(1-\widehat{s}_{HH}^\omega\,(s_{HH}^\omega)'\right)}{1-\delta\left(1-\left(s_{HH}^\omega\right)'\right)}-1\right\}+(\phi_{HF}^\omega)'\left(\widehat{s}_{HF}^\omega-1\right). \tag{A45}$$

Finally, we can calculate the increases in total gross profits, which are given by (8h). Multiplying and dividing $(\overline{\pi}_H^\omega)''$ by $(\overline{\pi}_H^\omega)'$, we obtain that $\left(\overline{\Pi}_H^{\mathscr{L}}\right)'' = \sum_{\omega\in\overline{\mathscr{L}}_H}\widehat{\overline{\pi}}_H^\omega\,(\overline{\pi}_H^\omega)'$. Therefore, dividing both sides by $\left(\overline{\Pi}_H^{\mathscr{L}}\right)'$, it is established that

$$\widehat{\overline{\Pi}}_H^{\mathscr{L}} = \sum_{\omega\in\overline{\mathscr{L}}_H}\psi_H^\omega\,\widehat{\overline{\pi}}_H^\omega,$$

where $\psi_H^{\omega} := \frac{\overline{\pi}_H^{\omega}}{\overline{\Pi}_H^{\mathscr{L}}}$.

B Parameters Calibration

For the computation of results, we need values for σ and δ. The latter comes from the estimates by Soderbery (2015). Thus, the rest of this appendix is devoted to the procedure for δ. Since our focus is on LFs, we calibrate this parameter to match features of these firms. Intuitively, we calibrate δ by fitting, as close as possible to the model, each LF's domestic market share variation not explained by its prices. This requires obtaining a measure of quality, for which we follow the intuitions in Khandelwal (2010).

We begin by explaining how we obtain prices, which are necessary for the estimation of δ. By exploiting that our datasets include information on quantities at the 8-digit Combined Nomenclature (henceforth CN8) level, we define prices as unit values. As is well known, unit values constitute an extremely noisy measure of prices. Moreover, as is common in Prodcom datasets, firms are not obliged to report quantities in Denmark. Thus, the data include both missing values and reports of quantities in different units of measure.

To reduce the noise, we clean the data by following standard procedures that use similar datasets (e.g., Amiti and Khandelwal 2013, Amiti et al. 2018, and Piveteau and Smagghue 2019). Using the logarithm of unit values as prices, this is accomplished by performing the following steps:

- by CN8 product, we drop prices that fall below the 5 percentile or above the 95 percentile, and
- by firm-CN8 product, we remove prices that are 150% greater or 66% lower than the previous or subsequent year.

Also, when units are expressed in different but comparable units, we express them in the same unit. For example, if some CN8 is expressed in kilograms and other CN8 in tons, we express both in kilograms.

The procedure defines prices at the firm-product level, whereas we perform an analysis at the firm-industry level. Due to this, it is necessary to aggregate prices at this level. To do this, we calculate prices as a weighted average of prices at the CN8 level for each firm-industry, with weights given by the contribution of each CN8 product to the firm's revenue. Also, we remove industries where at least one Danish LF does not report quantities or at least one CN8 is not expressed in comparable units.

With these prices, we proceed to estimate δ. To do this, we begin by expressing (2) in logarithms, which determines that the market share of a Danish LF producing variety ω in the industry n is

$$\ln s_{\omega n} = (1 - \sigma_n) \ln p_{\omega n} + \delta \ln z_{\omega n} - \ln \mathbb{A}_n. \tag{B1}$$

