Trade Liberalization with Granular Firms

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August 2021

Abstract

Relying on rich firm-product Danish data, we document that the bulk of manufacturing revenue comes from industries where large firms and numerous insignificant firms coexist. Given the importance of this market structure in the aggregate, we study its implications for gains of trade by embedding a set of oligopolistic firms into a monopolistic-competition model. In this setting, the idiosyncratic features of large firms become crucial for gains of trade, given the granular importance of their profits for aggregate income. In particular, gains of trade are negatively affected when a large firm has a pronounced home bias, since trade liberalization reduces its profit by increasing domestic competition. A calibration for Denmark reflects this feature: trade liberalization raises the profits of almost all large firms, but the fall in profit of one large firm almost completely offsets the gains in income from the profit channel.

Keywords: granularity, leaders, oligopolistic firms, firm heterogeneity, gains of trade.
JEL codes: F12, F10.

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1 Introduction

What is the typical market structure of high-revenue industries? How does trade liberalization quantitatively affect economies when that market structure is accounted for? By answering these questions, in this paper we underscore the coexistence of small and large firms within an industry. In particular, we highlight the presence of domestic leaders (henceforth, DLs): firms well-established in their industries, whose domestic and export revenues constitute a great bulk of each industry's income. Their incorporation determines that economies are "granular" (Gabaix, 2011), meaning that aggregate outcomes crucially depend on the idiosyncratic features of large firms.

Our analysis begins by identifying some empirical facts regarding market structure in Danish manufacturing. The information at our disposal is of high quality, since it constitutes the basis to construct Denmark's official statistics. Moreover, it is highly disaggregated at the firm-product level, thereby overcoming issues from datasets based on the firms' balance sheets, where firms are allocated to their main industry. This enables us to allocate each good sold by a domestic firm to a specific industry, and thus obtain domestic market shares at the firm-industry level. Furthermore, these market shares account for import competition accurately, exploiting that we observe almost the universe of imports in the country.

The analysis reveals that most of the manufacturing revenue comes from industries that are neither purely monopolistic nor purely oligopolistic. Rather, they consist of few firms with great domestic market shares and numerous insignificant firms.[1] While we denominate the former as DLs, we refer to the latter as domestic non-leaders (DNLs). We show in particular that, even when only half of the industries exhibit a coexistence of DNLs and DLs, they explain more than 80% of manufacturing revenue, with DLs as a group generating more than 50% of total income.

Guided by this empirical fact, in Section 3 we study how trade liberalization impacts an economy under this market structure. Our setting rests on the idea that firms are different in nature. Regarding DLs, there is evidence that firm leadership is persistent over time.[2] Bronnenberg et al. (2009) even show that many of the current leading brands in the US were originated as early as the late nineteenth century. Thus, DLs tend to be well-established in

[1] A similar pattern is observed by Bronnenberg et al. (2011) and Hottman et al. (2016) for consumer-packaged-good industries in the US, and Gaubert and Itskhoki (2021) for manufacturing industries in France. Also, there is an extensive literature regarding a highly skewed distribution of firm size (see, for instance, Axtell 2001 and di Giovanni and Levchenko 2013). These papers show a coexistence of large and small firms when all firms in a country are pooled, and take variables such as employment and total revenue as measures of firm size. Rather, we use expenditure-based market shares at the firm-industry level, which reflects the market structure of an industry.

[2] For instance, see Sutton (2007) for several industries from Japan, and in particular Bronnenberg et al. (2009; 2011) for the USA.

a country and less subject to extensive margin adjustments. As for DNLs, several patterns have been established in the literature.[3] Decker et al. (2014), among others, show that a typical small firm in the US starts its operations at a small scale and faces a high probability of exit. Furthermore, conditional on surviving, the overwhelming majority of these businesses remain small throughout their life cycle. Overall, these firms start small and either exit or keep operating at a low scale.

Based on this evidence, we set a model with coexistence of DNLs and DLs whose features reflect these regularities. Specifically, DNLs are modeled as in the monopolistic-competition setting by Melitz (2003). Thus, we conceive them as entrepreneurs that do not know their productivity and venture into markets to explore their possibilities in the industry. Their fate is such that they either do not succeed and exit the market, or stay active but negligible in size, with the most successful DNLs exporting. On the other hand, DLs are modeled as a fixed number of heterogeneous oligopolistic firms that know their productivity and earn positive profits, thereby affecting the country's income given their size.[4]

Using this setup, we study the impact of trade liberalization on welfare under the standard scenario of two symmetric countries. The goal is to identify channels operating under this market structure, which combines mechanisms arising under oligopoly and monopolistic competition. Specifically, as in monopolistic competition, trade liberalization increases the DNLs' expected profits and hence induces their entry, ultimately decreasing each country's price index. Simultaneously, the DLs' profits are affected by two opposing mechanisms that make their impact on income be ambiguous. On the one hand, trade liberalization provides DLs with better export conditions, which increases their profits and aggregate income. On the other hand, trade liberalization results in a tougher competitive environment for DLs in both the goods and labor market. Thus, the DLs' profits fall and the country's income decreases.

Given the countervailing effects on the DLs' profits, in Section 4 we investigate various scenarios. They allow us to describe outcomes depending on the DLs' idiosyncratic features and to identify observables that capture the strength of each mechanism. The first scenario considers DLs that exclusively serve the domestic market. This entails that DLs do not benefit from better export opportunities, but are exposed to tougher competition at home. Due to this, the DLs' profits fall, which reduces the country's income and hence affects welfare negatively. Under some extreme scenarios, this effect could be so pronounced that it leads to negative gains

[3] The patterns described have been documented several times for the US since, at least, Dunne et al. (1988) and observed for other countries (see, for instance, Schoar 2010 and La Porta and Shleifer 2008).

[4] To illustrate the kind of market structure we envision, the beer industry can be used as an example. This encompasses a few firms accruing top market shares in each country (e.g., Heineken in some European countries, Carlsberg in some Scandinavian countries), and a large pool of small firms (e.g., small brewpubs and microbreweries) producing at a low scale. Even though the latter are negligible when each firm is taken in isolation, they are non-trivial as a whole.

of trade.

The second scenario refers to a situation where DLs only export. In this case, DLs are negatively affected by the increase in competition in the foreign country, but they also benefit from better export opportunities. Our results establish that, overall, the positive effect of better export opportunities dominates, thus creating better export conditions. Consequently, DLs garner greater profits and the country's income increases, which guarantees positive gains of trade since the price index always decreases.

Finally, we investigate the general case where the features of DLs are left unspecified. Our focus is on observables that make it possible to infer whether a DL's profit is impacted positively or negatively by trade liberalization. We highlight in particular the role of a DL's export intensity, measured as the share of its exports in total sales: the higher a DL's export intensity, the higher the benefits from better export conditions and the lower the negative impact of tougher domestic competition. This implies that DLs with low home bias increase their profits and have a positive impact on the country's income. On the contrary, those with a significant home bias garner lower profits and have a negative impact on the country's income.

In Section 5, we conduct a numerical exercise to assess the effects of trade liberalization on welfare. This is done by calibrating the model to replicate key features of DLs and DNLs in Danish manufacturing. Our main finding is that the effect of trade liberalization on profits is positive, thereby ensuring the existence of gains of trade. Nonetheless, the increase in income through this channel is almost null, which is explained by the granularity of the economy. More precisely, almost all DLs benefit from trade liberalization and garner greater profits, given that their home bias is relatively low. Nonetheless, the second top DL exhibits a pronounced home bias, determining that its profit decreases to such an extent that it offsets the increase in profits by the rest of the DLs.

Related Literature and Contributions. Our paper contributes to a burgeoning literature on the granular importance of large firms, as in Gabaix (2011), Acemoglu et al. (2012), di Giovanni and Levchenko (2012), di Giovanni et al. (2014), and Gaubert and Itskhoki (2021). This literature highlights that the idiosyncratic way in which an aggregate shock affects a large firm has aggregate consequences for economies. The topic is closely related to an also growing literature on the rise of superstar firms, which documents that industries are becoming increasingly dominated by a few global firms (see, for instance, Autor et al. 2020; De Loecker et al. 2020; Bighelli et al. 2021; Díez et al. 2021).

Additionally, our paper is related to approaches accounting for granularity that go beyond a standard oligopoly. In this respect, it is worth mentioning Eaton et al. (2012) and Gaubert and Itskhoki (2021), who show the benefits of introducing a random number of oligopolistic firms to model granularity in general equilibrium.[5]

We contribute to this literature by providing a quantitative framework that incorporates granularity and is based on Shimomura and Thisse (2012) and Parenti (2018). These studies consider a market structure with coexistence of oligopolistic and monopolistic firms. Nonetheless, they focus on theoretical results and incorporate assumptions that hinder their application to tackle quantitative matters (e.g. firm homogeneity). Furthermore, none of them study trade liberalization in general equilibrium.

On the contrary, we document that industries with large and small firms coexisting explain the bulk of manufacturing income, thereby justifying their relevance for aggregate analyses, and for gains of trade in particular. Based on it, we set a model in general equilibrium that can be used for estimating gains of trade.[6] The approach is highly tractable, since it only requires knowledge of the DNLs' export intensity and each DL's revenue share. Furthermore, it accounts for heterogeneity of DLs, which is key to quantifying the effects in granular economies: it allows for a trade shock to impact each DL in an idiosyncratic way, thus capturing heterogeneous granular effects on welfare outcomes. This is clearly revealed in our calibration exercise, which shows that the losses of one DL almost completely offset the increases in profits by the rest of the DLs.

2 Empirical Facts

We present empirical evidence regarding market structure in Danish manufacturing. This has the goal of identifying features of Denmark's highest revenue industries. Given that the information at our disposal is the main source of the country's official statistics, it is of high quality. A further description of the data and measures used is included in Section 5.1.

We make use of two datasets for the year 2005, with similar patterns observed for other years. The first one is the Prodcom dataset, which provides information on production value of manufacturing firms. It encompasses 3,517 firms, and is collected by ensuring that at least 90% of the total production value in each 4-digit NACE industry is covered. The second dataset has data on exports and imports, and is collected by Danish Customs. Importantly, trade flows by both manufacturing and non-manufacturing firms are included, with almost the universe of transactions covered.[7] This allows us to obtain an accurate measure of import competition and

[5]For some other recent studies in International Trade accounting for granularity, see, for instance, Atkeson and Burstein (2008), di Giovanni and Levchenko (2013), and Edmond et al. (2015).

[6]For some of the most recent papers computing gains of trade through different methods, see Bertoletti et al. (2018), Feenstra (2018), Arkolakis et al. (2019), Balistreri and Tarr (2020), and Sun et al. (2020).

[7]The coverage is 95% for imports and 97% for exports with an EU trading partner, while the universe of transactions is covered for trade with non-EU countries.

export revenues.

Both datasets can be easily merged through a unique firm identifier. Moreover, the information on goods is disaggregated at the 8-digit Combined Nomenclature (CN) product code, whose first six digits are identical to the Harmonized System classification. Overall, the data provide information on total turnover, exports, and imports presented at the firm-product level.

Since the data are highly disaggregated, we are able to overcome restrictions imposed by datasets based on firms' balance sheets, where each firm's variables are allocated to its main industry. Thus, we define an industry by allocating each good at the CN 8-digit level to a 4-digit NACE industry. This leaves us with 203 industries out of 5,212 goods in the sample. With this information, we compute the domestic market share of each Danish firm-industry.[8] This is defined relative to industry expenditure, which comprises domestic sales and imports. Since Denmark is a small highly open economy, accounting for imports becomes crucial to obtaining market shares that capture the importance of a firm in an industry.

To classify industries according to their market structure, we split firms into DNLs and DLs using a domestic market of 3% as threshold. Similar results hold for other cutoffs. We begin by identifying industries that include a pool of DNLs. This guarantees the existence of several domestic firms with negligible market shares, which is a necessary condition (although not sufficient) to have a monopolistic competition market structure.

The results indicate that 107 industries out of 203 include a subset of DNLs, comprising an average number of 57 firms and a maximum above 330.[9] While these industries encompass a little bit more than half of the total, they account for 85% of the income generated by the manufacturing sector, where income is defined as the sum of domestic sales and exports by domestic firms.

In Figure 1a, we illustrate this by presenting the proportion of income accounted for industries comprising a group of DNLs. The results are aggregated at the 2-digit industry level using industry-revenue weights, with the last bar applying to the whole manufacturing sector.

We also inquire whether an industry that includes a set of DNLs fits a pure monopolistic competition market structure. This is the case when an industry does not additionally have DLs active in the market. The results indicate, out of the 107 industries having a pool of DNLs, 92 of them have at least one DL operating and generate 96% of the total income of

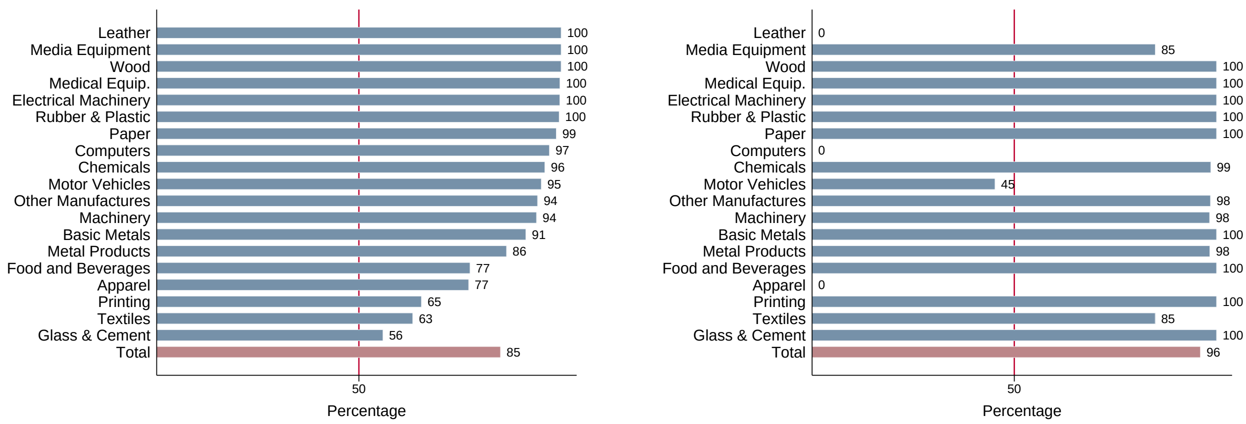
[8]We consider a firm Danish if it has production activities in Denmark (i.e., if it is included in the Danish Prodcom dataset). Instead, any firm that imports and has no production activity in Denmark is considered part of the import competition.

[9]The criteria we use to identify these industries are the following. First, we check that there are at least 10 firms in the market, and that the 10 firms or 20% of the firms with the lowest market share do not accumulate more than 6% of total market share. In addition, we only consider industries that are subject to import competition to account for markets with international trade. This is ensured by checking that at least 4% of an industry's market share corresponds to imports.

the industries with DNLs. This reveals that most of the manufacturing revenue comes from industries with coexistence of DLs and DNLs.[10] The results are presented in Figure 1b.[11]

Figure 1. *Proportion of Income by Industry*

(a) Industries with DNLs **(b)** Industries with DNLs and DLs



Note: In both figures, the results for each sector and the total are calculated using industry-revenue weights. In Figure 1a, the proportion of income for each sector is relative to the industries in the sector. In Figure 1b, the proportion is relative to the subset of industries with DNLs in the sector.

We also delve into the role of DLs in these industries. Figure 2a provides information about the proportion of industry revenue accrued by DLs as a group. The last bar in particular aggregates the information of each industry by using industry-revenue weights, revealing that more than 50% of total revenue in a typical industry is due to sales by DLs.

Additionally, Figure 2b describes the percentage of DLs that generate at least a certain proportion of total revenue in each industry. It provides evidence that the firms with the greatest domestic market share also rank high among the firms with highest revenues in each industry. Thus, not only DLs as a group generate a great part of the total revenue, but actually every single DL contributes significantly to it.

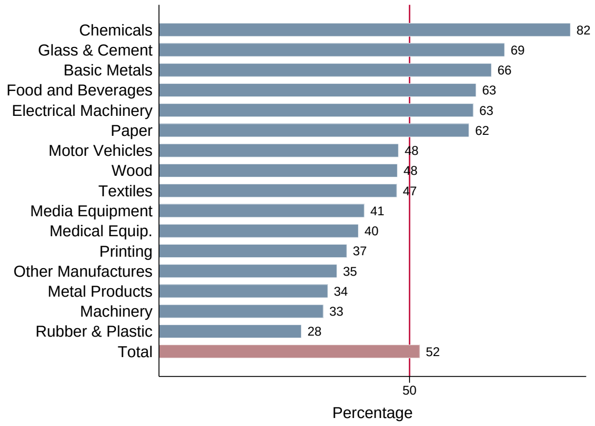
Specifically, in the case of our baseline definition of DLs (i.e., with a threshold of 3% for domestic market share), virtually every firm (98.79% of them) generates at least 3% of the industry revenue. Moreover, when we use 5% as domestic market share to define a DL, the minimum revenue generated by each firm is greater. Thus, almost all DLs have a revenue representing at least 5% of the industry revenue.

[10] If we define DLs as firms with a market share greater than 5%, industries with coexistence of DNLs and DLs encompass 81 industries and generate 90% of the income. Also, measuring the relative importance of each industry in terms of expenditure delivers similar conclusions. The numbers, nonetheless, are a little bit lower since some of the industries are almost entirely dominated by imports. Specifically, industries having DNLs cover around 82% of the manufacturing expenditures and, among these industries, 86% is generated by industries with coexistence of DNLs and DLs.

[11] While the figure indicates that Leather, Computers, and Apparel do not include industries with DLs, this has no major impact on total revenue. The reason is that these sectors are almost completely served by imports, rather than DNLs.

