The Microeconomics of New Trade Models*

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(*Final Version*)

Abstract

International trade can increase product market competition and hence be pro-competitive. Is this feature captured in New Trade Models? I study this question in a setting with firm heterogeneity à la Melitz, under any productivity distribution and standard demands (e.g., demands from an additively separable utility, linear, translog, Logit). My results indicate that better export opportunities are pro-competitive: they reduce the domestic firms' markups and induce the exit of the least productive domestic firms. But, surprisingly, tougher import competition is completely offset by a reduction in the mass of domestic incumbents, leaving the competitive environment unaffected. Thus, it does not impact the prices, quantities, or survival productivity cutoff of domestic firms. Consistent with previous studies, I also find that a reduction in import trade costs under two large countries and two-way trade always decreases competition. I show that this outcome can be rationalized as capturing worse export conditions exclusively.

Keywords: Melitz, pro-competitive effects, import competition, export opportunities.

JEL codes: F10, F12, D43, L13.

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1 Introduction

In the International-Trade field, it is common to study trade liberalization assuming monopolistic competition. In the standard versions of this model, trade can benefit an economy by (i) affecting industry conditions or (ii) affecting factor markets conditions. The latter requires that trade liberalization is country-wide, entailing variations in factor prices that affect a firm's behavior by impacting costs and income. On the contrary, (i) occurs even when an industry in isolation is liberalized, and hence ceteris paribus adjustments in the factor markets.

One benefit arising through (i) is caused by trade exposing domestic firms to tougher product market competition. This benefits industry consumers by lowering the average price of domestic firms, through both reductions in the domestic firms' markups and the exit of the least efficient domestic firms. This pro-competitive effect can come from either the import or export side. Regarding imports, the mechanism is more aggressive behavior of foreign firms, taking the form of entry and decreases in their prices. Instead, pro-competitive effects due to better export opportunities emerge by increasing industry profitability, which fosters entry of domestic firms and hence competition.

In this paper, I study these pro-competitive outcomes that are induced by trade increasing product market competition. Considering a setting with firm heterogeneity à la Melitz (2003) and standard demands, I show that better export opportunities decrease the average price of domestic firms. On the contrary, tougher import competition determines a null impact on competitive effects: more aggressive behavior of foreign firms is completely offset by a reduction in the mass of domestic incumbents. As a corollary, the domestic firms' decisions are not affected in equilibrium.

The result echoes Melitz's (2003) conjecture for a CES demand *that "the model should also be interpreted with caution as it precludes another potentially important channel for the effects of trade, which operates through increases in import competition."* My results formalize this intuition and show it actually holds for a large set of demands, including those that derive from an additively separable utility, the linear demand by Melitz and

Ottaviano (2008), the translog version by Feenstra (2003), and Logit.

Formally, the analysis considers a monopolistic competition market structure with heterogeneity à la Melitz, where ex-ante homogeneous firms do not know their productivity but can learn it by paying an entry cost. The setup allows for any productivity distribution and considers a demand system that summarizes market conditions through a single aggregate. This aggregate can be interpreted as a measure of competition, including examples such as a price index and a demand choke price.

To exclusively capture effects caused by product market competition, I consider an industry in isolation that is negligible for each country's aggregate conditions. Furthermore, I begin by studying a small country as defined by Demidova and Rodríguez-Clare (2009; 2013) and Melitz (2018). This supposes that any trade shock in the country studied does not affect the domestic market conditions of its trading partners, thus keeping unchanged the export conditions of the country analyzed. The investigation of this case is relevant on its own, since it characterizes outcomes in small economies. However, its main relevance for the paper's purposes is that reductions in import and export trade costs in a small country respectively isolate the effects of tougher import competition and better export opportunities.