Regarding each of these terms, the domestic market share and domestic prices, $s_{\omega n}$ and $p_{\omega n}$, are obtained from the Danish data. Moreover, σ_n comes from the estimates by Soderbery (2015) aggregated at the industry level by expenditure weights, while $\mathbb{A}_n$ is treated as fixed effect. As for $z_{\omega n}$, we make use of (6) and reexpress it in the following way:

$$z_{\omega n} := Y_n \frac{\delta}{f^z} \left[\frac{\widetilde{s}_{\omega n}^D}{\varepsilon (s_{\omega n})} (1 - s_{\omega n}) + \frac{\widetilde{s}_{\omega n}^X}{\sigma} \right],$$

where Y_n is the revenue in industry n, $\varepsilon (s_{\omega n})$ is the price elasticity of firm-industry (ω, n) at home, and $\widetilde{s}^{D}_{\omega n}$ and $\widetilde{s}^{X}_{\omega n}$ are the domestic and export revenue share of firm-industry (ω, n). Adding an error term $\varepsilon_{\omega n}$, this implies that

$$\ln \nu_{\omega n} = \delta \ln z_{\omega n} - \ln \mathbb{A}_n + \varepsilon_{\omega n}, \tag{B2}$$

where $\ln \nu_{\omega n} := \ln s_{\omega n} - (1-\sigma_n) \ln p_{\omega n}$. Finally, since some of the terms in $z_{\omega n}$ are industry-specific, (B2) can be equivalently expressed as

$$\ln \nu_{\omega n} = \Lambda_n + \delta \ln \xi_{\omega n} + \varepsilon_{\omega n}, \tag{B3}$$

where $\xi_{\omega n} := \frac{\widetilde{s}^{D}_{\omega n}}{\varepsilon(s_{\omega n})}(1-s_{\omega n}) + \frac{\widetilde{s}^{X}_{\omega n}}{\sigma}$ and $\Lambda_n := \delta \ln\left(\frac{Y_n \delta}{f^z}\right) - \ln \mathbb{A}_n$.

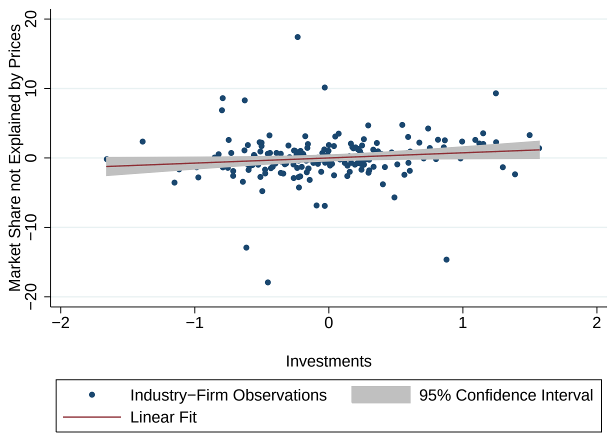
A value for δ is obtained by regressing (B3). The results of the fit are presented in Figure B.1, which indicates two results. The first one shows the estimation of δ when we utilize the values of σ by Soderbery (2015) for each industry. We also include results with a given sigma $\sigma := 3.53$ for comparison. This is the value used for the empirical analysis, which corresponds to manufacturing under industry-revenue weights.

Figure B.1. *Estimation of δ*

(a) Results

	$\ln \nu_{\omega n}$ (1)	$\ln \nu_{\omega n}$ (2)
$\ln \xi_{\omega n}$	0.677	0.742**
	(0.437)	(0.447)
Industry FE	Yes	Yes
σ	Variable	Fixed
N	211	211
R^2	0.991	0.882

(b) Scatter Plot with Industry-Demeaned Variables



C Machinery

In the main part of the paper, we provided results for the top sectors by their contribution to total manufacturing income, expenditures, and exports. Given the information provided in Table 1, these sectors were Food & Beverages, Chemicals, and Machinery. While results for the first two industries were included in the main body of the paper, it remains to present the outcomes for Machinery. We do this next.

Compared to manufacturing, concentration in machinery is substantially lower. This occurs due to the existence of a large number of SFs operating in the sector, with an average number of 76 firms in industries of the sector. In addition, the export intensity of each type of firm is high: SFs have a greater export intensity than even SFs in Chemicals, and each LF has an export intensity of at least 50%.

Export shocks to SFs and to all firms determine reductions in the price index that double the magnitude of manufacturing. Nevertheless, even when the characterization of this industry is somewhat different from manufacturing, the qualitative results for LFs are quite similar. In particular, concerning an export shock to all firms, all LFs increase their investments and garner greater profits. Moreover, the top firm increases its domestic presence and prices, while the opposite happens with the rest of the firms. The only difference relative to manufacturing is that, overall, this shock determines that LFs lose domestic market share as a group.