Figure 2. *Revenue Generated by DLs in Industries with Coexistence of DNLs and DLs*

(a) % of Revenue Generated by all DLs as Group



(b) % of DLs that Generate a Minimum Industry-Revenue Share

Threshold of Market Share defining a DL	Minimum Industry-Revenue Share Generated by each DL 3%	5%
3%	98.79	83.99
5%	100	99.46

Overall, the conclusions are twofold. First, the presence of negligible firms in an industry has to be accounted for in aggregate analyses—pure oligopolies only explain a small portion of total income in manufacturing. Second, the bulk of income is explained by industries with a simultaneous presence of DLs. Consequently, accounting for the idiosyncratic features of granular firms is also of first-order relevance for aggregate analyses.

3 Model Setup and Equilibrium

We consider a world economy with a single sector consisting of a horizontally differentiated good. The set of countries is $\mathcal{C} := \{H, F\}$, and we refer to H as the home country and F as the foreign country. Throughout the paper, we use the convention that a variable's subscript ij refers to i as the origin country and j as the destination country. For the setup description, we also consider countries i and j such that $i, j \in \mathcal{C}$, unless otherwise stated. All the derivations and proofs are relegated to Appendix A.

3.1 Types of Firms

In each country i, there is a set of firms $\overline{\Omega}$ that can potentially serve any country with a unique variety. We formalize the existence of two types of firms by partitioning $\overline{\Omega}$ into a finite set $\overline{\mathscr{L}}$ and a real interval $\overline{\mathcal{N}}$, whose letters are mnemonics for "large" and "negligible." A firm $\omega \in \overline{\mathscr{L}}$ is referred to as a DL, and we suppose that it is non-negligible for the industry. Likewise, a firm $\omega \in \overline{\mathcal{N}}$ is referred to as a DNL and supposed to be negligible.

To model each type of firm, we rely on several empirical regularities obtained in the literature. This leads us to characterize DNLs following Melitz (2003), and embed an exogenous number of oligopolistic firms to represent DLs. The characterization of DNLs is based on mounting evidence since at least Dunne et al. (1988), indicating that small firms operate with high uncertainty about their profitability. This results in high rates of entry and exit, reflecting the importance of accounting for extensive margin adjustments among small firms. Additionally, there is evidence that a typical small firm starts its operations at a small scale and, conditional on surviving, remains small throughout its life cycle (see, for instance, Decker et al. 2014).[12]

As for DLs, it has been documented that firm leadership is persistent over time.[13] In particular, Bronnenberg et al. (2009) provide the most systematic study up to this day in this respect. They show that leadership in several American industries exhibits geographical persistence, with several current leading brands launched more than a century ago. This implies that DLs tend to be well-established in a country, with their leadership only occasionally contested. Thus, supposing an exogenous number of DLs seems adequate as a stylized representation of these firms.

3.2 Supply Side

In each country i, there is a mass of identical agents L that are immobile across countries. Labor is the only production factor and each offers a unit of labor inelastically. Additionally, these agents are the owners of their own country's firms and get the same fraction of profits.

Following Shimomura and Thisse (2012) and Gaubert and Itskhoki (2021), we suppose that firms are income and wage takers.[14] Moreover, DNLs from i are modeled as ex-ante identical that do not know their productivity. These firms consider whether to pay a sunk entry cost, consisting of F units of home workers. If a firm does so, it receives a productivity draw φ and an assignation of a unique variety $\omega \in \overline{\mathcal{N}}$. Productivity draws come from a continuous random variable with non-negative support $\left[\underline{\varphi}, \overline{\varphi}\right]$ and cdf G. We denote the measure of DNLs that pay the entry cost by M_i^E.

As for DLs from i, there is an exogenous number of them. Moreover, each has assigned a unique variety $\omega \in \overline{\mathscr{L}}$ and productivity φ_ω that is common knowledge across the world. We suppose that $\varphi_\omega > \overline{\varphi}$ for any $\omega \in \overline{\mathscr{L}}$, so that any DL is more productive than the most productive DNL. To distinguish between the set of DLs in each country, we denote the set of active DLs from i by $\mathscr{L}_i$.

[12]While it is likely that differences in entrepreneurial skills play a role in this pattern, there is also evidence that other factors are at play. Hurst and Pugsley (2011) document that small entrepreneurs do not expect or have as a goal to grow big. In other words, DNLs do not make entry decisions expecting to be industry leaders. Moreover, even if DNLs have the goal of becoming leaders, there is evidence that DLs tend to engage in strategic moves when small firms start to loom large. In this way, DLs endogenously hinder the DNLs' chances to succeed in the market (see, for instance, D'Aveni 2002).

[13]For evidence on this matter, see for instance Sutton (2007) for Japanese industries, and Bronnenberg et al. (2009; 2011) for the USA.

[14]This assumption can be formally rationalized by supposing we study a representative industry among a continuum, as in Neary (2016).

Regarding costs, a DNL with productivity φ serving j from i has a constant marginal cost $c(\varphi, \tau_{ij}, w_i) := \frac{w_i}{\varphi}\tau_{ij}$, where τ_{ij} is a trade cost with $\tau_{ii} := 1$. Likewise, DL ω has a marginal cost c_{ij}^{ω} given by the same function $c\left(\varphi_{\omega}, \tau_{ij}^{\omega}, w_i\right)$, where $\tau_{ij}^{\omega} := \tau^{\omega}\tau_{ij}$ if $j \neq i$, and $\tau_{ii}^{\omega} := 1$. This implies that DLs have firm-specific trade costs, thereby allowing a more productive DL to have greater domestic revenues than any less productive DLs, without implying that its export revenues are greater too. Such a feature of the model makes it possible to let the data identify each DL's export intensity, allowing for a DL to be either domestic- or export-oriented. For instance, it rationalizes that, in our calibration based on Danish data, the second top DL exhibits a higher home bias than the rest of DLs.[15]

We also suppose that DLs from i and the mass M_i^E of DNLs have the option of not selling in j. If they do so, they have to pay an overhead fixed cost f_{ij} expressed in units of home workers, where $f_{ij} > f_{ii}$ for any $i \neq j$. Additionally, firm ω chooses prices p_{ij}^{ω} in each j, where we allow for the possibility that $p_{ij}^{\omega} = \infty$, to capture that ω possibly does not serve country j.

To reflect the coexistence of both types of firms, we suppose that there is always a subset of DNLs that are active in each country. Additionally, we assume that some DNLs export and find it profitable to serve their domestic market. Since DLs are also more productive than any DNL, then DLs always serve the domestic market.

In terms of notation, we denote by Ω_{ji} the set of varieties produced in j and sold in i, with Ω_i being the total varieties available in i. Likewise, $\Omega_{ji}^{\text{DNL}} := \overline{\mathscr{N}} \cap \Omega_{ji}$ and $\Omega_{ji}^{\text{DL}} := \overline{\mathscr{L}} \cap \Omega_{ji}$ are the subsets of varieties available in i produced by DNLs and DLs from j, respectively.

3.3 Demand Side

Preferences are represented by a CES utility function. Thus, the demand in j of firm ω from i is given by

$$Q_{ij}^{\omega} := Y_j \left(\mathbb{P}_j\right)^{\sigma-1} \left(p_{ij}^{\omega}\right)^{-\sigma},$$

where $\mathbb{P}_j$ is j's price index, Y_j is j's income, and $\sigma > 1$. Since firms can attain different masses, the expression of the price index in j is given by

$$\mathbb{P}_j = \left\{ \sum_{k\in\{H,F\}} \left[\int_{\omega\in\Omega_{kj}^{\text{DNL}}} \left(p_{kj}^{\omega}\right)^{1-\sigma} \mathrm{d}\omega + \sum_{\omega\in\Omega_{kj}^{\text{DL}}} \left(p_{kj}^{\omega}\right)^{1-\sigma} \right] \right\}^{\frac{1}{1-\sigma}}. \tag{1}$$

We define s_{ij}^{ω} as the revenue share by firm ω from i obtained through sales in j. It is given

[15] Assuming a firm-specific term in trade costs is isomorphic to adding a firm-specific demand shifter for foreign demand. Both assumptions allow us to rationalize that a DL could have any export intensity between 0 and 1.

by the following function:

$$s\left(p_{ij}^{\omega},\mathbb{P}_j\right):=\left(\frac{p_{ij}^{\omega}}{\mathbb{P}_j}\right)^{1-\sigma}. \tag{2}$$

Using this revenue share, we define the price elasticity of demand in j of firm ω from i by $\varepsilon\left(s_{ij}^{\omega}\right):=\sigma+s_{ij}^{\omega}\left(1-\sigma\right)$ if ω is a DL, and $\varepsilon_{ij}^{\omega}=\sigma$ if ω is a DNL.

3.4 Equilibrium

We now introduce the assumption of symmetric countries. This allows us to streamline notation by using D as shorthand for subscript ii, and X as shorthand for ij with $i\neq j$. Formally, symmetry means that for each DL $\omega\in\mathscr{L}_H$ with $(\varphi_\omega,\tau^\omega)$ there is a DL $\omega\in\mathscr{L}_F$ with the same productivity and trade costs. In addition, fixed costs and the common-component of trade costs are the same in each country, so that $\tau:=\tau_{ij}$ and $f_X:=f_{ij}$ for $i\neq j$, and $f_D:=f_{ii}$.

As for equilibrium variables, symmetry and balanced trade imply that wages in equilibrium are the same in each country, and we take them as the numéraire. Furthermore, the equilibrium price index and income are identical in both countries, and so we denote them without subscripts (i.e. by $\mathbb{P}$ and Y).

Consider DL ω from i that is active in j. Routine calculations determine that its optimal prices in j are given by

$$p_{ij}^{\omega}=m\left(s_{ij}^{\omega}\right)c_{ij}^{\omega}, \tag{3}$$

where $m\left(s_{ij}^{\omega}\right):=\frac{\varepsilon\left(s_{ij}^{\omega}\right)}{\varepsilon\left(s_{ij}^{\omega}\right)-1}$ is ω's markup. (3) determines an implicit solution for prices by using (2), which we denote by $p_{ij}^{\omega}\left(\mathbb{P}\right)$.

Moreover, the optimal revenue share of DL ω by sales in j is

$$s_{ij}^{\omega}\left(\mathbb{P}\right):=\left(\frac{p_{ij}^{\omega}\left(\mathbb{P}\right)}{\mathbb{P}}\right)^{1-\sigma}, \tag{4}$$

which establishes that ω's optimal revenue in j is a function $R_{ij}^{\omega}\left(\mathbb{P},Y\right):=Ys_{ij}^{\omega}\left(\mathbb{P}\right)$. Consequently, ω's total profit is $\pi_i^{\omega}:=\pi_D^{\omega}+\pi_X^{\omega}$, where its optimal profit in j is $\pi_{ij}^{\omega}\left(\mathbb{P},Y\right)=\frac{R_{ij}^{\omega}(\mathbb{P},Y)}{\varepsilon\left[s_{ij}^{\omega}(\mathbb{P})\right]}-f_{ij}$.

Total profits of DLs from i are defined as $\Pi_i:=\sum_{\omega\in\overline{\mathscr{L}}_i}\pi_i^{\omega}$, which are given in equilibrium by the following function:

$$\Pi_i\left(\mathbb{P},Y;\boldsymbol{\omega}\right):=\sum_{\omega\in\Omega_D^{\text{DL}}}\left(\frac{R_D^{\omega}\left(\mathbb{P},Y\right)}{\varepsilon\left[s_D^{\omega}\left(\mathbb{P}\right)\right]}-f_D\right)+\sum_{\omega\in\Omega_X^{\text{DL}}}\left(\frac{R_X^{\omega}\left(\mathbb{P},Y\right)}{\varepsilon\left[s_X^{\omega}\left(\mathbb{P}\right)\right]}-f_X\right). \tag{PROF}$$

The inclusion of $\boldsymbol{\omega}$ as an argument in (PROF) reflects that each DL is heterogeneous in terms of both φ_ω and τ^ω.

A DNL from i with productivity φ has no impact on the price index of any country, and so its price elasticity is given by σ. Due to this, its optimal price is $p_{ij}^{\text{DNL}}(\varphi)$, which is given by (3) but with markup $\frac{\sigma}{\sigma-1}$. Moreover, its profit in j is

$$\pi_{ij}^{\text{DNL}}(\mathbb{P}, Y; \varphi, \tau) := \frac{r_{ij}(\mathbb{P}, Y; \varphi, \tau_{ij})}{\sigma} - f_{ij},$$

where r_{ij} denotes its optimal revenue in j. By setting π_{ij}^{DNL} to zero, we obtain the domestic and export survival productivity cutoff of DNLs, φ_D^* and φ_X^*. They are determined by the following function:

$$\varphi_{ij}(\mathbb{P}, Y; \tau) = \frac{\sigma \tau_{ij}}{(\sigma-1)\mathbb{P}} \left(\frac{\sigma f_{ij}}{Y}\right)^{\frac{1}{\sigma-1}}.$$

Besides, given free entry, a DNL's expected profit equals the entry cost:

$$\pi_i^{\mathbb{E},\text{DNL}}(\mathbb{P}, Y; \tau) := \int_{\varphi_D^*}^{\overline{\varphi}} \left[\frac{r_D(\mathbb{P}, Y; \varphi)}{\sigma} - f_D\right] \mathrm{d}G(\varphi) + \int_{\varphi_X^*}^{\overline{\varphi}} \left[\frac{r_X(\mathbb{P}, Y; \varphi, \tau)}{\sigma} - f_X\right] \mathrm{d}G(\varphi) = F. \tag{FE}$$

With the characterization of optimal decisions by each type of firm, we can also define the condition for market clearing in i. This requires that (1) for i is consistent with firms' optimal decisions. To state this condition, define

$$\widetilde{\mathbb{P}}^{\text{DNL}}\left(\mathbb{P}, Y, M^E; \tau\right) := M^E \left[\int_{\varphi_D^*}^{\overline{\varphi}} \left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi}\right)^{\sigma-1} \mathrm{d}G(\varphi) + \int_{\varphi_X^*}^{\overline{\varphi}} \left(\frac{\sigma}{\sigma-1}\frac{\tau}{\varphi}\right)^{\sigma-1} \mathrm{d}G(\varphi)\right],$$

$$\widetilde{\mathbb{P}}^{\text{DL}}(\mathbb{P}, \boldsymbol{\omega}) := \sum_{\omega \in \Omega_D^{\text{DL}}} \left[\frac{m_D^\omega(\mathbb{P})}{\varphi_\omega}\right]^{\sigma-1} + \sum_{\omega \in \Omega_X^{\text{DL}}} \left[m_X^\omega(\mathbb{P}) \frac{\tau^\omega}{\varphi_\omega}\right]^{\sigma-1},$$

where $m_D^\omega(\mathbb{P})$ and $m_X^\omega(\mathbb{P})$ are DL ω's markups in each country evaluated at its optimal revenue shares, (4). Using these definitions, there is market clearing in i if the following condition is satisfied:

$$\mathbb{P}^{1-\sigma} = \widetilde{\mathbb{P}}^{\text{DNL}}\left(\mathbb{P}, Y, M^E; \tau\right) + \widetilde{\mathbb{P}}^{\text{DL}}(\mathbb{P}, \boldsymbol{\omega}), \tag{MS}$$

where "MS" refers to the fact that (MS) constitutes the equilibrium condition at the market stage (i.e., for given masses of DNLs in each country).

As for income, there is a continuum of DNLs with zero expected profits, so that their aggregate profits are zero. On the contrary, DLs have positive profits. Thus, by utilizing (PROF), total income in i is

$$Y = L + \Pi_i(\mathbb{P}, Y; \boldsymbol{\omega}). \tag{INC}$$

In summary, the equilibrium can be identified through a vector $\left(\mathbb{P}^*, Y^*, M^{E*}\right)$ that satisfies (MS), (FE), and (INC). Once we identify those values, any other equilibrium variable can be pinned down.

3.5 Trade Liberalization

Consider a small proportional reduction in τ, $\mathrm{d}\ln\tau \neq 0$. Our analysis focuses on the impact of trade liberalization on real income, $\mathbb{W} := \frac{Y}{\mathbb{P}}$, which is determined by

$$\mathrm{d}\ln\mathbb{W}^* := \mathrm{d}\ln Y^* - \mathrm{d}\ln\mathbb{P}^*. \tag{5}$$

Working each term out, we can express $\mathrm{d}\ln Y^*$ and $\mathrm{d}\ln\mathbb{P}^*$ in terms of observables, revealing that their magnitude depends on a few statistics: the export intensity of DNLs and the revenue shares of DLs. To show this, let R_D^{DNL} and R_X^{DNL} be the optimal revenues of all DNLs in each market. Furthermore, let $e^{\mathrm{DNL}} := \frac{R_X^{\mathrm{DNL}}}{R_D^{\mathrm{DNL}}+R_X^{\mathrm{DNL}}}$ be the export intensity of DNLs as a group, with their domestic intensity given by $d^{\mathrm{DNL}} := 1 - e^{\mathrm{DNL}}$. Then, the change in the price index can be expressed as

$$\mathrm{d}\ln\mathbb{P}^* = -\left(1 + \frac{\sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{s_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega} + \sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{s_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^\omega}{\varepsilon_D^\omega} - \sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}}\right)^{-1}\left(e^{\mathrm{DNL}} + \frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^\omega}{\varepsilon_D^\omega} - \sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}}\right). \tag{6}$$

Equation (6) establishes that *the price index always decreases following trade liberalization*, irrespective of the features of DNLs and DLs.

Likewise, the impact on income can be expressed as

$$\mathrm{d}\ln Y^* = (\sigma-1)\frac{\left(\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}\right) + \left(\sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{s_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega} + \sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{s_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}\right)\mathrm{d}\ln\mathbb{P}^*}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^\omega}{\varepsilon_D^\omega} - \sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}}. \tag{7}$$

The sign of $\mathrm{d}\ln Y^*$ is ambiguous, entailing that *trade liberalization can increase or reduce a country's income.* In fact, it is possible to conceive scenarios where the negative effect on income is so pronounced that there are losses from trade. As we show in the next section, this possibility arises when there are multiple DLs with a pronounced home bias. Nevertheless, it requires extreme assumptions and never occurs if there is one DL in each country.