To illustrate what the assumption accomplishes, consider trade between Iceland (a small country) and China (a large country) in some specific industry. In particular, I focus on the implications of the assumption for a reduction in Iceland's import trade costs. This trade shock entails that Chinese firms already serving Iceland decrease their prices, and that additional Chinese firms export to this country (i.e., the Chinese productivity cutoff to serve Iceland decreases). Thus, Iceland's home market is subject to tougher import competition. However, Iceland is an insignificant market, and hence better conditions to sell there do not affect China's home market. This can be rationalized in the model by a reduction in Iceland's import trade costs having a negligible impact on China's expected profits; consequently, more Chinese firms are not induced to enter the industry, precluding that competition in the Chinese market eventually increases. Overall, Iceland faces tougher import competition, but its export conditions are unchanged once China's home market is unaffected.

The results for a small country indicate that a reduction in import trade costs has a null impact on the competitive environment—tougher import competition is completely offset by a reduction in the mass of domestic incumbents. This implies that domestic competition does not vary, and so a domestic firm's quantity, price, markup, and survival productivity cutoff remain unaffected. The conclusion is robust to multidimensional firm heterogeneity (i.e., heterogeneity in demand and costs) and extends to any country-specific choice (e.g., quality and number of products). The latter entails that, actually, tougher import competition does not affect any decision made by an active domestic firm. On the contrary, better export opportunities are pro-competitive: reductions in export trade costs increase competition in the home market, thus decreasing the markups of active domestic firms and increasing the domestic survival productivity cutoff. The mechanism is an increase in expected profits that induces a greater mass of domestic firms to enter the industry, with a subset of them eventually surviving and serving home.

After this, I study the effects of a reduction in import trade costs in the standard scenario of two large countries. Unlike the small-country case, this trade shock now affects the home market of the trading partner. To illustrate why this is so, consider two large countries, H and F. Moreover, suppose that H is now the USA instead of Iceland, while F is still China. A reduction in H's import trade costs creates two effects. First, it exposes H's home market to tougher import competition, like in the small-country case. Additionally, the better export opportunities in China to serve a large country such as the USA imply a non-trivial increase in the Chinese firms' expected profits. This induces entry of domestic firms in China, making competition in F's home market increase.

Based on this intuition, I first analyze results under one-way trade, where no firm from H exports in the industry, but H is nonetheless served by firms from F. In this scenario, changes in the trading partner's home market are irrelevant for H's domestic market, since no firm from H exports in the industry. Due to this, a reduction in import trade costs only exposes H's domestic firms to tougher import competition. This determines that, like in the small-country case, the shock only reduces the mass of domestic incumbents from H, without any consequence for H's competitive environment.

On the contrary, a reduction in H's import trade costs under two-way trade (i.e., when both countries import and export simultaneously in the industry) always entails a "Metzler paradox", meaning that competition in H decreases.[1] My results indicate that the impact on H's competitive environment can be rationalized as exclusively capturing effects of worse export conditions. Specifically, considering trade between the US and China once again, the American home market would not be affected if American firms were only exposed to tougher import competition. However, since competition in China's home market increases, American firms now face tougher competition when they export. This reduces the expected profits of American firms, inducing exit of American firms serving the USA and hence decreasing competition in the American home market.

Contributions and Related Literature. My paper is related to a literature identifying robust results in monopolistic competition with heterogeneous firms.[2] It is closely related to Arkolakis et al. (2019), who investigate the differential impact of trade liberalization when demands allow for variable markups. Their focus is on a comparison of *total* effects in general equilibrium, relative to scenarios with constant markups. This implies that they account for effects caused by trade simultaneously changing the conditions of the factor and product markets, remaining agnostic about the specific channels impacting the economy.

On the contrary, the goal of my paper is to analyze pro-competitive channels in Melitz, exclusively focusing on the effects caused by trade changing the conditions of the product market. By definition, these effects are triggered ceteris paribus changes in the factor markets, and hence arise even if an industry is negligible for aggregate conditions. They are caused by the exposure of domestic firms to tougher product market competition, which reduces the domestic firms' market power and induces the exit of the least productive

[1] The existence of a Metzler paradox was shown under heterogeneity à la Melitz and specific demands in Melitz and Ottaviano (2008), Spearot (2014), Demidova (2017), and Bagwell and Lee (2018; 2020). In the Krugman model, which can be understood as a particular case of Melitz, a Metzler paradox was shown in Venables (1987).