Table C.1. *Impact of a 1% Reduction in Export Trade Costs - Machinery*

(a) Impact on each LF

Better Export Access	**Firm**	**Domestic Market Share** Change (p.p.)	**Domestic Prices/Markups** Change (%)	**Quality Investments** Change (%)	**Export Revenues** Change (%)	**Total Gross Profits** Change (%)
For All Firms	Top 1	-0.02	-0.01	38.08	62.57	34.58
	Top 2	-0.74	-0.22	5.20	35.13	2.40
	Top 3	-0.39	-0.12	12.84	41.73	11.00
	Top 4	-0.15	-0.04	24.68	51.67	23.48
Only For LFs	Top 1	2.81	0.94	65.00	83.51	67.49
	Top 2	1.17	0.26	42.91	66.42	44.31
	Top 3	0.94	0.28	49.87	71.89	50.98
	Top 4	0.81	0.24	59.33	79.19	60.21
Only For SFs	Top 1	-2.27	-0.72	-20.22	-14.24	-23.09
	Top 2	-1.61	-0.49	-29.95	-21.50	-32.28
	Top 3	-1.09	-0.32	-28.30	-20.24	-29.82
	Top 4	-0.75	-0.22	-25.31	-18.00	-26.30

(b) Impact on LFs as Group

Better Export Access	**Domestic Market Share** Change (p.p.)	**Total Gross Profits** Change (%)
For All Firms	-1.31	23.08
Only For LFs	5.72	59.36
Only For SFs	-5.72	-26.38

D Sensitivity Analysis

We have computed the quantitative results relying on two assumptions: a discrete productivity distribution for SFs and an arbitrary number of firms taken as large. The goal was to compute results through a few statistics, and so lay bare the crucial role of the firms' export intensity in determining outcomes. In the following, we illustrate that the results are robust to these aspects, by recomputing outcomes for manufacturing.

D.1 Cutoff of Firms

We start by showing that aggregate outcomes are insensitive to the cutoff defining a firm as large. This occurs because the magnitude in which a LF affects the industry depends on its size, measured through its market and revenue shares. As a consequence, except for the very top firms, the rest of the firms have a small impact on aggregate outcomes. This can be observed in Table D.1, where we present results restricting LFs to the top 3 and top 2 firms.[18]

Table D.1. *Aggregate Outcomes Following a 10% Reduction in Export Trade Costs for all Firms - Different Sets of Large Firms*

Better Export Access for All Firms	**Baseline**	**Top 3 LFs**	**Top 2 LFs**
Domestic Market Share (Change p.p)	0.82	0.84	0.94
Total Gross Profits (Change %)	20.48	21.03	22.57

D.2 Bounded Pareto Distribution

Next, we show that the same qualitative conclusions as in the baseline model hold under a bounded Pareto distribution for SFs: better export opportunities imply a reallocation of domestic market share towards LFs and an increase in the profits of LFs as a group. In fact, each LF is qualitatively impacted in the same way as in the baseline model.

The result is explained because the specific choice of productivity distribution for SFs has second-order effects under small changes in export trade costs. Recall that bounding the variations in export trade costs is necessary for our results. Otherwise, either all SFs or LFs could stop operating, whereas our results correspond to a market structure with coexistence of both types of firms.

To see why this occurs, notice that the features of SFs only matter insofar as they affect H's price index. This variable is in turn identified by (FE), where $\mathrm{d}\ln \mathbb{P}_H = \frac{e_H^{\mathcal{N}}}{d_H^{\mathcal{N}}} \mathrm{d}\ln \tau_{HF}^{\mathcal{N}}$ for an infinitesimal change of export trade costs. Thus, once we calibrate the export intensity of SFs, the magnitude in which H's price index decreases is independent of the SFs' productivity distribution. The intuition for why the SFs' productivity distribution has a second-order effect is similar to what occurs with trade liberalization between two symmetric large countries (see, for instance, Melitz and Redding 2015 and Arkolakis et al. 2019). Conditional on $e_H^{\mathcal{N}}$, the SFs' productivity distribution only affects (FE) through changes in the productivity cutoffs. However, marginal entrants have zero profits and hence a negligible impact on expected profits.