To provide some intuition regarding why the sign of $\mathrm{d}\ln Y^*$ is ambiguous, we can decompose the total impact on income in several terms. This establishes that income increases or decreases according to the following

$$\operatorname{sgn}\left(\mathrm{d}\ln Y^*\right) = \operatorname{sgn}\left[\underbrace{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}}_{(i)} + \underbrace{\left(\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}\right)\mathrm{d}\ln\mathbb{P}^*}_{(ii)} + \underbrace{\left(\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega}\right)\mathrm{d}\ln\mathbb{P}^*}_{(iii)}\right], \tag{8}$$

where $\mathrm{d}\ln\mathbb{P}^* < 0$ by (6). Equation (8) exclusively reflects how the DLs' profits are impacted, since wages are taken as the numéraire and the DNLs' aggregate profits are zero. It rationalizes that the impact on income is ambiguous due to the existence of opposing channels affecting profits. Term (i) captures that trade liberalization has a positive effect on each DL's profit, by providing better export opportunities. On the contrary, the terms (ii) and (iii) capture the negative impact of trade liberalization on each DL's profit due to tougher competition at home and abroad, where increase in competition is reflected through a reduction in the price index.

4 Welfare Analysis

Gains of trade, (5), are calculated through the variation in the price index and income, respectively given by (6) and (7). Under coexistence of DNLs and DLs, analyzing the determinants of gains of trade is more complex than under a pure oligopoly or monopolistic competition. This is because the setting incorporates channels operating under both market structures. Due to this, we begin by analyzing several cases that lay bare the mechanisms of adjustment.

More precisely, we derive results under absence of DLs, for DLs that are domestic-oriented, and for DLs that are export-oriented. These cases have specific implications for the computation of (6) and (7). In particular, they make it possible to isolate the terms (i), (ii), and (iii) given in (8). This allows us to illustrate how trade liberalization differentially impacts profits, depending on the DLs' features. The results simultaneously unveil the granular importance of DLs, where the idiosyncratic impact on each DL can entail starkly different outcomes for an economy.

The section concludes by analyzing the general case where the DLs both serve home and export, whose effects are a combination of the mechanisms analyzed in the previous cases. The conclusions of this scenario become useful for interpreting the outcomes arising with Danish data.

4.1 No Leaders

We begin by considering a benchmark scenario with no DLs, in which case our setting collapses to the Melitz model. In terms of our setup, this scenario captures effects that operate exclusively through the impact of trade liberalization on DNLs. Furthermore, it determines that aggregate profits are always zero and hence not affected by trade liberalization. Thus, since wages are taken as the numéraire, income does not vary and gains of trade equal the variation in the price index. Formally, gains of trade are given by (6) with $s_D^\omega = s_X^\omega = 0$ for any DL ω, yielding

$$\mathrm{d}\ln \mathbb{W}^* = -\mathrm{d}\ln \mathbb{P}^* = e^{\mathrm{DNL}}. \tag{9}$$

The result indicates that there are always gains of trade under absence of DLs, and their magnitude equals the DNLs' export intensity. This statistic acts as a proxy for the magnitude in which better export opportunities increase expected profits, and hence induce entry. Consequently, when DNLs have higher export intensity, entry following trade liberalization is more pronounced, ultimately reducing the price index more markedly. While this has a positive effect on welfare, we will see next that it also entails greater competition for DLs in both the goods and labor market, thus reducing each DL's profit and so the country's income.

4.2 Domestic-Oriented Leaders

We incorporate the existence of DLs by assuming that they do not export, so that their revenues come exclusively from sales at home.[16] This case determines gains of trade computed through the terms (6) and (7) with $s_X^\omega = 0$ for each DL ω. It yields in particular that the impact on income reduces to

$$\mathrm{d}\ln Y^* = \frac{(\sigma-1)\left(\sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{s_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega}\right)}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^\omega}{\varepsilon_D^\omega}}\,\mathrm{d}\ln\mathbb{P}^* < 0, \tag{10}$$

where recall that $\mathrm{d}\ln\mathbb{P}^* < 0$. By using the decomposition of effects on income given by (8), this case determines that only the term (iii) is different from zero. This term reflects that trade liberalization impacts DLs' profits negatively through increased domestic competition, without any benefit from better export opportunities. Consequently, income always decreases, leading us to the following result.

Proposition 4.1. *When DLs exclusively serve the domestic market, trade liberalization decreases each country's income. Furthermore, gains of trade are not guaranteed.*

Proposition 4.1 indicates that, if the reduction in income is quite pronounced, negative gains of trade are even possible. From (10), this could arise if DLs generate a substantial share of aggregate revenue, so that their decreases in profits have a significant impact in aggregate terms. Nonetheless, as we formally show in Appendix D, losses from trade only arise under quite extreme scenarios, requiring multiple domestic-oriented DLs accruing disproportionately high market shares.

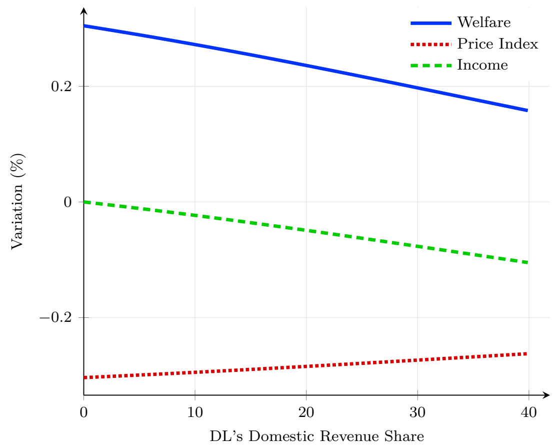
Even when losses from trade are unlikely, decreases in the DLs' profits can still affect gains of trade considerably. This is illustrated in Figure 3, which shows that a greater domestic revenue share of a DL results in diminished gains of trade.[17] The outcome arises since the profit of a domestic-oriented DL is always affected negatively by trade liberalization, in a magnitude

[16] Scenarios where DLs are primarily domestic-oriented cover several cases. One occurs when the DLs' products are only successful at their home markets. Additionally, it arises when DLs are foreign-owned firms that set operations in the country with the sole goal of serving the local market.

[17] Some perverse scenarios arise when sigma is close to one and both the DNLs' export intensity and the DL's revenue share take extreme values. In those cases, it is possible that a higher revenue share of a DL generates outcomes that differ from those we present. Nonetheless, this only arises under implausible industry features.

that depends on its revenue share: the fall in profits is more pronounced when a DL has a greater revenue share, which affects welfare negatively by reducing income, but also indirectly by mitigating the decrease in the price index. The latter follows since decreases in income lower the DNLs' expected profits and hence the DNLs' incentives to enter, which ultimately erodes the effect of trade liberalization on competition.

Figure 3. *1% Reduction in Trade Costs: Role of a DL's Domestic Revenue Share (DL Only Serving Home)*



4.3 Export-Oriented Leaders

Consider now a situation where DLs are mainly export-oriented, by making the extreme assumption that their domestic market share tends to zero.[18] In this scenario, the impact of trade liberalization on the price index and income is respectively given by (6) and (7) with $s_D^\omega \to 0$ for each DL ω. This determines that income varies according to

$$\mathrm{d}\ln Y^* = \left[\frac{(\sigma-1)\left(\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}\right)}{1-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}}\right] + \left[\frac{(\sigma-1)\left(\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}\right)}{1-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}}\right]\mathrm{d}\ln\mathbb{P}^* > 0. \qquad (11)$$

This case guarantees positive gains of trade, since the price index always decreases and income always increases. We formalize this in the following proposition.

Proposition 4.2. *When DLs exclusively serve the foreign market, trade liberalization increases each country's income. This implies that there are always positive gains of trade in each country.*

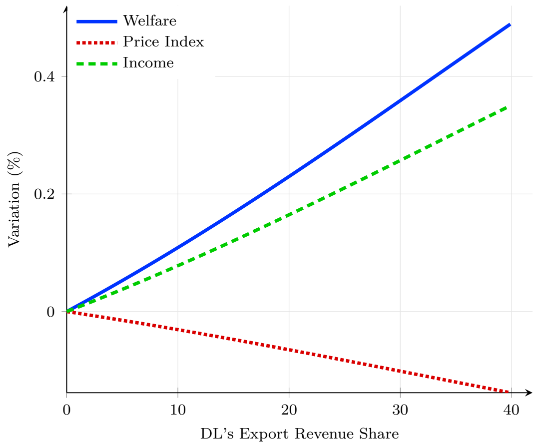
Variations in income in this scenario occur through the terms (i) and (ii) in (8). These terms capture opposing effects on profits: the term (i) captures that the DLs' profits increase due to better export opportunities, whereas (ii) captures that the DLs' profits decrease due to tougher

[18]Cases like this arise when, for instance, large firms establish operations in the country to use it as an export platform.

competition in the foreign country. Overall, the proposition establishes that the positive effect dominates, and so trade liberalization represents an improvement in export conditions for DLs. Due to this, the DLs' profits always increase, and so does income.[19]

Figure 4 illustrates that a higher export revenue share of a DL is associated with higher gains of trade.[20] It reflects that the positive impact on a country's income is more pronounced when a DL's export revenue share is greater.

Figure 4. *1% Reduction in Trade Costs*
Role of a DL's Export Revenue Share (DL Only Exporting and DNLs only Serving Home)



The graph illustrates this by considering one DL that exports, and DNLs only serving their home market (i.e., $e^{\text{DNL}} = 0$). The latter shuts the variations in the price index due to better export opportunities for DNLs, so that the curves only reflect effects caused by the changes in the DL's profits. By this, the graph demonstrates the positive effect of trade liberalization on profits and hence income, which indirectly reduces the price index more by its positive effect on the DNLs' expected profits.

4.4 General Case

Next, we study the determinants of welfare when DLs possibly serve the domestic and foreign markets simultaneously. The impact on the price index, income, and welfare can be understood as a combination of the effects arising in the cases analyzed previously. Thus, trade liberalization activates several mechanisms operating concurrently. For one thing, it decreases the price index by inducing entry of DNLs. For another thing, it affects the DLs' profits positively through better export conditions, but also negatively by increasing competition at home. This entails

[19] Income increases since, by substituting in for $\mathrm{d}\ln \mathbb{P}^*$ and working out the expression, $\operatorname{sgn}(\mathrm{d}\ln Y^*) = \operatorname{sgn}\left[\sum_{\omega\in\Omega_X^{\text{DL}}} \frac{s_X^\omega}{\varepsilon_X^\omega}\left(1 - \frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega} e^{\text{DNL}}\right)\right]$. Given that $s_X^\omega < 1$ and $e^{\text{DNL}} < 1$, then $\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega} e^{\text{DNL}} < 1$, and so the result follows.

[20] The same caveat as in Footnote 17 applies.

that the impact of trade liberalization on the DLs' profits is ambiguous, explaining why income can increase or decrease.

Formally, the impact of trade liberalization on the price index is given by (6), implying that the price index always decreases. Likewise, the impact on income is given by (7), and, substituting in $\mathrm{d}\ln \mathbb{P}^*$ by (6), can be expressed as

$$\mathrm{d}\ln Y^* = (1-\sigma)\frac{\left(\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{s_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega}+\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}\right)e^{\text{DNL}}-\left(\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}\right)}{1-\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{s_D^\omega}{\varepsilon_D^\omega}-\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}+\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{s_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega}+\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{s_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}} \gtreqless 0. \tag{12}$$

Next, we proceed to analyze the welfare determinants in terms of observables, with a focus on the ambiguous impact on income. The cases previously analyzed have shown the importance of the domestic and export revenue shares of DLs, with each having opposing effects on profits. We will show that the relative importance of these effects is reflected through a DL's export intensity, allowing us to use this statistic to know which effect dominates.

To see this, it can be shown that the sign of (12) is given by

$$\text{sgn}\;(\mathrm{d}\ln Y^*) = \text{sgn}\left(\sum_{\omega\in\Omega_D^{\text{DL}}}\alpha^\omega\eta^\omega\right), \tag{13}$$

where $\alpha^\omega := \frac{R_D^\omega + R_X^\omega}{Y}$ is DL ω's income share and

$$\eta^\omega := e^\omega\left(\frac{1}{\varepsilon_X^\omega}-\frac{1}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}e^{\text{DNL}}\right)-d^\omega\left(\frac{1}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega}e^{\text{DNL}}\right),$$

where $e^\omega := \frac{R_X^\omega}{R_D^\omega + R_X^\omega}$ and $d^\omega := 1-e^\omega$ is DL ω's export and domestic intensity, respectively. In words, $\alpha^\omega > 0$ represents the importance of DL ω's for total income, while $\eta^\omega \gtreqless 0$ reflects whether trade liberalization impacts DL ω's profits positively or negatively.

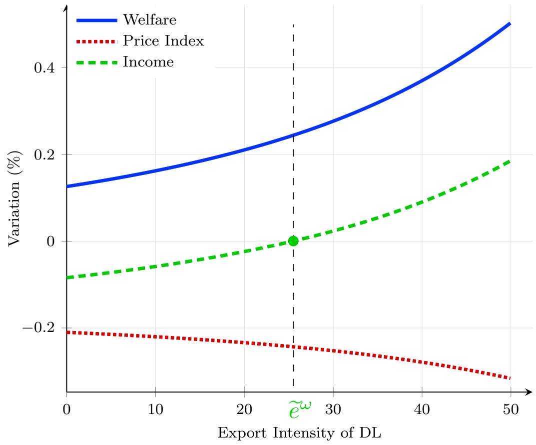
In terms of observables, DL ω's export intensity can be used to identify the sign of η^ω. This follows since higher export intensity of a DL determines that a DL is more benefited from better export conditions and less negatively impacted by increases in competition at home. As a corollary, a DL with a low home bias garners higher profits following trade liberalization, while the opposite happens when a DL has a pronounced home bias.[21]

We illustrate this relation in Figure 5, by considering some DL ω. The graph identifies the critical level of export intensity, $\widetilde{e}^\omega$, that defines a zero contribution to income by ω. In particular, the effects of better export conditions dominate if $e^\omega > \widetilde{e}^\omega$, determining that income increases. Instead, DL ω is highly affected by tougher domestic competition if $e^\omega < \widetilde{e}^\omega$, in

[21] The term η^ω also depends on the DNLs' export intensity. This acts as a proxy for the magnitude in which entry of DNLs occurs, and hence in which competition increases and negatively affects a DL's profit. Specifically, higher export intensity of DNLs entails more pronounced decreases in the price index. We focus on a DL's export intensity since our analysis with Danish data identifies one specific value for the DNLs' export intensity, and so the heterogeneous effects on the DLs' profit are due to the export intensity of each DL.

which case income lowers.

Figure 5. *1% Reduction in Trade Costs: The Role of a DL's Export Intensity (DLs and DNLs Serving Both Markets)*



5 Numerical Analysis

In this section, we perform a numerical analysis based on a calibration for Denmark. We consider an initial scenario with trade costs τ', and a counterfactual situation where trade costs become τ''. For the computation of results, we utilize the "hat-algebra" procedure as in, for instance, Dekle et al. (2008). Following this approach, we respectively denote by x' and x'' the equilibrium in each scenario of any variable x, and its proportional change by $\widehat{x} := \frac{x''}{x'}$. Our analysis considers a proportional variation in trade costs, $\widehat{\tau}$, with results expressed as proportional changes.

Unlike the case of an infinitesimal change in trade costs, the quantification of results requires specifying a productivity distribution for DNLs. Our baseline choice considers that the productivity of DNLs is a random variable with support $\{\varphi^I, \varphi^D, \varphi^X\}$, where $\varphi^I < \varphi^D < \varphi^X$. The superscripts are mnemonics for "inactive", "domestic", and "exporters", according to the role of DNLs in equilibrium: a DNL does not serve any country if it gets φ^I, only serves home if it gets φ^D, and serves both the domestic and foreign market if it gets φ^X.

The productivity distribution chosen allows us to compute the impact on welfare, the price index, and income with the same information as in the case with infinitesimal trade shocks. Consistent with that case too, changes in the DNLs' survival productivity cutoffs have negligible effects on welfare. This makes it possible to interpret results through the analysis in Section 4, and simultaneously keep the procedure parsimonious.[22]

[22]Incorporating a continuous productivity distribution barely changes the results. We formally show this under a bounded Pareto distribution in Section 6.3. We have opted for a simple productivity distribution as a baseline case, since computing gains of trade under a continuous distribution requires making various additional

For the computation of effects in terms of observables, consider DL ω. Respectively define its gross profit in the domestic and exports markets by $\overline{\pi}_D^\omega$ and $\overline{\pi}_X^\omega$, with total gross profit $\overline{\pi}^\omega$. Also, let the fraction of domestic and export revenues in its gross profit be respectively $\phi_D^\omega =: \frac{\overline{\pi}_D^\omega}{\overline{\pi}^\omega} := \frac{d^\omega/\varepsilon_D^\omega}{d^\omega/\varepsilon_D^\omega + e^\omega/\varepsilon_X^\omega}$ and $\phi_X^\omega := 1 - \phi_D^\omega$. Finally, define $\psi^\omega := \frac{\overline{\pi}^\omega}{Y} = \frac{s_D^\omega}{\varepsilon_D^\omega} + \frac{s_X^\omega}{\varepsilon_X^\omega}$, which is the contribution of ω's gross profit to the country's total income.

As we show in Appendix A.3, the computation of effects can be obtained by solving the following system:

$$\widehat{\mathbb{P}} = \left[\frac{1 + \frac{\left(e^{\text{DNL}}\right)'}{\left(d^{\text{DNL}}\right)'}}{\widehat{Y}\left(1 + \frac{\left(e^{\text{DNL}}\right)'}{\left(d^{\text{DNL}}\right)'}\widehat{\tau}^{1-\sigma}\right)}\right]^{\frac{1}{\sigma-1}}, \tag{14a}$$

$$\widehat{Y} = 1 + \sum_{\omega \in \overline{\mathscr{L}}_i} \psi^\omega \left(\widehat{\overline{\pi}}^\omega - 1\right), \tag{14b}$$

$$\widehat{\overline{\pi}}^\omega = 1 + \left(\phi_D^\omega\right)' \left(\widehat{Y}\frac{\widehat{s}_D^\omega}{\widehat{\varepsilon}_D^\omega} - 1\right) + \left(\phi_X^\omega\right)' \left(\widehat{Y}\frac{\widehat{s}_X^\omega}{\widehat{\varepsilon}_X^\omega} - 1\right), \tag{14c}$$

$$\widehat{s}_D^\omega = \frac{\left(\widehat{m}_D^\omega\right)^{1-\sigma}}{\left(\widehat{\mathbb{P}}\right)^{1-\sigma}}, \tag{14d}$$

$$\widehat{s}_X^\omega = \left(\frac{\widehat{m}_X^\omega \widehat{\tau}}{\widehat{\mathbb{P}}}\right)^{1-\sigma}, \tag{14e}$$

$$\widehat{\varepsilon}_{ij}^\omega = \frac{\sigma - \left(s_{ij}^\omega\right)'(\sigma - 1) + \left(1 - \widehat{s}_{ij}^\omega\right)\left(s_{ij}^\omega\right)'(\sigma - 1)}{\sigma - \left(s_{ij}^\omega\right)'(\sigma - 1)}, \tag{14f}$$

$$\widehat{m}_{ij}^\omega = \widehat{\varepsilon}_{ij}^\omega \frac{(\sigma - 1)\left[1 - \left(s_{ij}^\omega\right)'\right]}{\widehat{\varepsilon}_{ij}^\omega\left[\sigma - \left(s_{ij}^\omega\right)'(\sigma - 1)\right] - 1}, \tag{14g}$$

for $i, j \in \mathcal{C}$, each DL ω, and given $\widehat{\tau}$. In turn, this allows us to calculate the impact on welfare through

$$\widehat{\mathbb{W}} = \frac{\widehat{Y}}{\widehat{\mathbb{P}}}. \tag{15}$$

Finally, we can calculate the variation in total gross profits of DLs from i, $\overline{\Pi}_i$, through

$$\widehat{\overline{\Pi}}_i = \sum_{\omega \in \mathscr{L}_i} \left(\lambda^\omega\right)' \widehat{\overline{\pi}}^\omega, \tag{16}$$

where $\lambda^\omega := \frac{\psi^\omega}{\sum_{\omega \in \mathscr{L}_i} \psi^\omega}$ is DL ω's contribution to the total gross profits of DLs from i.

assumptions, identifying additional statistics, and calibrating several additional variables. These aspects make it harder to identify the main drivers of the results.

5.1 Calibration

As indicated in Section 2, the information at our disposal is at the firm-product level. Moreover, we aggregate it to the 4-digit NACE industry level to express it at the firm-industry level. Overall, the data provide us with information on turnover and exports of manufacturing firms, along with imports by both manufacturing and non-manufacturing firms. The calibration employed matches average features of manufacturing industries, using industry-revenue weights. Moreover, we identify variables following standard practices applied to similar European datasets (e.g., Amiti et al. 2018 and Gaubert and Itskhoki 2021).

We consider a firm as domestic if it is included in the Danish Prodcom dataset, so that the classification relies on whether a firm has production activities in Denmark. Also, to identify average features of Danish manufacturing, the calculations are based on a set of industries that are consistent with our theoretical framework. To accomplish this, we first partition firms in each industry into DLs and DNLs. The criterion used is based on each firm's domestic market share, where we employ a 3% as threshold. Additionally, we only keep industries where there is at least one DL and a pool of DNLs, which is ensured following the procedure outlined in Footnote 9.

Using domestic market shares instead of revenue shares to define a DL turns to be a more stringent condition empirically, and also avoids some potential issues.[23] In fact, the criterion used leads us to take the four top firms as DLs, and each of these firms generates at least 5% of the total income. In Appendix B, we show that similar results hold by directly using revenue shares as a criterion to classify firms. Likewise, the specific domestic market share cutoff to define a DL is unimportant for our results, because DLs affect the economy according to their importance for aggregate conditions. Thus, taking firms with low domestic market shares as DLs have a small impact on outcomes. We illustrate this in Section 6.2, where we recalculate results by taking only the top two firms as DLs. The outcomes are virtually the same.

The computation of domestic market shares in each industry requires calculating expenditures, which are defined as the sum of domestic sales and imports. Each firm's domestic revenue is computed as the difference between a firm's total turnover and its export value.[24]

[23] This follows by the results in Section 2, which indicate that almost all of the firms with a domestic market share greater than 3% also have a revenue share greater than 3%. Instead, a classification purely based on revenue shares faces the problem that, since Denmark is a small highly open economy, some of the industries are almost exclusively served by imports. Thus, a domestic firm could accumulate a high revenue share in the industry simply because domestic firms have a low level of operation.

[24] Total turnover is defined by economic ownership of the goods sold and produced by Danish firms. Thus, its definition is not related to the physical territory of the production. Specifically, turnover includes sales of own goods (either produced, processed, or assembled by the firm), goods produced by a subcontractor established abroad (if the firm owns the inputs of the subcontracted firm), and resales of goods bought from other domestic firms and sold with any processing. However, it excludes sales of goods imported and produced by foreign firms not owned by the Danish firm. Due to this, in Appendix C, we define domestic sales of a firm as the sum of

Likewise, we take as imports those acquired by domestic firms inactive in the industry considered. They cover imports by non-manufacturing firms and by domestic firms not producing in that industry.

Finally, we take total turnover as revenue, and split it into domestic and export sales to construct the domestic and export intensity of DNLs. These terms are respectively calculated as the domestic and export sales by DNLs relative to the DNLs' total income. We additionally use the decomposition of revenues to calculate the income shares of each DL, whose values are expressed relative to the industry income.

Using industry revenues as weights, we end up considering an industry whose features are presented in Table 1.

Table 1. *Statistics of Danish Manufacturing*

(a) Revenue Shares (in %)

	Domestic Revenues as % of the country's income	Export Revenues as % of the country's income
Top 1	18.90	11.11
Top 2	8.20	2.00
Top 3	5.14	3.41
Top 4	3.75	2.75

(b) Revenue Intensities (in %)

	Domestic Intensity	Export Intensity
DNLs	68.62	31.38
Top 1	62.98	37.02
Top 2	80.40	19.60
Top 3	60.12	39.88
Top 4	57.63	42.37

Note: Calculations based on industries with coexistence of DNLs and DLs, and using industry revenues as weights. Domestic and export intensity calculated, respectively, as domestic and export sales relative to the total sales of the DNLs or DL considered.

Regarding parameters, we only need to calibrate σ. We utilize the estimates by Soderbery (2015), who employs an augmented version of the methodology by Broda and Weinstein (2006) that accounts for small-sample biases. Averaging across industries with revenue weights, we obtain $\sigma := 3.53$, which we use throughout the paper.

5.2 Welfare Results

Figure 6a presents the impact of trade liberalization for reductions in trade costs between 0% and 10%.[25] The results are computed by solving the system (14). The graph shows that the variation in income is positive, thereby ensuring positive gains of trade. However, the changes in income are almost null, and so the increases in welfare are almost completely explained by the reduction in the price index. Specifically, a reduction in 10% of trade costs increases welfare by 3.87%, with the price index decreasing by 3.60% and income having a modest increase of

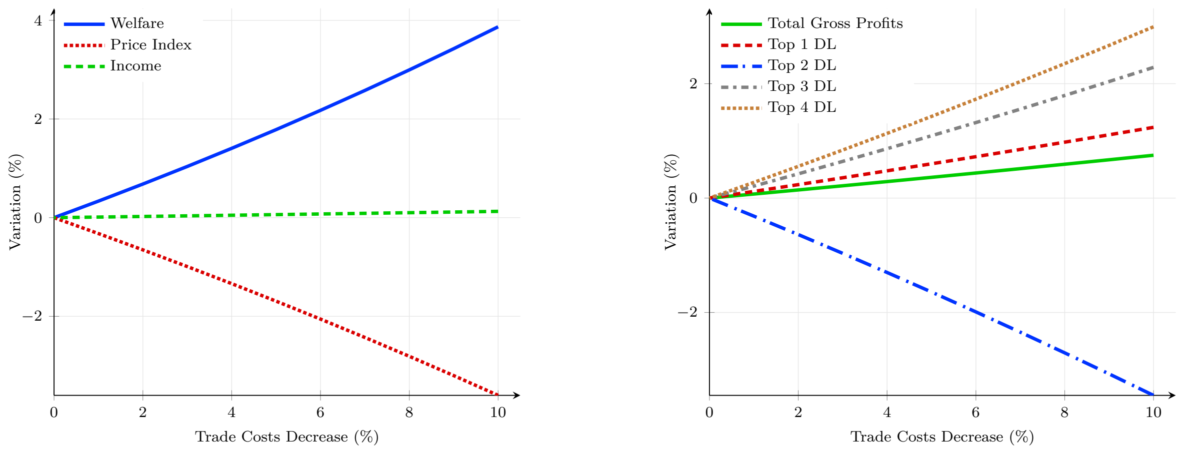
a firm's total turnover and its imports. We show that, by proceeding in this way, market shares are virtually identical.

[25]The range of trade costs chosen is due to two reasons. First, our results are valid as long as there is coexistence of DNLs and DLs. Larger changes in trade costs could imply that one of these groups stops serving the market, entailing a different market structure. Second, our sensitivity results consider a bounded Pareto distribution for the DNLs' productivity, and a solution for larger changes in trade costs under this distribution does not necessarily exist.

0.13%. In terms of their log contribution to welfare, which is almost constant for the range of trade costs considered, the price-index variation accounts for 96.5%, while income only for 3.5%.[26]

Figure 6. *Impact of Trade Liberalization*

(a) Welfare, Price Index, and Income **(b)** DL's Profit



The fact that the variation in income is almost null can be explained by how each DL's profit is affected by trade liberalization. All Danish DLs are exporters, and so trade liberalization entails opposing effects for them: each DL is positively impacted by better export conditions, but also negatively by the increase in domestic competition. Our theoretical results have shown that a DL's export intensity provides information about which of these effects dominates. More precisely, a DL with lower export intensity is less benefited by better export conditions, and simultaneously more exposed to tougher competition at home. Thus, the lower a DL's export intensity, the lower the benefits of trade liberalization that a DL reaps. In fact, trade liberalization can decrease a DL's profit if its export intensity is sufficiently low.

In this respect, Table 1b shows that DLs starkly differ in terms of their export intensity, and so they are differentially impacted by trade liberalization. In particular, the export intensity of the second top DL is relatively low compared with the rest of the DLs. This implies that the second top DL is primarily affected by tougher competition at home, while the rest of the DLs mainly benefit from better export conditions. This explains the results in Figure 6b, where the profit of the second top DL decreases following trade liberalization, whereas the profits of the rest of the DLs increase. Overall, both effects are almost completely offset in terms of aggregate profits, determining increases in income that are positive but almost null.

The result highlights the granularity of Danish manufacturing, where the idiosyncratic features of even one DL can substantially alter outcomes. To put the result in context, the

[26] Formally, we express (15) as $\ln \widehat{\mathbb{W}} = \ln \widehat{Y} - \ln \widehat{\mathbb{P}}$, and determine the contribution of each term by using that $\frac{\ln \widehat{Y}}{\ln \widehat{\mathbb{W}}} - \frac{\ln \widehat{\mathbb{P}}}{\ln \widehat{\mathbb{W}}} = 1$.

International-Trade literature has documented several times the high correlation between exporting and productivity (see, for instance, Mayer and Ottaviano 2008 and Bernard et al. 2012). While this result holds as an average feature, it does not rule out that one highly profitable firm can be domestic-oriented. This is key in our model, since a firm with this feature is negatively affected by trade liberalization and, depending on its size, can have a significant impact on gains of trade. Indeed, the pronounced home bias of the second top Danish DL critically affects the gains of trade through the profit channel.

6 Additional Results and Sensitivity Analysis

In this section, we begin by comparing gains of trade relative to monopolistic competition. Then, we show that our numerical results are not sensitive to either the cutoff to define a firm as a DL or whether the productivity distribution for DNLs is continuous.

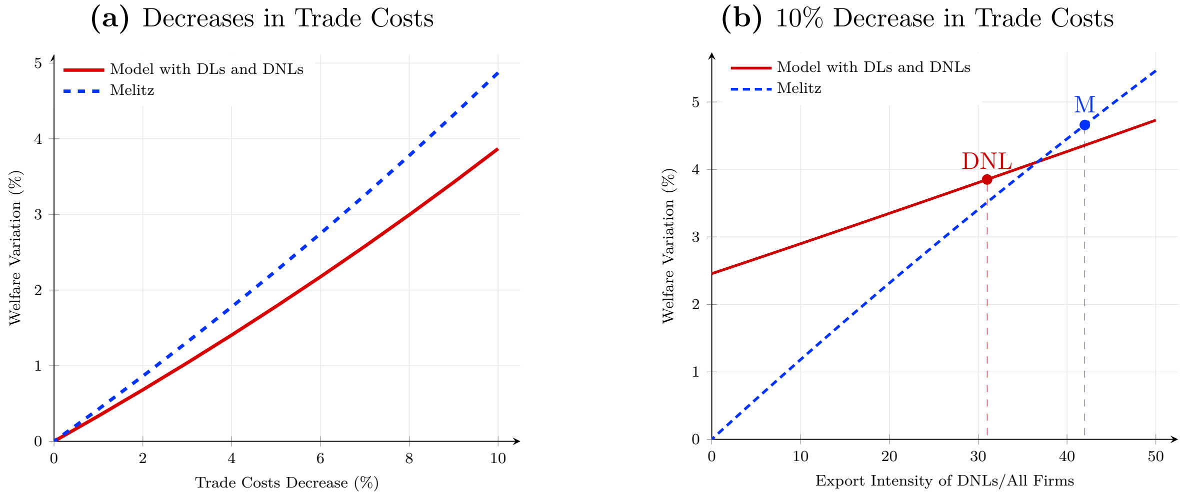
6.1 Monopolistic Competition

One important message from Section 2 is that the bulk of income is explained by industries comprising a set of insignificant firms. This feature is consistent with monopolistic competition, which is the standard market structure assumed in the International-Trade field. Nonetheless, a pure monopolistic competition means that we actually treat all firms as insignificant, including DLs. Thus, it ignores that DLs are better described as oligopolistic firms. To study the consequences of this, next we compare the results in our setting relative to a scenario where all firms are considered monopolistic.

With this goal, we exploit that our setting collapses to the Melitz model when the set of DLs is assumed empty. This implies that income is not impacted by trade liberalization, since wages are taken as the numéraire and aggregate profits are always zero. A corollary of this is that gains of trade are exclusively given by the variation in the price index, and this is computed by (14a) under $\widehat{Y} = 1$.

Welfare outcomes under each model are presented in Figure 7a, and reveal that gains of trade in Melitz are somewhat greater than in our setting. To explain this, we can rely on the analysis of Section 4.1, where all firms are treated as DNLs and so our model collapses to Melitz. It establishes that the price index's variation equals the DNLs' export intensity, where the relevant export intensity is computed by considering all firms (i.e. both DNLs and DLs). In terms of the Danish data, this export intensity is 41.9%, which is higher than the DNLs' export intensity in our model. The reason is that the export intensity of all firms includes that of DLs, which are firms that on average exhibit a higher export intensity than DNLs.

Figure 7. *Welfare in each Setting*



Taking this into account, there are two potential sources explaining the differences in welfare gains between our model and Melitz:

[i] *Changes in income*: Trade liberalization in Melitz has no impact on aggregate profits, while the model accounting for DNLs and DLs entails positive aggregate profits. Moreover, changes in aggregate profits affect welfare directly through their impact on income, and also indirectly through their impact on the price index.

[ii] *Changes in the average export intensity of firms*: DNLs in our model discard from consideration the productivity draws of DLs. Consequently, it becomes more likely that DNLs eventually only serve the domestic market or export at a low scale. This affects the DNLs' entry choices, since they are made by expecting a lower export intensity relative to Melitz. Ultimately, this is reflected in Melitz entailing a greater impact of trade liberalization on the price index.

Next, we argue that the difference in results is explained by *ii)*. To do this, we utilize Figure 7b, which allows us to isolate *i)* and *ii)*. The graph computes welfare in our setting for different values of DNLs' export intensity, and also welfare in Melitz for different values of all firms' export intensity. The red dot indicates the change in welfare in our setting, since it is computed with the export intensity observed for Danish DNLs. Likewise, the blue dot corresponds to the welfare variation under Melitz, which is computed considering the export intensity of all Danish firms.

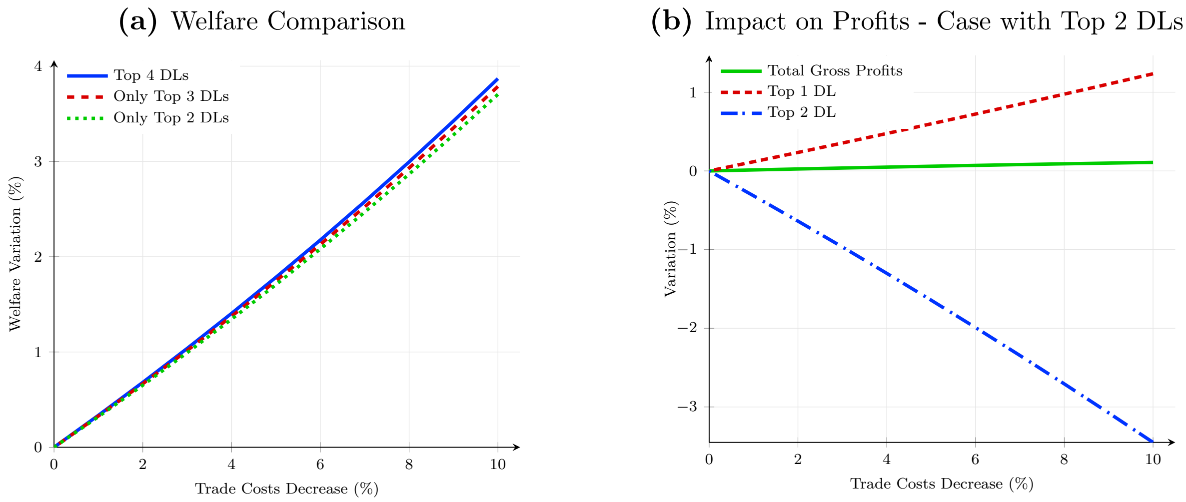
To isolate the effects due to *i)*, suppose that the export intensity of all firms is equal to the DNLs' export intensity (i.e., 31.4%). Under this assumption, the results across models only differ because our setting accounts for the impact of the DLs' profits on income. Figure 7b reveals that welfare evaluated at this export intensity predicts higher gains of trade in our model, although the difference is quite small. This is consistent with our setup predicting increases in income, but in an insignificant magnitude.