Quantifying results under a continuous distribution has the issue that the tractable case of an unbounded Pareto distribution is not feasible, since no SF can be more productive than a LF. Taking this into account, we choose a bounded Pareto distribution. Dispensing with an unbounded Pareto makes the calibration more challenging, as Head et al. (2014) notice for the canonical Melitz model. It implies in particular that several additional variables and parameters need to be calibrated, and some of them are hard to identify in the data. Additionally, the application of the hat-algebra procedure ends up determining a quite complex system of equations.

[18] The same conclusion holds for reductions in export trade costs affecting one type of firm in isolation. For instance, consider the top 3 firms as large. A 10% reduction in the export trade costs of LFs determines that variations of the LFs' domestic market share and total gross profits are respectively 5.2 p.p. and 43.1%. These numbers are 6.9 p.p. and 41.3% in our baseline case. Furthermore, these numbers for a 10% reduction in the export trade costs of SFs become -4.8 p.p. and 16.7%, while they are -5.5 p.p. and -16.8% in the baseline case.

For the analysis, consider H's export trade costs initially given by $\left(\tau_{HF}^{\mathcal{N}}\right)'$ for SFs and $(\tau_{HF}^{\omega})'$ for each LF ω, with common component τ_{HF}'. This scenario is compared relative a situation where export trade costs become $\left(\tau_{HF}^{\mathcal{N}}\right)''$ for SFs and $(\tau_{HF}^{\omega})''$ for each LF ω, with common component τ_{HF}''.

The assumption of a bounded Pareto determines that $a := 1/\varphi$ is distributed with support $[\underline{a}, \overline{a}]$ and cdf $\widetilde{G}(a) := \frac{(a)^k-(\underline{a})^k}{(\overline{a})^k-(\underline{a})^k}$. Moreover, it implies that $\int_{a_1}^{a_2} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a) = \frac{k}{\gamma} \frac{(\underline{a})^{\gamma}}{\left[(\overline{a})^k-(\underline{a})^k\right]} \left[\left(\frac{a_2}{\underline{a}}\right)^{\gamma} - \left(\frac{a_1}{\underline{a}}\right)^{\gamma}\right]$ where $\gamma := k + \frac{1-\sigma}{1-\delta}$. We denote the inverse of the productivity cutoff in country $j \in \mathcal{C}$ by a_{Hj}.

Equation (FE) for each set of trade costs implies

$$\begin{aligned}\int_{a_{HF}'}^{a_{HH}'} \left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} a^{\frac{1-\sigma}{1-\delta}} \left(\mathcal{D}_H'\right)^{\frac{1}{1-\delta}} - f_{HH}\right] \mathrm{d}\widetilde{G}(a) + \int_{\underline{a}}^{a_{HF}'} \left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} a^{\frac{1-\sigma}{1-\delta}} \left(\mathcal{D}_H' + \mathcal{X}_H'\right)^{\frac{1}{1-\delta}} - f_{HH} - f_{HF}\right] \mathrm{d}\widetilde{G}(a) = \\ \int_{a_{HF}''}^{a_{HH}''} \left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} a^{\frac{1-\sigma}{1-\delta}} \left(\mathcal{D}_H''\right)^{\frac{1}{1-\delta}} - f_{HH}\right] \mathrm{d}\widetilde{G}(a) + \int_{\underline{a}}^{a_{HF}''} \left[\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} a^{\frac{1-\sigma}{1-\delta}} \left(\mathcal{D}_H'' + \mathcal{X}_H''\right)^{\frac{1}{1-\delta}} - f_{HH} - f_{HF}\right] \mathrm{d}\widetilde{G}(a).\end{aligned} \tag{D1}$$