On the contrary, once we let the export intensity vary, welfare in Melitz is higher. This reflects that the export intensity of all firms is greater than that of DNLs, and so the reductions in the price index under Melitz are more pronounced. It captures that trade liberalization increases more a firm's expected profits in Melitz and hence induces more pronounced entry, since firms consider that becoming a DL is possible.

6.2 Alternative Cutoffs to Define Domestic Leaders

The procedure for computing results requires splitting firms into DLs and DNLs. Here, we show that the outcomes are not sensitive to the cutoff considered for the partition. With this goal, in Figure 8a we replicate the analysis by only considering the top three or top two firms as DLs. The graph demonstrates that the results slightly change. For instance, the increases in welfare for a reduction in 10% of trade costs are 3.79% taking the top three firms as DLs, and 3.70% taking the top two firms as DLs. This contrasts with the 3.86% obtained in the baseline model with the top four firms as DLs.

Figure 8. *Impact of Trade Liberalization*



The small differences are explained because the lower a firm's revenue share, the lower its influence on aggregate outcomes. As a corollary, results are primarily driven by the top two firms, which are the firms that generate the bulk of profits and therefore have the greatest impact on income. This is demonstrated in Figure 8b, which establishes that the variation in income is almost zero. This follows for the same reasons as in the baseline case: trade liberalization reduces the second top firm's profit in a magnitude that almost completely offsets the increase in profit of the top first firm.[27]

[27] In Appendix B, we recalculate all the empirical results for a calibration where DLs and DNLs are classified

6.3 Bounded Pareto Productivity Distribution for Domestic Non-Leaders

Under infinitesimal variations in trade costs, the choice of DNLs' productivity distribution has no bearing in the results—the variation in welfare is independent of it. This explains why we did not have to choose a specific distribution for the analysis in Section 4, where a small change in trade costs is considered. The reason is that the exit of the least-productive DNLs has a trivial effect on the gains from trade. This occurs since the DNLs' productivity distribution only affects the magnitude in which the survival productivity cutoff changes; and firms in the neighborhood of this productivity level have a negligible impact on the DNLs' expected profits, and hence on the price index.[28]

As for the empirical analysis, the impact on welfare through the exit of the least productive firms was also absent, even when we considered a discrete change in trade costs: we chose a simple discrete productivity distribution for DNLs, entailing no changes in the survival productivity cutoffs. Next, we show that our results are virtually the same if we allow for a continuous productivity distribution of DNLs. Consequently, accounting for changes in welfare due to the exit of the least-productive DNLs does not modify our conclusions.[29]

Consider that the productivity of DNLs follows a bounded Pareto distribution. Following Helpman et al. (2008), we express the model in terms of $a := 1/\varphi$, whose distribution has finite support $[a_L, a_H]$ and cdf given by

$$G(a) := \frac{(a)^k - (a_L)^k}{(a_H)^k - (a_L)^k},$$

where the inverse of the survival productivity cutoff in each market is respectively denoted by a_D and a_X.

The computation of outcomes for this case requires calibrating several additional parameters and addressing several issues regarding existence of a solution. Due to this, we relegate the details to Appendix E, and here present the results directly.

Under this productivity distribution, gains of trade are obtained by solving the system of

according to total revenue, rather than domestic market shares. This broadens the scope of large firms to include big exporters. Under this alternative, we show that similar qualitative conclusions are obtained. This is consistent with the empirical fact stated in Section 2, which indicates that being a DL is an almost sufficient condition for being among the firms with highest revenues in each industry.

[28] This property also arises in the Melitz model under symmetric countries, as Melitz and Redding (2015) and Arkolakis et al. (2019) notice. In fact, it follows for the same reasons.

[29] This is in part because we consider relatively small changes in trade costs. This is necessary to obtain results (see Footnote 25).

equations (14b)-(14g), and substituting (14a) by the following equation:

$$\widehat{Y}\left(\widehat{\mathbb{P}}\right)^{\sigma-1}\left[\left(\widehat{\tau}\right)^{1-\sigma}\left(e^{\mathrm{DNL}}\right)'\frac{\left(\frac{1}{\tau'}\frac{\widehat{\mathbb{P}}}{\widehat{\tau}}\right)^{k-\sigma+1}\left(\frac{f_D}{f_X}\widehat{Y}\right)^{\frac{k-\sigma+1}{\sigma-1}}\left(a'_D/a_L\right)^{k-\sigma+1}-1}{\left(\frac{1}{\tau'}\right)^{k-\sigma+1}\left(\frac{f_D}{f_X}\right)^{\frac{k-\sigma+1}{\sigma-1}}\left(a'_D/a_L\right)^{k-\sigma+1}-1}+\left(d^{\mathrm{DNL}}\right)'\frac{\left(\widehat{\mathbb{P}}\right)^{k-\sigma+1}\left(\widehat{Y}\right)^{\frac{k-\sigma+1}{\sigma-1}}\left(a'_D/a_L\right)^{k-\sigma+1}-1}{\left(a'_D/a_L\right)^{k-\sigma+1}-1}\right]-1=$$

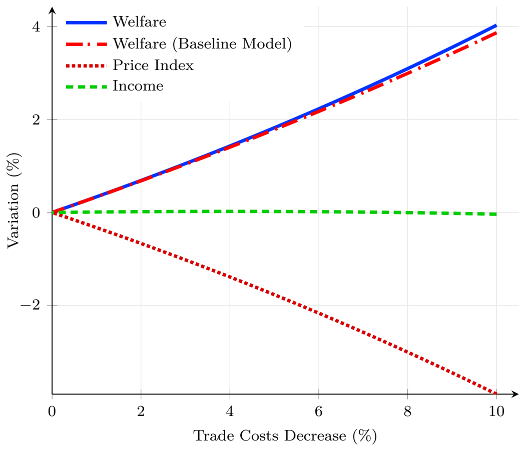
$$\frac{\sigma F_D}{\left(R^{\mathrm{DNL}}\right)'}\left[\left(\frac{f_X}{f_D}\frac{\left(\tau'\right)^{-k}\left(\frac{f_D}{f_X}\right)^{\frac{k}{\sigma-1}}\left(a'_D/a_L\right)^{k}-1}{\left(a'_D/a_L\right)^{k}-1}\right)\left(\frac{\left(\frac{1}{\tau'}\frac{\widehat{\mathbb{P}}}{\widehat{\tau}}\right)^{k}\left(\frac{f_D}{f_X}\widehat{Y}\right)^{\frac{k}{\sigma-1}}\left(a'_D/a_L\right)^{k}-1}{\left(\frac{1}{\tau'}\right)^{k}\left(\frac{f_D}{f_X}\right)^{\frac{k}{\sigma-1}}\left(a'_D/a_L\right)^{k}-1}-1\right)+\left(\frac{\left(\widehat{\mathbb{P}}\right)^{k}\left(\widehat{Y}\right)^{\frac{k}{\sigma-1}}\left(a'_D/a_L\right)^{k}-1}{\left(a'_D/a_L\right)^{k}-1}-1\right)\right],$$

where F_D is the total domestic fixed costs spent by the active DNLs, and R^{DNL} is the DNLs' total revenues.

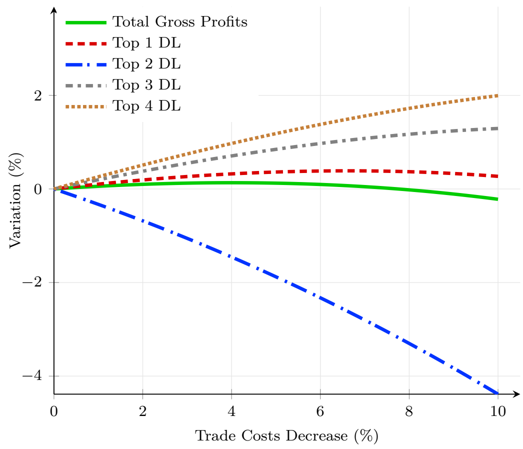
The results are presented in Figure 9. They indicate that the increase in welfare is 4.03% following a reduction in 10% of trade costs, instead of 3.86% as in the baseline model.

Figure 9. *Impact of Trade Liberalization - Bounded Pareto Distribution for DNLs*

(a) Welfare, Price Index, and Income



(b) Profits



7 Conclusion

In this paper, we documented the relevance of industries comprising a few large firms and numerous negligible firms for aggregate analyses. Considering Danish manufacturing, we showed that these industries explain the bulk of revenue, even when they cover a little bit more than half of the total. A corollary of this is that the economy is "granular", so that the idiosyncratic way in which a shock impacts each large firm has aggregate consequences.

Based on this evidence, we analyzed the implications of this market structure for gains of trade. We proposed a monopolistic-competition model à la Melitz, with a set of non-negligible firms embedded that impact aggregate conditions and earn positive profits. Our framework highlights how the specific features of large firms are crucial for the impact on profits and hence income. In particular, we established that a large firm with a low home bias/great export intensity benefits from trade, given that it is primarily benefited from better export opportunities. On the contrary, trade liberalization increases competition, and so the opposite occurs if a large firm exhibits a high home bias/low export intensity—this firm would be predominantly affected by tougher domestic competition, with small benefits from better export opportunities.

Given the granular importance of large firms, our framework implies that the effects of trade on even one firm can substantially affect welfare through its impact on profits. This feature was reflected in the gains of trade of Danish manufacturing. Specifically, trade liberalization raises aggregate profits, thereby increasing welfare through the profit channel. However, the variation in aggregate profits was almost null, since trade liberalization entails a heterogeneous impact on large firms: the second top firm has a fall in profit that almost entirely offsets the increases in profits by the rest of DLs.

Our findings provide some caution about policy recommendations that account for superstar firms. Some authors have recently remarked that several interventions could entail magnified positive effects once we consider the impact on large firms (e.g., Freund and Pierola 2015; Gaubert and Itskhoki 2021). In this respect, our results suggest that the impact of a policy depends crucially on the large firms' idiosyncratic features, precluding the prescription of identical policies for every country. In other words, by the very definition of granularity, the set of large firms should not be treated as a group of homogeneous objects—they are firms with specific features, and the assessment of a policy can be starkly different even if only one firm is differentially impacted.

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Online Appendix - not for publication

In Appendix A, we provide derivations and proofs for the results in the main body of the paper. In Appendix B, we calibrate the model to an alternative where top Danish firms are defined by total industry revenues. In Appendix C, we consider alternative approaches to compute domestic market shares to show the robustness of our conclusions regarding the importance of domestic leaders. In Appendix D, we provide an example of negative gains of trade. Finally, in Appendix E, we recalculate all the results by assuming that the productivity distribution of DNLs follows a bounded Pareto.

A Derivations and Proofs

In Appendix A.1, we provide some intermediate results that help us derive final results more easily. To keep notation simple, we occasionally omit arguments from the functions.

A.1 Intermediate Results

Next, we derive some intermediate results for prices and market shares (Appendix A.1.1) and total profits of DLs (Appendix A.1.2).

A.1.1 Prices and Market Shares

Given optimal prices, we proceed to determine the partial effect of the price index on the market share and price elasticity of a DL ω from $i \in \mathcal{C}$ in $j \in \mathcal{C}$. Regarding the effect of market share on prices, by (3), we can express domestic prices by $\ln p_{ij}^{\omega} = \ln m_{ij}^{\omega} + \ln c_{ij}^{\omega}$. Therefore,

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{\partial \ln m_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{\partial \ln m_{ij}^{\omega}}{\partial \ln \varepsilon_{ij}^{\omega}} \frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}}.$$

In turn, $\ln m_{ij}^{\omega} = \ln \varepsilon_{ij}^{\omega} - \ln \left(\varepsilon_{ij}^{\omega} - 1\right)$ and $\varepsilon_{ij}^{\omega} = \sigma + s_{ij}^{\omega}(1-\sigma)$. Consequently, $\frac{\partial \ln m_{ij}^{\omega}}{\partial \ln \varepsilon_{ij}^{\omega}} = 1 - \frac{\varepsilon_{ij}^{\omega}}{\varepsilon_{ij}^{\omega}-1} = 1 - m_{ij}^{\omega}$, and $\frac{\partial \varepsilon_{ij}^{\omega}}{\partial s_{ij}^{\omega}} = 1-\sigma$ so that $\frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{s_{ij}^{\omega}(1-\sigma)}{\varepsilon_{ij}^{\omega}}$. Thus,

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \left(1 - m_{ij}^{\omega}\right) \frac{s_{ij}^{\omega}(1-\sigma)}{\varepsilon_{ij}^{\omega}},$$

which, by using that $1 - m_{ij}^{\omega} = \frac{-1}{\varepsilon_{ij}^{\omega}-1}$ and $\varepsilon_{ij}^{\omega} - 1 = (\sigma - 1)\left(1 - s_{ij}^{\omega}\right)$, becomes

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{s_{ij}^{\omega}}{\left(1 - s_{ij}^{\omega}\right)\varepsilon_{ij}^{\omega}}. \tag{A1}$$

Substituting (3) into (2), market shares satisfy $s_{ij}^{\omega} = \left(\frac{p_{ij}^{\omega}(s_{ij}^{\omega})}{\mathbb{P}_j}\right)^{1-\sigma}$, which determines an implicit function $s_{ij}^{\omega}(\mathbb{P}_i)$. Differentiating it, we obtain $\mathrm{d}\ln s_{ij}^{\omega}\left[1-(1-\sigma)\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}}\right] = (1-\sigma)\,\mathrm{d}\ln \mathbb{P}_j$. Working out the expression, and using (A1) and that $(\sigma-1)\,s_{ij}^{\omega} = \sigma - \varepsilon_{ij}^{\omega}$, we obtain the following

$$\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} = (\sigma - 1)\frac{\varepsilon_{ij}^{\omega} - \varepsilon_{ij}^{\omega} s_{ij}^{\omega}}{\sigma - \varepsilon_{ij}^{\omega} s_{ij}^{\omega}}. \tag{A2}$$

Moreover, using (A2), we establish that

$$\frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} = \frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} \frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} = \frac{s_{ij}^{\omega}\left(1-\sigma\right)}{\varepsilon_{ij}^{\omega}} \frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j}. \tag{A3}$$

As for a DNL ω, by using (3), the partial effects of price index and trade costs on its market share in j is

$$\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} = -\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \tau_{ij}} = \sigma - 1. \tag{A4}$$

A.1.2 Total Profits of DLs

Next, we determine the partial effects of expenditure, price index, and trade costs on total profits of DLs. The sum of profits of DLs from $i \in \mathcal{C}$ can be expressed as

$$\Pi_i = \sum_{\omega \in \Omega_{ii}^{\mathrm{DL}}} \left(\frac{Y_i s_{ii}^{\omega}\left(\mathbb{P}_i\right)}{\varepsilon_{ii}^{\omega}\left(\mathbb{P}_i\right)} - w_i f_{ii} \right) + \sum_{j \in \mathcal{C} \backslash \{i\}} \sum_{\omega \in \Omega_{ij}^{\mathrm{DL}}} \left(\frac{Y_j s_{ij}^{\omega}\left(\mathbb{P}_j; \tau_{ij}\right)}{\varepsilon_{ij}^{\omega}\left(\mathbb{P}_j\right)} - w_i f_{ij} \right), \tag{A5}$$

which determines that if $\mathrm{d} \ln Y_k \neq 0$, $\mathrm{d} \ln \mathbb{P}_i \neq 0$, $\mathrm{d} \ln \mathbb{P}_j \neq 0$, and $\mathrm{d} \ln \tau_{ij} \neq 0$, with $j \neq i$ and $k \in \mathcal{C}$, then

$$\mathrm{d}\Pi_i = \left(\sum_{\omega \in \Omega_{ik}^{\mathrm{DL}}} \frac{R_{ik}^{\omega}}{\varepsilon_{ik}^{\omega}} \right) \mathrm{d} \ln Y_k + \left[\sum_{\omega \in \Omega_{ii}^{\mathrm{DL}}} \frac{R_{ii}^{\omega}}{\varepsilon_{ii}^{\omega}} \left(\frac{\partial \ln s_{ii}^{\omega}}{\partial \ln \mathbb{P}_i} - \frac{\partial \ln \varepsilon_{ii}^{\omega}\left(\mathbb{P}_i\right)}{\partial \ln \mathbb{P}_i} \right) \right] \mathrm{d} \ln \mathbb{P}_i + \left[\sum_{\omega \in \Omega_{ij}^{\mathrm{DL}}} \frac{R_{ij}^{\omega}}{\varepsilon_{ij}^{\omega}} \left(\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} - \frac{\partial \ln \varepsilon_{ij}^{\omega}\left(\mathbb{P}_j\right)}{\partial \ln \mathbb{P}_j} \right) \right] \mathrm{d} \ln \mathbb{P}_j + \left[\sum_{\omega \in \Omega_{ij}^{\mathrm{DL}}} \frac{R_{ij}^{\omega}}{\varepsilon_{ij}^{\omega}} \frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \tau_{ij}} \right] \mathrm{d} \ln \tau_{ij}.$$

To obtain each of the partial derivatives of this expression, by (A2), (A3) and working out the expression, it is determined that, for $k \in \mathcal{C}$,

$$\frac{\partial \ln s_{ik}^{\omega}\left(\mathbb{P}_k\right)}{\partial \ln \mathbb{P}_k} - \frac{\partial \ln \varepsilon_{ik}\left[s_{ik}^{\omega}\left(\mathbb{P}_k\right)\right]}{\partial \ln \mathbb{P}_k} = \left(\sigma - 1\right) \frac{\varepsilon_{ik}^{\omega} - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}{\sigma - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}} \left[1 - \frac{s_{ik}^{\omega}\left(1-\sigma\right)}{\varepsilon_{ik}^{\omega}} \right],$$

which, using that $s_{ik}^{\omega}\left(1-\sigma\right) = \varepsilon_{ik}^{\omega} - \sigma$, becomes

$$\frac{\partial \ln s_{ik}^{\omega}\left(\mathbb{P}_k\right)}{\partial \ln \mathbb{P}_k} - \frac{\partial \ln \varepsilon_{ik}\left[s_{ik}^{\omega}\left(\mathbb{P}_k\right)\right]}{\partial \ln \mathbb{P}_k} = \frac{\sigma\left(\sigma - 1\right)}{\varepsilon_{ik}^{\omega}} \frac{\varepsilon_{ik}^{\omega} - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}{\sigma - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}} = \left(\sigma - 1\right) \frac{\sigma - \sigma s_{ik}^{\omega}}{\sigma - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}.$$

In addition, by using (A4) and for $k \in \mathcal{C}$ and $j \neq \{i\}$, it is determined that

$$\frac{\partial \Pi_i}{\partial \ln Y_k} = \sum_{\omega \in \Omega_{ik}^{\mathrm{DL}}} \frac{R_{ik}^{\omega}}{\varepsilon_{ik}^{\omega}}, \tag{A6}$$

$$\frac{\partial \Pi_i}{\partial \ln \mathbb{P}_k} = \left(\sigma - 1\right) \sum_{\omega \in \Omega_{ik}^{\mathrm{DL}}} \frac{R_{ik}^{\omega}}{\varepsilon_{ik}^{\omega}} \frac{\sigma - \sigma s_{ik}^{\omega}}{\sigma - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}, \tag{A7}$$

$$\frac{\partial \Pi_i}{\partial \ln \tau_{ij}} = -\left(\sigma - 1\right) \sum_{\omega \in \Omega_{ij}^{\mathrm{DL}}} \frac{R_{ij}^{\omega}}{\varepsilon_{ij}^{\omega}}. \tag{A8}$$

A.2 Section 3.5

Next, we provide derivations for the results included in Section 3.5, where trade liberalization between two symmetric countries is considered. We perform the calculations to obtain the partial effects of the trade shock on income (Appendix A.2.1), the total effects on price index and income (Appendix A.2.2), and the condition for the sign of the income effect (Appendix A.2.3).

A.2.1 Partial Effects on Income

We start by obtaining the partial effects of the price index and trade costs on income. This requires characterizing $Y\left(\mathbb{P}; \tau\right)$, which is the implicit solution to (INC). In turn, (INC) for i depends on the total profits of its DLs, which are given by (PROF).

Given the symmetry of countries and wages chosen as the numéraire, the total profit of DLs can be reexpressed as

$$\Pi_i = \left[\sum_{\omega \in \Omega_D^{\mathrm{DL}}} \left(\frac{Y(\mathbb{P};\tau)\, s_D^\omega(\mathbb{P})}{\varepsilon\left[s_D^\omega(\mathbb{P})\right]} - f_D \right) + \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \left(\frac{Y(\mathbb{P};\tau)\, s_X^\omega(\mathbb{P};\tau)}{\varepsilon\left[s_X^\omega(\mathbb{P};\tau)\right]} - f_X \right) \right]. \tag{A9}$$

By (A6), (A7), and (A8), we can establish that, given $i, j \in \mathcal{C} := \{H, F\}$ with $i \neq j$, the partial effects on total profits are

$$\frac{\partial \Pi_i}{\partial \ln Y} = \frac{\partial \Pi_i}{\partial \ln Y_i} + \frac{\partial \Pi_i}{\partial \ln Y_j} = \sum_{\omega \in \Omega_D^{\mathrm{DL}}} \frac{R_D^\omega}{\varepsilon_D^\omega} + \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \frac{R_X^\omega}{\varepsilon_X^\omega},$$

$$\frac{\partial \Pi_i}{\partial \ln \mathbb{P}} = \frac{\partial \Pi_i}{\partial \ln \mathbb{P}_i} + \frac{\partial \Pi_i}{\partial \ln \mathbb{P}_j} = (\sigma - 1) \left[\sum_{\omega \in \Omega_D^{\mathrm{DL}}} \frac{R_D^\omega}{\varepsilon_D^\omega} \frac{\sigma - \sigma s_D^\omega}{\sigma - \varepsilon_D^\omega s_D^\omega} + \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \frac{R_X^\omega}{\varepsilon_X^\omega} \frac{\sigma - \sigma s_X^\omega}{\sigma - \varepsilon_X^\omega s_X^\omega} \right],$$

$$\frac{\partial \Pi_i}{\partial \ln \tau} = \sum_{\omega \in \Omega_X^{\mathrm{DL}}} (1 - \sigma) \frac{R_X^\omega}{\varepsilon_X^\omega}.$$

Differentiating (INC), the partial effects on income are given by

$$\mathrm{d}\ln Y \left(Y - \frac{\partial \Pi_i}{\partial \ln Y} \right) = \frac{\partial \Pi_i}{\partial \ln \mathbb{P}} \mathrm{d}\ln \mathbb{P},$$

$$\mathrm{d}\ln Y \left(Y - \frac{\partial \Pi_i}{\partial \ln Y} \right) = \frac{\partial \Pi_i}{\partial \ln \tau} \mathrm{d}\ln \tau,$$

and, so,

$$\frac{\partial \ln Y}{\partial \ln \mathbb{P}} = (\sigma - 1) \frac{\sum_{\omega \in \Omega_D^{\mathrm{DL}}} \frac{R_D^\omega}{\varepsilon_D^\omega} \frac{\sigma - \sigma s_D^\omega}{\sigma - \varepsilon_D^\omega s_D^\omega} + \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \frac{R_X^\omega}{\varepsilon_X^\omega} \frac{\sigma - \sigma s_X^\omega}{\sigma - \varepsilon_X^\omega s_X^\omega}}{Y - \sum_{\omega \in \Omega_D^{\mathrm{DL}}} \frac{R_D^\omega}{\varepsilon_D^\omega} - \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \frac{R_X^\omega}{\varepsilon_X^\omega}}, \tag{A10}$$

$$\frac{\partial \ln Y}{\partial \ln \tau} = (1 - \sigma) \frac{\sum_{\omega \in \Omega_X^{\mathrm{DL}}} \frac{R_X^\omega}{\varepsilon_X^\omega}}{Y - \sum_{\omega \in \Omega_D^{\mathrm{DL}}} \frac{R_D^\omega}{\varepsilon_D^\omega} - \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \frac{R_X^\omega}{\varepsilon_X^\omega}}. \tag{A11}$$

A.2.2 Total Impact on Price Index and Income

Substituting in $Y^* := Y(\mathbb{P}^*;\tau)$, (FE) for $i \in \mathcal{C}$ becomes

$$\pi_i^{\mathbb{E},\mathrm{DNL}} := \int_{\varphi_D^*}^{\overline{\varphi}} \left[\frac{R_D(\mathbb{P}^*, Y^*, \varphi)}{\sigma} - f_D \right] \mathrm{d}G(\varphi) + \int_{\varphi_X^*}^{\overline{\varphi}} \left[\frac{R_X(\mathbb{P}^*, Y^*, \varphi; \tau)}{\sigma} - f_X \right] \mathrm{d}G(\varphi) = F, \tag{A12}$$

where $\varphi_D^* := \varphi_D(\mathbb{P}^*, Y^*)$ and $\varphi_X^* := \varphi_X(\mathbb{P}^*, Y^*; \tau)$.

Differentiating (A12), the total effect of τ on the price index is given by

$$\frac{\mathrm{d}\ln \mathbb{P}^*}{\mathrm{d}\ln \tau} = -\left(\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \tau} \right) \left(\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \mathbb{P}} \right)^{-1},$$

where

$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \mathbb{P}} = \frac{\partial \pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial \ln \mathbb{P}} + \frac{\partial \pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial \ln Y} \frac{\partial \ln Y^*}{\partial \ln \mathbb{P}},$$

$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \tau} = \frac{\partial \pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial \ln \tau} + \frac{\partial \pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial \ln Y} \frac{\partial \ln Y^*}{\partial \ln \tau}.$$

To get an expression for each term, let

$$r_{ij}^{\mathrm{DNL}} := \frac{R_{ij}^{\mathrm{DNL}}}{M_i^E}$$

where R_{ij}^{DNL} are the total revenues in j of DNLs as a group:

$$R_{ij}^{\mathrm{DNL}}\left(\mathbb{P},Y,M^{E};\tau\right):=M^{E}\int_{\varphi_{ij}^{*}}^{\overline{\varphi}}R\left(\mathbb{P},Y,\varphi;\tau\right)\,\mathrm{d}G\left(\varphi\right).$$

It can be shown that $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln\mathbb{P}}=\frac{\sigma-1}{\sigma}\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)$, $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln\tau}=\frac{1-\sigma}{\sigma}r_X^{\mathrm{DNL}}$, and $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln Y}=\frac{r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}}{\sigma}$. Thus,

$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln\mathbb{P}}=\frac{\sigma-1}{\sigma}\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)\left[1+\frac{\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}+\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}\right],$$

$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln\tau}=\frac{1-\sigma}{\sigma}\left[r_X^{\mathrm{DNL}}+\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)\frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}\right].$$

Therefore,

$$\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}=\frac{r_X^{\mathrm{DNL}}+\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)\frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}}{\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)\left[1+\frac{\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}+\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}\right]}.$$

To obtain (6) included in the main body of the paper, we have to take the following steps. First, we divide numerator and denominator by $r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}$ and Y:

$$\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}=\frac{\frac{r_X^{\mathrm{DNL}}}{r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}}+\frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^{\omega}}{\varepsilon_X^{\omega}}}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^{\omega}}{\varepsilon_X^{\omega}}}}{1+\frac{\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}+\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^{\omega}}{\varepsilon_X^{\omega}}}}.$$

After this, for the term $\frac{r_X^{\mathrm{DNL}}}{r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}}$, we divide numerator and denominator by the revenue generated by DNLs divided by M^{E*}. Then, (6) is obtained.

Regarding the total effect of τ on the equilibrium income, given by (7), this is determined by

$$\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}=\frac{\partial\ln Y^{*}}{\partial\ln\tau}+\frac{\partial\ln Y^{*}}{\partial\ln\mathbb{P}}\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}.$$

By using (A10) and (A11), we obtain

$$\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}=(1-\sigma)\frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}+(\sigma-1)\frac{\left(\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}+\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}\right)\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}},$$

and, dividing and multiplying numerator and denominator by Y and gathering terms, we end up with (7).

A.2.3 Derivations and Proofs for Section 4

The derivations of the expressions in Section 4 and the proofs of their propositions can be established by reexpressing the effect on the price index and income. Regarding the former, this is given by (6). As for the impact on income, we need to show that (7) can be expressed as (12), and that its sign is given by (13).

To streamline notation, we first define some variables. Let $\chi_1:=\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^{\omega}}{\varepsilon_X^{\omega}}$, $\chi_2:=\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}$, $\delta_1:=\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^{\omega}}{\varepsilon_D^{\omega}}$, and $\delta_2:=\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}$. Therefore, the impact on the price index is given by

$$\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}=\frac{e^{\mathrm{DNL}}+\frac{\chi_1}{1-\delta_1-\chi_1}}{1+\frac{\delta_2+\chi_2}{1-\delta_1-\chi_1}}=\frac{\left(1-\delta_1-\chi_1\right)e^{\mathrm{DNL}}+\chi_1}{1-\delta_1-\chi_1+\delta_2+\chi_2}, \tag{A13}$$

where the second equality follows by multiplying numerator and denominator by $1-\delta_1-\chi_1$.

Furthermore, the effect on the equilibrium income is

$$\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}=\frac{(\sigma-1)}{1-\delta_1-\chi_1}\left[\left(\delta_2+\chi_2\right)\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}-\chi_1\right], \tag{A14}$$

which establishes that

$$\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}>0 \text{ iff } \left(\delta_2+\chi_2\right)\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}>\chi_1.$$

Substituting in by (A13), the condition is

$$\left(\delta_2+\chi_2\right)\left(\frac{\left(1-\delta_1-\chi_1\right)e^{\mathrm{DNL}}+\chi_1}{1-\delta_1-\chi_1+\delta_2+\chi_2}\right)>\chi_1,$$

and, by working it out, it is established that it is reduced to

$$\left(\delta_2+\chi_2\right)e^{\mathrm{DNL}}-\chi_1>0.$$

Thus,

$$\begin{aligned}
\operatorname{sgn}\left(\mathrm{d}\ln Y^{*}\right)&=\operatorname{sgn}\left[\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^{\omega}}{\varepsilon_X^{\omega}}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{s_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}e^{\mathrm{DNL}}-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{s_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}e^{\mathrm{DNL}}\right],\\
&=\operatorname{sgn}\left[\frac{1}{Y}\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}-\frac{1}{Y}\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}e^{\mathrm{DNL}}-\frac{1}{Y}\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}e^{\mathrm{DNL}}\right].
\end{aligned}$$

Multiplying and dividing each term in the sum by $R_D^{\omega}+R_X^{\omega}$, and using that $d^{\omega}=1-e^{\omega}$, then we obtain (13).

Next, we show that the impact on income can be expressed as in (12). In terms of the streamlined notation, (7) is (A14). Moreover, by using (A13) and working out the expression, this becomes

$$\begin{aligned}
\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}&=\frac{(\sigma-1)}{1-\delta_1-\chi_1}\left[\frac{\left(\delta_2-\delta_1\delta_2+\chi_2-\chi_2\delta_1-\delta_2\chi_1-\chi_2\chi_1\right)e^{\mathrm{DNL}}+\delta_2\chi_1+\chi_2\chi_1}{1-\delta_1-\chi_1+\delta_2+\chi_2}-\chi_1\right]\\
&=\frac{(\sigma-1)}{1-\delta_1-\chi_1}\left[\frac{\left(\delta_2\left(1-\delta_1-\chi_1\right)+\chi_2\left(1-\delta_1-\chi_1\right)\right)e^{\mathrm{DNL}}-\chi_1\left(1-\delta_1-\chi_1\right)}{1-\delta_1-\chi_1+\delta_2+\chi_2}\right],\\
&=(\sigma-1)\left(\frac{\left(\delta_2+\chi_2\right)e^{\mathrm{DNL}}-\chi_1}{1-\delta_1-\chi_1+\delta_2+\chi_2}\right),
\end{aligned}$$

and so

$$\mathrm{d}\ln Y^{*}=(1-\sigma)\left(\frac{\left(\delta_2+\chi_2\right)e^{\mathrm{DNL}}-\chi_1}{1-\delta_1-\chi_1+\delta_2+\chi_2}\right). \tag{A15}$$

Finally, the proof of Proposition 4.1 directly follows by using (A15) and that $\chi_1=\chi_2=0$ when DLs do not export. Moreover, counterexamples to show that gains of trade are not guaranteed are presented in Appendix D. As for Proposition 4.2, it follows by using (A15), that $\delta_1=\delta_2=0$ when DLs only export, and $\chi_2<\chi_1$.

A.3 Computation of Counterfactual

We consider scenarios with common component of trade costs given by τ'' and τ'. As in the main body of the paper, for any variable x, we denote its equilibrium under each set of export trade costs by x' and x'', and express the results by $\widehat{x}:=\frac{x''}{x'}$.

We begin by establishing (14a). To do this, we reexpress (FE) with the productivity distribution we have assumed:

$$\left[\frac{r\left(\mathbb{P},\varphi^{D}\right)}{\sigma}-f_{D}\right]\left[\Pr\left(\varphi^{D}\right)+\Pr\left(\varphi^{X}\right)\right]+\left[\frac{r\left(\mathbb{P};\varphi^{X}\tau\right)}{\sigma}-f_{X}\right]\Pr(\varphi^{X})=F. \tag{A16}$$

Thus, since this equation has to hold in both scenarios,

$$\left[Y'\frac{\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi^{D}}\right)^{1-\sigma}}{\sigma\left(\mathbb{P}'\right)^{1-\sigma}}-f_{D}\right]\Pr\left(\varphi^{D}\right)+\left[Y'\frac{\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi^{X}}\right)^{1-\sigma}}{\sigma\left(\mathbb{P}'\right)^{1-\sigma}}-f_{D}\right]\Pr(\varphi^{X})+\left[Y'\frac{\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi^{X}}\right)^{1-\sigma}}{\sigma\left(\mathbb{P}'\right)^{1-\sigma}}\left(\tau'\right)^{1-\sigma}-f_{X}\right]\Pr(\varphi^{X})=$$

$$\left[Y''\frac{\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi^{D}}\right)^{1-\sigma}}{\sigma\left(\mathbb{P}''\right)^{1-\sigma}}-f_{D}\right]\Pr\left(\varphi^{D}\right)+\left[Y''\frac{\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi^{X}}\right)^{1-\sigma}}{\sigma\left(\mathbb{P}''\right)^{1-\sigma}}-f_{D}\right]\Pr(\varphi^{X})+\left[Y''\frac{\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi^{X}}\right)^{1-\sigma}}{\sigma\left(\mathbb{P}''\right)^{1-\sigma}}\left(\tau''\right)^{1-\sigma}-f_{X}\right]\Pr(\varphi^{X}).$$

Gathering terms and after some algebraic manipulation, this can be reexpressed as

$$\gamma\left(\widehat{Y}\widehat{\mathbb{P}}^{\sigma-1}-1\right)+\chi\left(\widehat{Y}\widehat{\mathbb{P}}^{\sigma-1}\widehat{\tau}^{1-\sigma}-1\right)=0,$$

where $\gamma:=\frac{r_{D}\left(\mathbb{P}',\varphi^{D}\right)}{\sigma}\Pr\left(\varphi^{D}\right)+\frac{r_{D}\left(\mathbb{P}',\varphi^{X}\right)}{\sigma}\Pr(\varphi^{X})$ and $\chi:=\frac{r_{X}\left(\mathbb{P}',\varphi^{X};\tau'\right)}{\sigma}\Pr(\varphi^{X})$. Using that $(M_{D})'=\left(M^{E}\right)'\left[\Pr\left(\varphi^{D}\right)+\Pr\left(\varphi^{X}\right)\right]$ and $(M_{X})'=\left(M^{E}\right)'\Pr\left(\varphi^{X}\right)$, we can multiply γ and χ by $\left(M^{E}\right)'$ to obtain $\left(M^{E}\right)'\gamma=\frac{\left(R_{HH}^{\text{DNL}}\right)'}{\sigma}$ and $\left(M^{E}\right)'\chi=\frac{\left(R_{HF}^{\text{DNL}}\right)'}{\sigma}$. This determines that

$$\widehat{\mathbb{P}}=\left[\frac{1+\frac{\chi}{\gamma}}{\widehat{Y}\left(1+\frac{\chi}{\gamma}\widehat{\tau}^{1-\sigma}\right)}\right]^{\frac{1}{\sigma-1}},$$

where, using that $\frac{\left(e^{\text{DNL}}\right)'}{\left(d^{\text{DNL}}\right)'}=\frac{\chi(M^{E})'/\left[\left(R_{D}^{\text{DNL}}\right)'+\left(R_{X}^{\text{DNL}}\right)'\right]}{\gamma(M^{E})'/\left[\left(R_{D}^{\text{DNL}}\right)'+\left(R_{X}^{\text{DNL}}\right)'\right]}=\frac{\chi}{\gamma}$, we obtain (14a).