To reexpress this, we use that

$$\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \left\{\int_{a_{HF}'}^{a_{HH}'} \left[a^{\frac{1-\sigma}{1-\delta}} \left(\mathcal{D}_H'\right)^{\frac{1}{1-\delta}}\right] \mathrm{d}\widetilde{G}(a) - \int_{a_{HF}''}^{a_{HH}''} \left[a^{\frac{1-\sigma}{1-\delta}} \left(\mathcal{D}_H''\right)^{\frac{1}{1-\delta}}\right] \mathrm{d}\widetilde{G}(a)\right\} = \left[\left(\widehat{\mathbb{P}}_H\right)^{\frac{\sigma-1}{1-\delta}} \frac{\int_{a_{HF}''}^{a_{HH}''} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)}{\int_{a_{HF}'}^{a_{HH}'} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)} - 1\right] \frac{1-\delta}{\sigma} \frac{\left(R_H^d\right)'}{\left(M_H^E\right)'},$$

$$\left(\alpha\beta^{\delta}\right)^{\frac{1}{1-\delta}} \left\{\int_{\underline{a}}^{a_{HF}''} \left[a^{\frac{1-\sigma}{1-\delta}} \left(\mathcal{D}_H'' + \mathcal{X}_H''\right)^{\frac{1}{1-\delta}}\right] \mathrm{d}\widetilde{G}(a) - \int_{\underline{a}}^{a_{HF}'} \left[a^{\frac{1-\sigma}{1-\delta}} \left(\mathcal{D}_H' + \mathcal{X}_H'\right)^{\frac{1}{1-\delta}}\right] \mathrm{d}\widetilde{G}(a)\right\} = \left\{\left[\frac{\left(R_{HH}^x\right)' \left(\widehat{\mathbb{P}}_H\right)^{\sigma-1} + \left(R_{HF}^x\right)' \left(\widehat{\tau}_{HF}^{\mathcal{N}}\right)^{1-\sigma}}{\left(R_H^x\right)'}\right]^{\frac{1}{1-\delta}} \frac{\int_{\underline{a}}^{a_{HF}''} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)}{\int_{\underline{a}}^{a_{HF}'} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)} - 1\right\} \frac{1-\delta}{\sigma} \left[\frac{\left(R_H^x\right)'}{\left(M_H^E\right)'}\right],$$

$$\int_{a_{HF}''}^{a_{HH}''} f_{HH} \mathrm{d}\widetilde{G}(a) - \int_{a_{HF}'}^{a_{HH}'} f_{HH} \mathrm{d}\widetilde{G}(a) + \int_{\underline{a}}^{a_{HF}''} \left(f_{HH} + f_{HF}\right) \mathrm{d}\widetilde{G}(a) - \int_{\underline{a}}^{a_{HF}'} \left(f_{HH} + f_{HF}\right) \mathrm{d}\widetilde{G}(a) = \left\{\left(\frac{\left(\frac{a_D''}{a_D'}\right)^k - \left(\frac{a_{HF}''}{a_{HH}'}\right)^k}{1 - \left(\frac{a_{HF}'}{a_{HH}'}\right)^k} - 1\right) \frac{\int_{a_{HF}'}^{a_{HH}'} f_{HH} \mathrm{d}\widetilde{G}(a)}{\left(R_H^d\right)'} \left(R_H^d\right)' + \left[\frac{\left(\frac{a_{HF}''}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^k - 1}{\left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^k - 1} - 1\right] \frac{\int_{\underline{a}}^{a_{HF}'} \left(f_{HH} + f_{HF}\right) \mathrm{d}\widetilde{G}(a)}{\left(R_H^x\right)'} \left(R_H^x\right)'\right\},$$