Regarding income, (14b) is obtained by using that $Y=L+\sum_{\omega\in\overline{\mathscr{L}}_{i}}\pi^{\omega}$, so that

$$Y''-Y'=\sum_{\omega\in\overline{\mathscr{L}}_{i}}\left[(\overline{\pi}_{i}^{\omega})''-(\overline{\pi}_{i}^{\omega})'\right].$$

Since $x''-x'=x'\left(\widehat{x}-1\right)$ for any variable x, this can be reexpressed as

$$Y'\left(\widehat{Y}-1\right)=\sum_{\omega\in\overline{\mathscr{L}}_{i}}(\overline{\pi}^{\omega})'\left(\widehat{\overline{\pi}}^{\omega}-1\right).$$

Thus, given $\widehat{Y}-1=\sum_{\omega\in\overline{\mathscr{L}}_{i}}\frac{(\overline{\pi}^{\omega})'}{Y'}\left(\widehat{\overline{\pi}}^{\omega}-1\right)$ and letting $\psi^{\omega}:=\frac{(\overline{\pi}^{\omega})'}{Y'}=\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}}+\frac{s_{X}^{\omega}}{\varepsilon_{X}^{\omega}}$, we obtain $\left(\widehat{Y}-1\right)=\sum_{\omega\in\overline{\mathscr{L}}_{i}}\psi^{\omega}\left(\widehat{\overline{\pi}}^{\omega}-1\right)$. In addition, this can be alternatively expressed by $\widehat{Y}-1=\frac{(\overline{\Pi}_{i})'}{Y'}\left(\widehat{\overline{\Pi}}_{i}-1\right)$, where $\frac{(\overline{\Pi}_{i})'}{Y'}=\sum_{\omega\in\overline{\mathscr{L}}_{i}}\left(\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}}+\frac{s_{X}^{\omega}}{\varepsilon_{X}^{\omega}}\right)$. This also shows (16).

As for (14c), the difference of profits for DL ω are given by

$$(\overline{\pi}^{\omega})''-(\overline{\pi}^{\omega})'=Y''\frac{\left(s_{D}^{\omega}\right)''}{\left(\varepsilon_{D}^{\omega}\right)''}-Y'\frac{\left(s_{D}^{\omega}\right)'}{\left(\varepsilon_{D}^{\omega}\right)'}+Y''\frac{\left(s_{X}^{\omega}\right)''}{\left(\varepsilon_{X}^{\omega}\right)''}-Y'\frac{\left(s_{X}^{\omega}\right)'}{\left(\varepsilon_{X}^{\omega}\right)'}.$$

The left-hand side can be reexpressed by $(\overline{\pi}^{\omega})''-(\overline{\pi}^{\omega})'=\left(\widehat{\overline{\pi}}^{\omega}-1\right)(\overline{\pi}^{\omega})'$. Likewise, the terms of the right-hand side can be reexpressed as $Y''\frac{\left(s_{D}^{\omega}\right)''}{\left(\varepsilon_{D}^{\omega}\right)''}-Y'\frac{\left(s_{D}^{\omega}\right)'}{\left(\varepsilon_{D}^{\omega}\right)'}=Y'\frac{\left(s_{D}^{\omega}\right)'}{\left(\varepsilon_{D}^{\omega}\right)'}\left(\widehat{Y}\frac{\widehat{s}_{D}^{\omega}}{\widehat{\varepsilon}_{D}^{\omega}}-1\right)$ and $Y''\frac{\left(s_{X}^{\omega}\right)''}{\left(\varepsilon_{X}^{\omega}\right)''}-Y'\frac{\left(s_{X}^{\omega}\right)'}{\left(\varepsilon_{X}^{\omega}\right)'}=Y'\frac{\left(s_{X}^{\omega}\right)'}{\left(\varepsilon_{X}^{\omega}\right)'}\left(\widehat{Y}\frac{\widehat{s}_{X}^{\omega}}{\widehat{\varepsilon}_{X}^{\omega}}-1\right)$. Therefore,

$$\widehat{\overline{\pi}}^{\omega}=1+\frac{(\overline{\pi}_{D}^{\omega})'}{(\overline{\pi}^{\omega})'}\left(\widehat{Y}\frac{\widehat{s}_{D}^{\omega}}{\widehat{\varepsilon}_{D}^{\omega}}-1\right)+\frac{(\overline{\pi}_{X}^{\omega})'}{(\overline{\pi}^{\omega})'}\left(\widehat{Y}\frac{\widehat{s}_{X}^{\omega}}{\widehat{\varepsilon}_{X}^{\omega}}-1\right).$$

We have defined in the main body of the paper that $\phi_{D}^{\omega}:=\frac{\overline{\pi}_{D}^{\omega}}{\overline{\pi}^{\omega}}$, so that $\phi_{D}^{\omega}=\frac{R_{D}^{\omega}/\varepsilon_{D}^{\omega}}{R_{D}^{\omega}/\varepsilon_{D}^{\omega}+R_{X}^{\omega}/\varepsilon_{X}^{\omega}}$. Furthermore, we obtain $\phi_{D}^{\omega}=\frac{s_{D}^{\omega}/\varepsilon_{D}^{\omega}}{s_{D}^{\omega}/\varepsilon_{D}^{\omega}+s_{X}^{\omega}/\varepsilon_{X}^{\omega}}$ by multiplying and dividing by income, and we can recover ϕ_{X}

by the fact that $\phi_X^\omega = 1 - \phi_D^\omega$.

Equations (14d) and (14e) follow directly by using that $s_{ij}^\omega = \left(\frac{p_{ij}^\omega}{\mathbb{P}}\right)^{1-\sigma}$ and $p_{ij}^\omega = m_{ij}^\omega c_{ij}^\omega$. Moreover, for (14f), consider countries $i, j \in \mathcal{C}$. Given that the elasticity is given by the function $\varepsilon\left(s_{ij}^\omega\right) = \sigma + s_{ij}^\omega (1-\sigma)$, we can reexpress it in differences by

$$\left(\varepsilon_{ij}^\omega\right)'' - \left(\varepsilon_{ij}^\omega\right)' = \left[\left(s_{ij}^\omega\right)'' - \left(s_{ij}^\omega\right)'\right](1-\sigma),$$

or, using that $x'' - x' = x'(\widehat{x} - 1)$ for any variable x, this can be reexpressed as

$$\widehat{\varepsilon}_{ij}^\omega = 1 + \left(1 - \widehat{s}_{ij}^\omega\right) \frac{\left(s_{ij}^\omega\right)'(\sigma-1)}{\sigma - \left(s_{ij}^\omega\right)'(\sigma-1)}.$$

As for (14g), we know that $m_{ij}^\omega := \frac{\varepsilon_{ij}^\omega}{\varepsilon_{ij}^\omega - 1}$ for any $i, j \in \mathcal{C}$. Thus,

$$\widehat{m}_{ij}^\omega = \frac{\left(\varepsilon_{ij}^\omega\right)''}{\left(\varepsilon_{ij}^\omega\right)'' - 1} \frac{\left(\varepsilon_{ij}^\omega\right)' - 1}{\left(\varepsilon_{ij}^\omega\right)'},$$

and, given that $\left(\varepsilon_{ij}^\omega\right)'' = \widehat{\varepsilon}_{ij}^\omega \left(\varepsilon_{ij}^\omega\right)'$, we obtain the result.

B Large Firms Defined By Revenue

The calibration of the model requires splitting firms into DNLs and DLs, and so taking a stand on what constitutes being a well-established oligopolistic firm. In the main body of the paper, we used a classification of firms based on domestic market shares. Implicitly, we were relying on the results in Section 2, which indicates that a high domestic market share acts as an almost sufficient condition for having great revenues in an industry.

Next, we recalculate the results by classifying DLs and DNLs according to total revenue, rather than domestic market shares. Basically, this broadens the scope of large firms to include big exporters whose domestic sales are insignificant. To distinguish this case relative to the baseline scenario, we refer to each type of firm in the setting considered as high-revenue firms (HRFs) and low-revenue firms (LRFs). Moreover, we keep referring to DNLs and DLs when we deal with the baseline definition.

To construct a representative industry, we follow similar steps to those in Section 5. First, we only consider those industries with coexistence of DLs and DNLs. For these industries, we define the domestic market and income shares of each HRFs, and the domestic and export intensities of LRFs. After this, we define a representative industry by taking the top four HRFs and using industry revenues as weights. Notice that the elasticity of substitution is still $\sigma := 3.53$, since this value does not depend on how we classify firms. The description of the representative industry is presented in Table 2. We also incorporate the export intensity of all firms for comparison.

Table 2. *Statistics of Danish Manufacturing*

(a) Features of HRFs (in %)

	Domestic Revenues as % of the country's income	Export Revenues as % of the country's income
Top 1	17.46	15.23
Top 2	8.11	4.48
Top 3	5.38	2.56
Top 4	3.68	2.03

(b) Revenue Intensities (in %)

	Domestic Intensity	Export Intensity
LRFs	75.82	24.18
DNLs	68.62	31.38
All Firms	58.12	41.88

Note: Calculations based on industries with coexistence of DNLs and DLs, and using industry revenues as weights. Domestic market shares are calculated relative to expenditures, which includes both domestic sales and imports. Domestic and export intensity calculated, respectively, as domestic and export sales (i.e., LRFs, DNLs, and all firms) relative to the total sales of the group considered.

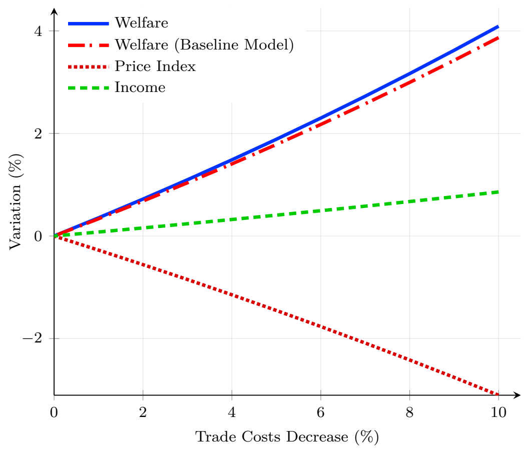
Comparing the description in Table 2a with our baseline scenario, we obtain two conclusions. First, domestic market and income shares of HRFs are quite similar to those corresponding to DLs. Second, the export shares are somewhat different, with the top two HRFs exhibiting a higher contribution in this respect.

As for the information in Table 2b, we have included the export intensity of DNLs to compare it with that of LRFs. The numbers indicate that LRFs as a group have a greater home bias, so that their export intensity is lower. This implies a more pronounced entry of negligible firms following trade liberalization.

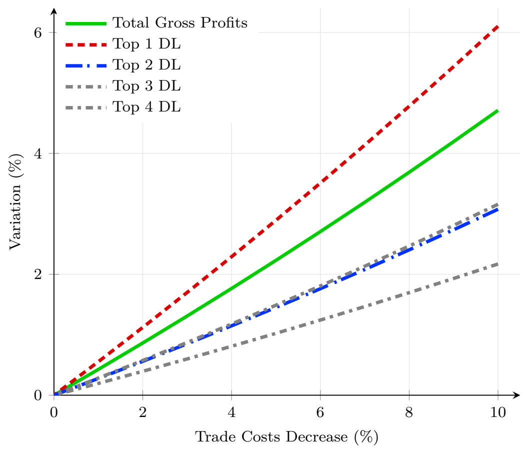
The results regarding welfare, income, and the price index are depicted in Figure 10a. For a reduction in trade costs of 10%, they predict an increase in welfare of 4.09%, relative to 3.87% under DLs and DNLs. Thus, the increase in welfare predicted is 5.73% higher, which represents a small difference relative to the baseline case. Furthermore, the price index decreases by 3.10% and income increases by 0.85%. While the magnitude in which income varies is still relatively small, it is higher than the 0.13% obtained in the baseline case. On the contrary, the decrease in the price index is less pronounced, relative to the 3.60% of the baseline case.

Figure 10. *Impact of Trade Liberalization*

(a) Impact on Income



(b) Impact on Welfare



In Figure 10b, we delve into how the increase in income takes place by presenting the variation in profits of the HRFs. Relative to the baseline scenario, the profits of all HRFs become greater. This arises because HRFs incorporate the presence of large firms due to a great level of exports, which makes HRFs have higher export intensity than DLs. In particular, the second top HRF increases its profits following trade liberalization, which contrasts with the second top DL that ends up with lower profits.

Nonetheless, since HRFs incorporate firms with higher export intensity, LRFs exhibit a more pronounced home bias. This occurs because now firms with great domestic intensity are part of the LRFs. Therefore, the decrease in the price index due to entry of LRFs is less marked than what is predicted with DNLs. Overall, this offsets the slightly higher increase in income predicted, thereby determining similar variations in gains of trade relative to the baseline case.

C Market Shares Based on Alternative Definitions of Sales

In this appendix, we show that domestic market shares are not sensitive to how we define sales to compute them. Ideally, total domestic sales of a firm should include those goods supplied to the market, irrespective if they are produced by itself or other firms. As a baseline case, we have used total turnover as the firm's domestic supply, which already includes some imports. Specifically, in our data, total turnover includes sales of goods produced by the firm itself, goods produced by a subcontractor established abroad when the firm owns the inputs of the subcontracted firm, and resales of goods bought from other domestic firms if they are sold with any processing. Nonetheless, there is a portion of total supply that is not covered by turnover: goods bought to firms established abroad that the firm does not own.

The reason to use total turnover as a baseline scenario is that, even though we have information about firms' imports, we do not know whether they are inputs or final goods for the firm. Thus, by defining sales as total turnover we are taken a conservative position: we are implicitly assuming that any import is either an input or a final good that has been reprocessed by a firm, which in our data is already included in total turnover.

Next, we show that, if we use measures of DLs sales that account for imported goods, domestic market shares are almost identical to those in the baseline case. To illustrate the results, we present the domestic market share of the top four firms by using industry-revenue weights. This also allows us to show that the domestic revenue shares and domestic market shares are quite similar.

Since we do not have information regarding whether each import constitutes an input, we consider two alternatives.

- **Alternative A**: domestic sales of a firm are defined as the sum of its turnover and imports minus its exports of any 8-digit product that belongs to the firm's industry and is also produced by the firm.
- **Alternative B**: domestic sales of a firm are defined as the sum of its turnover and imports minus its exports of any 8-digit product that belongs to the firm's industry.

Alternative A is based on the assumption that, if a firm produces and sells a good domestically, an import of this good constitutes a product to be resold without further processing, rather than an input. Regarding Alternative B, it supposes that any good imported that belongs to the industry is resold.

The results are presented in Table 3. We also include the case of high-revenue firms as leaders, as we did in the previous appendix. This is defined as Alternative C. The outcomes indicate that the domestic shares of DLs are virtually identical.

Table 3. *Domestic Market Share of DLs*

	Baseline Case (Turnover)	Alternative A	Alternative B	Alternative C
Top 1	17.83	17.96	17.84	16.31
Top 2	7.27	7.26	7.24	7.28
Top 3	4.65	4.89	4.92	4.89
Top 4	3.36	3.55	3.73	3.38

D Welfare Losses

In the main body of the paper, we claimed that trade liberalization could lead to negative gains of trade. Specifically, even when trade liberalization always decreases the price index, income could reduce enough to such an extent that it decreases real income. The goal of this appendix is twofold. First, to provide an example where this is indeed the case. Second, to show that this outcome only arises under quite extreme scenarios. In fact, we begin by establishing that there cannot be welfare losses if there is only one DL in each country.

Proposition D.1. *If there is only one DL in each country, there are always positive gains of trade.*

Proof of Proposition D.1. A DL that exclusively serves the domestic market represents the worst-case scenario in terms of income and therefore welfare. Thus, if we can prove that there are positive gains of trade when the DL only serves home, the result follows.