After some algebra and using the results just stated, (D1) becomes

$$\begin{aligned}&\left[\left(\widehat{\mathbb{P}}_H\right)^{\frac{\sigma-1}{1-\delta}} \frac{\int_{a_{HF}''}^{a_{HH}''} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)}{\int_{a_{HF}'}^{a_{HH}'} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)} - 1\right] \frac{1-\delta}{\sigma} \frac{\left(R_H^d\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'} + \left\{\left[\frac{\left(R_{HH}^x\right)' \left(\widehat{\mathbb{P}}_H\right)^{\sigma-1} + \left(R_{HF}^x\right)' \left(\widehat{\tau}_{HF}^{\mathcal{N}}\right)^{1-\sigma}}{\left(R_H^x\right)'}\right]^{\frac{1}{1-\delta}} \frac{\int_{\underline{a}}^{a_{HF}''} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)}{\int_{\underline{a}}^{a_{HF}'} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)} - 1\right\} \frac{1-\delta}{\sigma} \frac{\left(R_H^x\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'} = \\ &\qquad \left(\frac{\left(\frac{a_D''}{a_D'}\right)^k - \left(\frac{a_{HF}''}{a_{HH}'}\right)^k}{1 - \left(\frac{a_{HF}'}{a_{HH}'}\right)^k} - 1\right) \frac{F_H^d}{\left(R_H^d\right)'} \frac{\left(R_H^d\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'} + \left[\frac{\left(\frac{a_{HF}''}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^k - 1}{\left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^k - 1} - 1\right] \frac{\left(F_H^x\right)'}{\left(R_H^x\right)'} \frac{\left(R_H^x\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'},\end{aligned} \tag{D2}$$

where F_H^d and F_H^x are the total fixed costs incurred by only domestic and exporter SFs, respectively. Given the bounded Pareto distribution, $\frac{\int_{\underline{a}}^{a_{HF}''} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)}{\int_{\underline{a}}^{a_{HF}'} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)} = \frac{\left(\frac{a_{HF}''}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - 1}{\left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - 1}$, $\frac{\int_{a_{HF}''}^{a_{HH}''} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)}{\int_{a_{HF}'}^{a_{HH}'} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)} = \frac{\left(\frac{a_{HH}''}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - \left(\frac{a_{HF}''}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma}}{\left(\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - \left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma}}$, and $\frac{\int_{a_{HF}'}^{a_{HH}'} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)}{\int_{\underline{a}}^{a_{HF}'} a^{\frac{1-\sigma}{1-\delta}} \mathrm{d}\widetilde{G}(a)} = \frac{\left(\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - \left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma}}{\left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - 1}$. Incorporating this, (D2) becomes

$$\begin{aligned}&\left[\left(\widehat{\mathbb{P}}_H\right)^{\frac{\sigma-1}{1-\delta}} \frac{\left(\widehat{\mathbb{P}}_H \frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - \left(\frac{a_{HF}''}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma}}{\left(\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - \left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma}} - 1\right] \frac{1-\delta}{\sigma} \frac{\left(R_H^d\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'} + \left\{\left[\frac{\left(R_{HH}^x\right)' \left(\widehat{\mathbb{P}}_H\right)^{\sigma-1} + \left(R_{HF}^x\right)' \left(\widehat{\tau}_{HF}^{\mathcal{N}}\right)^{1-\sigma}}{\left(R_H^x\right)'}\right]^{\frac{1}{1-\delta}} \frac{\left(\frac{a_{HF}''}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - 1}{\left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^{\gamma} - 1} - 1\right\} \frac{1-\delta}{\sigma} \frac{\left(R_H^x\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'} = \\ &\qquad \left(\frac{\left(\frac{a_D''}{a_D'}\right)^k - \left(\frac{a_{HF}''}{a_{HH}'}\right)^k}{1 - \left(\frac{a_{HF}'}{a_{HH}'}\right)^k} - 1\right) \frac{\left(F_H^d\right)'}{\left(R_H^d\right)'} \frac{\left(R_H^d\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'} + \left[\frac{\left(\frac{a_{HF}''}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^k - 1}{\left(\frac{a_{HF}'}{a_{HH}'}\frac{a_{HH}'}{\underline{a}}\right)^k - 1} - 1\right] \frac{\left(F_H^x\right)'}{\left(R_H^x\right)'} \frac{\left(R_H^x\right)'}{\left(R_H^d\right)' + \left(R_H^x\right)'}.\end{aligned} \tag{D3}$$