When the DL and this exclusively sells the domestic market, a variation in trade costs determines that

$$\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}=(\sigma-1)\frac{\left(\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}}\frac{\sigma-\sigma s_{D}^{\omega}}{\sigma-\varepsilon_{D}^{\omega}s_{D}^{\omega}}\right)\left(\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}\right)}{1-\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}}},$$

which implies that welfare is

$$\frac{\mathrm{d}\ln\mathbb{W}^{*}}{\mathrm{d}\ln\tau}=(\sigma-1)\frac{\left(\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}}\frac{\sigma-\sigma s_{D}^{\omega}}{\sigma-\varepsilon_{D}^{\omega}s_{D}^{\omega}}\right)\left(\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}\right)}{1-\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}}}-\left(\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}\right).$$

Towards a contradiction, suppose that $\frac{\mathrm{d}\ln\mathbb{W}^{*}}{\mathrm{d}\ln\tau}>0$. Since $\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}>0$ and $\varepsilon_{D}^{\omega}>1$, then $\frac{\mathrm{d}\ln\mathbb{W}^{*}}{\mathrm{d}\ln\tau}>0$ iff

$$\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}}(\sigma-1)\left(\frac{\sigma-\sigma s_{D}^{\omega}}{\sigma-\varepsilon_{D}^{\omega}s_{D}^{\omega}}\right)>1-\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}} \tag{D1}$$

Using $(\sigma-1)(\sigma-\sigma s_{D}^{\omega})=(\sigma-1)(1-s_{D}^{\omega})=\varepsilon_{D}^{\omega}-1$, (D1) becomes

$$\frac{s_{D}^{\omega}}{\varepsilon_{D}^{\omega}}\left[\frac{\sigma(\varepsilon_{D}^{\omega}-1)}{\sigma-\varepsilon_{D}^{\omega}s_{D}^{\omega}}+1\right]>1.$$

Working out the expression, this becomes $s_D^\omega \sigma - (s_D^\omega)^2 > \sigma - \varepsilon_D^\omega s_D^\omega$. Besides, using that $\varepsilon_D^\omega = \sigma + s_D^\omega (1 - \sigma)$, this is equivalent to

$$s_D^\omega (2 - s_D^\omega) > 1.$$

Since $s_D^\omega < 1$, the left-hand side is always increasing in s_D^ω, which allows us to determine that its maximum value is strictly lower than 1. Therefore, a contradiction. ■

To identify an example with negative gains of trade, trade liberalization necessarily has to impact income negatively. Additionally, this effect has to be pronounced enough to surmount any reduction of the price index. This is more likely to arise if DLs do not export (so that they do not benefit from better export opportunities) and generate a substantial part of the country's income, entailing that the negative impact of tougher competition on profits has significant consequences for welfare.

Based on this, Table 4 presents an example where the revenue shares of DLs determine negative gains of trade. The outcomes are depicted in Figure 12 for different levels of domestic intensity of DNLs and assuming $\sigma := 3.53$ as in the baseline case.

Table 4. *Features of DLs*

	Domestic Revenues as % of the country's income	Export Revenues as % of the country's income
Top 1	30	0
Top 2	30	0
Top 3	30	0

Figure 11. *Welfare Losses*

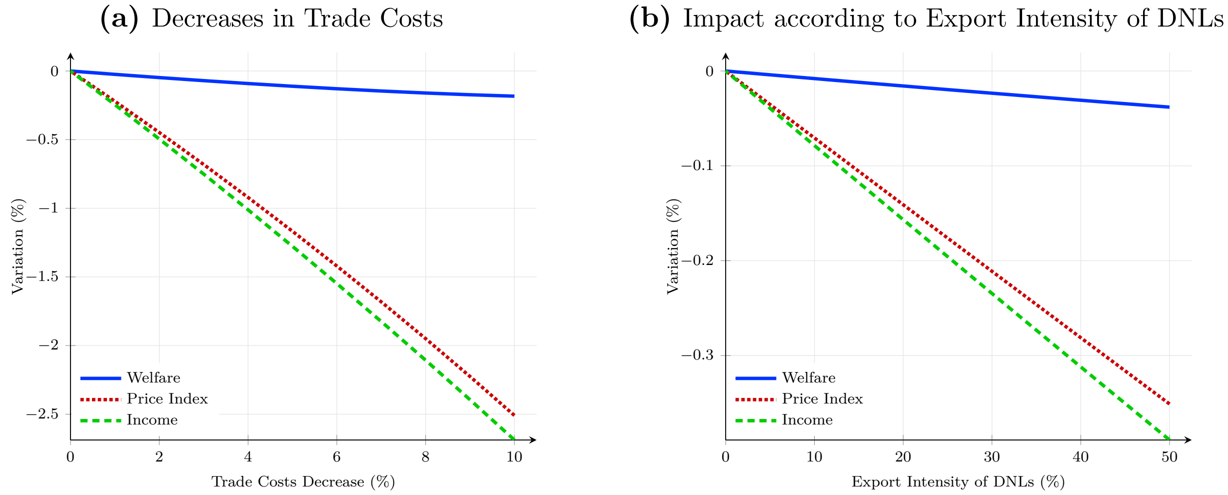


Figure 12 establishes that there are welfare losses, irrespective of the DNLs' domestic intensity. This occurs because the fall in income is more significant for welfare than the reduction in the price index. The example also reveals that losses from trade require quite extreme revenue shares. Thus, since there are always positive gains of trade when there is only one DL in each country, we conclude that the model tends to generate positive gains of trade. In other words, cases with negative gains of trade are implausible.

E Bounded Pareto Distribution for DNLs

In this appendix, we expand upon the analysis in Section 6.3 and quantify gains of trade under a more general productivity distribution of DNLs. This allows us to incorporate welfare effects due to changes in the survival productivity cutoffs. They act by inducing the exit of the least-productive DNLs, and hence reallocating production towards more productive firms.

In particular, we suppose that the DNLs' productivity follows a bounded Pareto distribution. Following Helpman et al. (2008), we incorporate this by expressing the model in terms of $a := 1/\varphi$, with finite support $[a_L, a_H]$ and cdf given by

$$G(a) := \frac{(a)^k - (a_L)^k}{(a_H)^k - (a_L)^k}.$$

We suppose that $k > \sigma - 1$ to ensure that some of the integrals calculated below are finite. Furthermore, we denote the inverse of the survival productivity cutoff in each market by $a_D := \frac{\sigma-1}{\sigma}\mathbb{P}\left(\frac{Y}{\sigma f_D}\right)^{\frac{1}{\sigma-1}}$ and $a_X := \frac{\sigma-1}{\sigma}\frac{\mathbb{P}}{\tau}\left(\frac{Y}{\sigma f_X}\right)^{\frac{1}{\sigma-1}}$, respectively.

E.1 Procedure for Computation

The procedure to compute the results is similar to the baseline model, which requires solving the system (14). Specifically, equations (14b)-(14g) remain the same, while the change in the DNLs' productivity distribution requires substituting (14a) by

$$\left[\frac{Y''\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\sigma(\mathbb{P}'')^{1-\sigma}}(\tau'')^{1-\sigma}\int_{a_L}^{a''_X} a^{1-\sigma}\mathrm{d}G(a) - \int_{a_L}^{a''_X} f_X\,\mathrm{d}G(a)\right] + \left[\frac{Y''\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\sigma(\mathbb{P}'')^{1-\sigma}}\int_{a_L}^{a''_D} a^{1-\sigma}\mathrm{d}G(a) - \int_{a_L}^{a''_D} f_D\,\mathrm{d}G(a)\right] =$$
$$\left[\frac{Y'\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\sigma(\mathbb{P}')^{1-\sigma}}(\tau')^{1-\sigma}\int_{a_L}^{a'_X} a^{1-\sigma}\mathrm{d}G(a) - \int_{a_L}^{a'_X} f_X\,\mathrm{d}G(a)\right] + \left[\frac{Y'\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\sigma(\mathbb{P}')^{1-\sigma}}\int_{a_L}^{a'_D} a^{1-\sigma}\mathrm{d}G(a) - \int_{a_L}^{a'_D} f_D\,\mathrm{d}G(a)\right], \tag{E1}$$

which arises by taking differences of (FE) between the two equilibria. By working (E1) out,

$$\widehat{Y}\left(\widehat{\mathbb{P}}\right)^{\sigma-1}\left[(\widehat{\tau})^{1-\sigma}\frac{\int_{a_L}^{a''_X} a^{1-\sigma}\mathrm{d}G(a)}{\int_{a_L}^{a'_X} a^{1-\sigma}\mathrm{d}G(a)}\left(r_X^{\mathrm{DNL}}\right)' + \frac{\int_{a_L}^{a''_D} a^{1-\sigma}\mathrm{d}G(a)}{\int_{a_L}^{a'_D} a^{1-\sigma}\mathrm{d}G(a)}r_D^{\mathrm{DNL}}\right] - \left[\left(r_D^{\mathrm{DNL}}\right)' + \left(r_X^{\mathrm{DNL}}\right)'\right] =$$
$$\sigma f_X G(a'_X)\left[\frac{G(a''_X)}{G(a'_X)} - 1\right] + \sigma f_D G(a'_D)\left[\frac{G(a''_D)}{G(a'_D)} - 1\right],$$

where the bounded Pareto distribution implies that

$$\int_{a_L}^{a'_X} a^{1-\sigma}\mathrm{d}G(a) = \frac{k}{k-\sigma+1}\frac{(a_L)^{k-\sigma+1}}{(a_H)^k - (a_L)^k}\left[\left(\frac{a'_X}{a_L}\right)^{k-\sigma+1} - 1\right].$$

After some algebra, and using that $\frac{a''_X}{a'_D} := \frac{1}{\tau'}\frac{\widehat{\mathbb{P}}}{\widehat{\tau}}\left(\frac{f_D}{f_X}\widehat{Y}\right)^{\frac{1}{\sigma-1}}$, $\frac{a'_X}{a'_D} := \frac{1}{\tau'}\left(\frac{f_D}{f_X}\right)^{\frac{1}{\sigma-1}}$, and $\frac{a''_D}{a'_D} := \widehat{\mathbb{P}}\left(\widehat{Y}\right)^{\frac{1}{\sigma-1}}$, we end up expressing (E1) by

$$\widehat{Y}\left(\widehat{\mathbb{P}}\right)^{\sigma-1}\left[(\widehat{\tau})^{1-\sigma}\left(e^{\mathrm{DNL}}\right)'\frac{\left(\frac{1}{\tau'}\frac{\widehat{\mathbb{P}}}{\widehat{\tau}}\right)^{k-\sigma+1}\left(\frac{f_D}{f_X}\widehat{Y}\right)^{\frac{k-\sigma+1}{\sigma-1}}\left(a'_D/a_L\right)^{k-\sigma+1}-1}{\left(\frac{1}{\tau'}\right)^{k-\sigma+1}\left(\frac{f_D}{f_X}\right)^{\frac{k-\sigma+1}{\sigma-1}}\left(a'_D/a_L\right)^{k-\sigma+1}-1}+\left(d^{\mathrm{DNL}}\right)'\frac{\left(\widehat{\mathbb{P}}\right)^{k-\sigma+1}\left(\widehat{Y}\right)^{\frac{k-\sigma+1}{\sigma-1}}\left(a'_D/a_L\right)^{k-\sigma+1}-1}{\left(a'_D/a_L\right)^{k-\sigma+1}-1}\right]-1=$$

$$\frac{\sigma F_D}{\left(R^{\mathrm{DNL}}\right)'}\left[\left(\frac{f_X}{f_D}\frac{G\left(a'_X\right)}{G\left(a'_D\right)}\right)\left(\frac{\left(\frac{1}{\tau'}\frac{\widehat{\mathbb{P}}}{\widehat{\tau}}\right)^{k}\left(\frac{f_D}{f_X}\widehat{Y}\right)^{\frac{k}{\sigma-1}}\left(a'_D/a_L\right)^{k}-1}{\left(\frac{1}{\tau'}\right)^{k}\left(\frac{f_D}{f_X}\right)^{\frac{k}{\sigma-1}}\left(a'_D/a_L\right)^{k}-1}-1\right)+\left(\frac{\left(\widehat{\mathbb{P}}\right)^{k}\left(\widehat{Y}\right)^{\frac{k}{\sigma-1}}\left(a'_D/a_L\right)^{k}-1}{\left(a'_D/a_L\right)^{k}-1}-1\right)\right], \tag{E2}$$

where F_D are the total domestic fixed costs spent by the active DNLs, and $\left(R^{\mathrm{DNL}}\right)'$ are the DNLs' total revenues in the equilibrium with trade costs τ'. (E2) is the equation that replaces (14a).

In summary, the system of equations to obtain the results are given by (14b)-(14g) and (E2).

E.2 Calibration

We utilize $\sigma := 3.53$, along with the calibration for the DLs' revenue shares and the export intensity of DNLs employed in the main body of the paper. Furthermore, it is necessary to calibrate some additional parameters of DNLs to take equation (E2) to the data. First, we assume that $f_D := 1$. This is without loss of generality, since only f_D/f_X matters for the calibration. For k, we use estimates provided in the literature, whose range is between 4 and 5.[30] Consistent with this, we take $k := 4.5$. Finally, we also take $F_D/R^{\mathrm{DNL}} := 0.1$ as a baseline value. The results are not sensitive to this term, as we formally show below. This becomes important since the term is subject to different interpretations, making its identification in terms of observable hard.

The rest of the parameters are calibrated to match some key moments of the Danish data. Specifically, we need to calibrate τ', f_X, and a_L. As in di Giovanni and Levchenko (2013), we ensure a solution exists by fixing one of these parameters. We choose τ' since, given the features of Danish DNLs, the existence of a solution crucially depends on it. In particular, there exists a solution only for values $\tau' < 1.2$, while it is necessary that $\tau' > 1$ by definition of trade costs. Taking into account that we consider reductions in trade costs up to 10%, we choose $\tau' := 1.15$ to ensure that both conditions are met.

As for f_X and a_L, we have expressed (E2) in a way that a_L only affects this equation through a'_D/a_L. Therefore, we choose a value a_L that generates a specific a'_D/a_L. Thus, we choose f_X and a'_D/a_L such that the model can generate the proportion of DNLs that export and the total export intensity of DNL in the Danish data. They are 48% and 68.62%, respectively.[31] By using this, we obtain $f_X := 1.033$ and $a'_D/a_L := 2.002$. Notice that the proportion of exporters among DNLs also determines the value $G\left(a'_X\right)/G\left(a'_D\right)$ in (E2).

[30] See, for instance, Head et al. (2014), Melitz and Redding (2015), Feenstra (2018), and Gaubert and Itskhoki (2021).

[31] In terms of the model, they correspond to $\frac{G\left(a'_X\right)}{G\left(a'_D\right)} = \frac{\left[\frac{1}{\tau'}\left(\frac{f_D}{f_X}\right)^{\frac{1}{\sigma-1}}\right]^{k}\left(a'_D/a_L\right)^{k}-1}{\left(a'_D/a_L\right)^{k}-1}$ and $\frac{\left(r_X^{\mathrm{DNL}}\right)'}{\left(r_D^{\mathrm{DNL}}\right)'} = (\tau')^{1-\sigma}\frac{\left[\frac{1}{\tau'}\left(\frac{f_D}{f_X}\right)^{\frac{1}{\sigma-1}}\right]^{k-\sigma+1}\left(a'_D/a_L\right)^{k-\sigma+1}-1}{\left(a'_D/a_L\right)^{k-\sigma+1}-1}$, respectively.

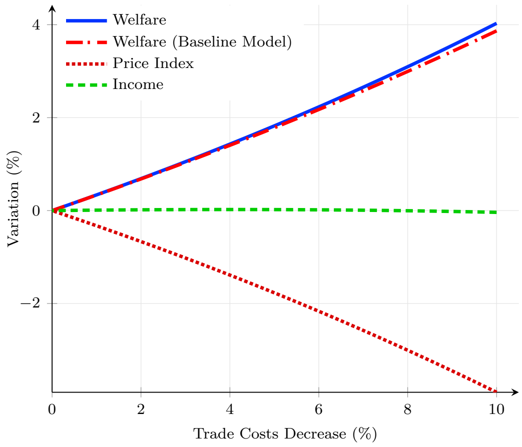
E.3 Results

The results regarding welfare are presented in Figure 12a. They indicate that the increase in welfare is 4.03% for a reduction in 10% of trade costs, in contrast to 3.86% in the baseline model. Thus, welfare gains due to inter-firm reallocations are quite small for the range of reduction in trade costs considered. This occurs because only marginal entrants exit when a trade shock is small. Thus, since by definition marginal entrants have zero profits, they have a negligible effect on expected profits. As a corollary, the consequences of these firms exiting barely influence the magnitude in which the price index has to decrease to restore zero expected profits.

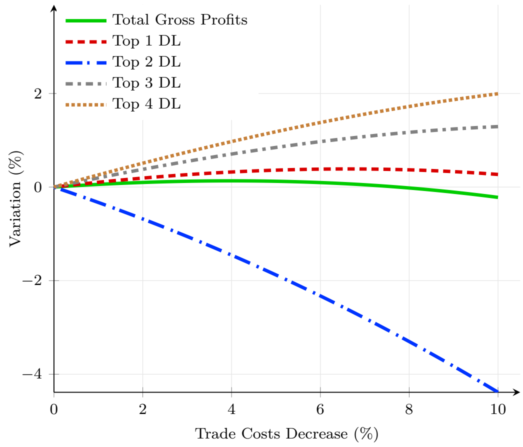
In Figure 12c, we also present a sensitivity analysis of the results. This reveals that the specific calibration of $\left(F_D/R^{\text{DNL}}\right)'$ used slightly changes the outcomes.

Figure 12. *Welfare under a Bounded Pareto for DNLs*

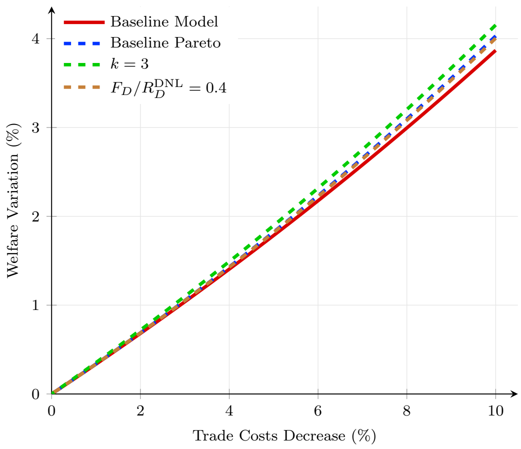
(a) Welfare, Income, and Price Index



(b) Profits



(c) Sensitivity



Furthermore, relative to the baseline case, the differences in welfare under a bounded Pareto distribution are increasing in trade costs: this is 0.66% and 4.18% following a 1% and 10% reduction in trade costs, respectively. The property also holds for the variations in the price index: relative to the baseline case, the decrease in the price index is 1.40% and 8.57% greater under a reduction in trade costs of 1% and 10% reduction in trade costs. This also explains why income ends up even decreasing for large changes in trade costs, as it can be observed in Figure 12b.