Equation (D2) makes use of ratios between inverses of productivity cutoffs. They are given by

$$\frac{a_{HH}''}{a_{HH}'} = \widehat{\mathbb{P}}_H, \tag{D4a}$$

$$\frac{a''_{HF}}{a'_{HF}} = \left\{ \frac{\left(\left(\widehat{\mathbb{P}}_H\right)^{\sigma-1} + \left(\widehat{\tau}^{\mathcal{N}}_{HF}\right)^{1-\sigma} \frac{\left(R^x_{HF}\right)'}{\left(R^x_{HH}\right)'}\right)^{\frac{1}{1-\delta}} - \left[\left(\widehat{\mathbb{P}}_H\right)^{\sigma-1}\right]^{\frac{1}{1-\delta}}}{\left(1 + \frac{\left(R^x_{HF}\right)'}{\left(R^x_{HH}\right)'}\right)^{\frac{1}{1-\delta}} - 1} \right\}^{\frac{1-\delta}{\sigma-1}}, \tag{D4b}$$

$$\frac{a'_{HF}}{a'_{HH}} = \left(\frac{f_{HH}}{f_{HF}}\right)^{\frac{1-\delta}{\sigma-1}} \left[\left(1 + \frac{\left(R^x_{HF}\right)'}{\left(R^x_{HH}\right)'}\right)^{\frac{1}{1-\delta}} - 1\right]^{\frac{1-\delta}{\sigma-1}}, \tag{D4c}$$

$$\frac{a''_{HF}}{a'_{HH}} = \frac{a''_{HF}}{a'_{HF}} \frac{a'_{HF}}{a'_{HH}}. \tag{D4d}$$

In summary, the system of equations to take to the data comprises equations (8b)–(8j), (D3), and (D4). The computation requires calibrating several parameters and terms. First, $\frac{\left(R^d_H\right)'}{\left(R^d_H\right)' + \left(R^x_H\right)'}$, $\frac{\left(R^x_H\right)'}{\left(R^d_H\right)' + \left(R^x_H\right)'}$, $\frac{\left(R^x_{HH}\right)'}{\left(R^x_H\right)'}$, and $\frac{\left(R^x_{HF}\right)'}{\left(R^x_H\right)'}$ are identified exactly as we have shown for the case of choices in quality by SFs. This is because they are the analogous to $\left(\lambda^d_H\right)'$, $\left(\lambda^x_H\right)'$, $\left(d^x_H\right)'$ and $\left(e^x_H\right)'$. Additionally, we need to calibrate f_{HH}/f_{HF} and $\frac{a'_{HH}}{\underline{a}}$. This can be accomplished by using

$$\frac{\left(R^d_H\right)'}{\left(R^x_H\right)'} = \frac{1}{\rho} \frac{\left(\frac{a'_{HH}}{\underline{a}}\right)^{\gamma} \left(1 - \psi^{\gamma}\right)}{\left(\psi \frac{a'_{HH}}{\underline{a}}\right)^{\gamma} - 1},$$

$$\frac{\widetilde{G}\left(a'_{HF}\right)}{\widetilde{G}\left(a'_{HH}\right)} = \frac{\left(\psi \frac{a'_{HH}}{\underline{a}}\right)^{k} - 1}{\left(\frac{a'_{HH}}{\underline{a}}\right)^{k} - 1},$$

where $\rho := \left(1 + \frac{\left(R^x_{HF}\right)'}{\left(R^x_{HH}\right)'}\right)^{\frac{1}{1-\delta}}$ and $\psi := \left[\frac{f_{HH}}{f_{HF}} (\rho - 1)\right]^{\frac{1-\delta}{\sigma-1}}$. The term $\frac{\widetilde{G}(a'_{HF})}{\widetilde{G}(a'_{HH})}$ is the proportion of exporters among SFs, which in our data is 48%. Furthermore, $\frac{R^d_H}{R^x_H}$ is the sales by SFs only serving home relative to the total sales by the SFs that export. This can be identified through the use of previous calibrations, and determines that $\frac{\left(R^d_H\right)'}{\left(R^x_H\right)'} = 0.16$.

Finally, we also need to calibrate $\frac{\left(F^d_H\right)'}{\left(R^d_H\right)'}$ and $\frac{\left(F^x_H\right)'}{\left(R^x_H\right)'}$. They are the proportion of overhead costs relative to the total sales by domestic and exporter SFs, respectively. These terms can be inferred by information for profit shares of one type of firm. In particular, we do it for SFs and based on values reported in the literature. Specifically, De Loecker and Eeckhout (2020) report an 8% of firm's profit rate relative to sales, which is weighted by firm revenue. De Loecker et al. (2020) also obtain aggregate profit rates for Europe and North America, which are around 6% and 8% respectively. Based on this and that we deal with SFs, we take a revenue-weighted profit rate of 3%, so that it is half of what occurs in Europe.

Given $y := 0.03$, we can compute $\frac{\left(F^d_H\right)'}{\left(R^d_H\right)'}$ and $\frac{\left(F^x_H\right)'}{\left(R^x_H\right)'}$ by

$$y = \left(\frac{1-\delta}{\sigma} - \frac{\left(F^d_H\right)'}{\left(R^d_H\right)'}\right) \frac{\left(R^d_H\right)'}{\left(R^d_H\right)' + \left(R^x_H\right)'} + \left(\frac{1-\delta}{\sigma} - \frac{\left(F^x_H\right)'}{\left(R^x_H\right)'}\right) \frac{\left(R^x_H\right)'}{\left(R^d_H\right)' + \left(R^x_H\right)'},$$

$$\frac{\left(F^d_H\right)'}{\left(R^d_H\right)'} \frac{\left(R^d_H\right)'}{\left(R^d_H\right)' + \left(R^x_H\right)'} = \frac{\left(1 - \frac{\widetilde{G}(a'_{HF})}{\widetilde{G}(a'_{HH})}\right)}{\frac{\widetilde{G}(a'_{HF})}{\widetilde{G}(a'_{HH})}} \left(1 + \frac{f_{HF}}{f_{HH}}\right)^{-1} \frac{\left(F^x_H\right)'}{\left(R^x_H\right)'} \frac{\left(R^x_H\right)'}{\left(R^d_H\right)' + \left(R^x_H\right)'},$$

where the second equality follows by using relations implied by the model. Given a calibration for these additional values, we can solve for (8b)–(8j), (D3), and (D4). The results are presented in Table D.2, indicating that outcomes are quite similar to the baseline case.

Table D.2. *Impact of a 10% Reduction in Export Trade Costs - With Choices in Quality by All Firms and Bounded Pareto for SFs*

(a) Impact on each LF

Better Export Access	**Firm**	**Domestic Market Share** Change (p.p.)	**Domestic Prices/Markups** Change (%)	**Quality Investments** Change (%)	**Export Revenues** Change (%)	**Total Gross Profits** Change (%)
For All Firms	Top 1	0.88	0.34	55.35	29.14	24.98
	Top 2	-0.16	-0.05	42.78	14.08	11.57
	Top 3	-0.24	-0.07	38.74	9.36	7.50
	Top 4	-0.09	-0.03	42.26	13.47	12.25
Only For LFs	Top 1	3.69	1.48	42.54	66.13	45.54
	Top 2	1.51	0.49	34.39	59.60	36.16
	Top 3	0.96	0.30	31.79	57.50	33.02
	Top 4	0.77	0.23	36.25	61.11	37.24
Only For SFs	Top 1	-2.60	-0.97	-8.18	-11.79	-16.37
	Top 2	-1.49	-0.47	-12.04	-17.19	-19.94
	Top 3	-1.07	-0.33	-13.30	-18.94	-20.95
	Top 4	-0.74	-0.22	-13.12	-18.68	-20.04

(b) Impact on LFs as Group

Better Export Access	**Domestic Market Share** Change (p.p.)	**Total Gross Profits** Change (%)
For All Firms	0.39	19.04
Only For LFs	6.93	41.39
Only For SFs	-5.90	-17.97

As we have argued above, the reason why results are similar is that we consider small reductions in export shocks, which implies that the characterization of SFs only affects the price index directly. Figure D.1 compares how this variable changes with a bounded Pareto relative to our baseline scenario. It reveals that the differences increase when we consider larger changes in export trade costs. However, they are still quite small for the range of trade costs we consider: even for a reduction in 10% of export trade costs, our baseline model predicts that H's price index decreases 4.06%, while this becomes 4.36% under a bounded Pareto distribution.

Figure D.1. *Impact on the Price Index - Different Productivity Distributions of SFs*