[2] See, for instance, Arkolakis et al. (2012), Zhelobodko et al. (2012), Bertoletti and Epifani (2014), Bertoletti and Etro (2015), and Neary and Mrazova (2017).

domestic firms by decreasing profits.

Even when I focus on the effects caused by the impact of trade on the product market, my findings are relevant for a setting à la Melitz in general equilibrium. They imply that, in this setting and under standard demands, total competitive effects do not capture the effects caused by tougher import competition in the product market. Thus, for instance, any competitive effect in a small country following a reduction in import trade costs is necessarily triggered by imports changing the conditions of the factor markets. The requirements for these changes to take place are different from the ones I focus on. They require that the trade shock is country-wide (so that general-equilibrium mechanisms are activated) and influences a domestic firm's decisions by triggering changes in costs and income through factor prices.

I also show that my conclusions regarding pro-competitive effects are quite robust. They hold under any continuous productivity distribution, standard demands, asymmetric countries, small or large trade shocks, any form of trade costs, and independently of whether country-specific overhead costs exist. This contrasts with competitive effects encompassing adjustments in the factor markets, which are usually quite sensitive to simplifying assumptions (e.g., a Pareto productivity distribution, iceberg trade costs, infinitesimal trade shocks, absence of country-specific fixed costs).

Finally, I also contribute by providing an approach for analyzing Melitz under demands that depend on a single aggregate. While the use of this type of demand is not new in the trade literature,[3] the innovation of my paper lies in interpreting the setting as a large aggregative economy in the sense of Acemoglu and Jensen (2010; 2015).[4] I make use of their techniques, and extend them to account for an endogenous number of agents to cope with monopolistic competition. One advantage of this approach is that, by employing monotone comparative statics, it identifies results under a set of critical sufficient conditions, thus

[3]See, for instance, Neary and Mrazova (2017), Parenti et al. (2017), Arkolakis et al. (2019), and Fally (2019).

[4]Large aggregative *economies* are broader than large aggregative *games*. Both refer to models where agents are atomistic and market conditions are captured through an aggregator. Nonetheless, the latter assumes additive separability of the aggregator. I do not impose this condition, which considerably broadens the range of demands covered.

ignoring ancillary assumptions.

The paper proceeds as follows. Section 2 illustrates the main results of the paper by using Melitz and Ottaviano's (2008) linear demand. Section 3 outlines the model setup. Section 4 considers a small country. Section 5 analyzes the case of two large countries. Section 6 concludes.

2 An Illustration

In this section, I illustrate the main results of the paper by utilizing Melitz and Ottaviano's (2008) linear demand. This demand has been a common choice to introduce endogenous markups, making it particularly adequate for the purpose.

Consider a set of countries $\mathcal{C}$, and an industry that is insignificant for any country's aggregate conditions. The setup is described through countries i and j such that $i, j \in \mathcal{C}$. Moreover, I utilize the convention that a variable's subscript ij refers to i as the origin country and j as the destination country.

In each country i, there is a mass of firms that are ex-ante identical and do not know their productivity. These firms can get assigned a unique variety ω and a draw of productivity by paying a sunk cost F_i. Following Melitz and Ottaviano (2008), I describe the productivity distribution through the marginal cost distribution that it defines, denoting their cdf by G_i with support $[\underline{c}_i, \bar{c}_i]$. Moreover, firms do not incur overhead costs, and the cost in i to have one unit arrive at j is $c\tau_{ij}$, where c is the marginal cost and τ_{ij} are trade costs with $\tau_{ii} := 1$.

The mass of domestic firms in i that decide to pay the entry cost is denoted by M_i^E, and the total mass of varieties available in j by $M_j := \sum_{k \in \mathcal{C}} M_{kj}$. Additionally, $\Omega_{ij} := [0, M_{ij}]$ refers to the set of varieties produced in i and consumed in j.

Assuming a unitary mass of identical agents, the demand in country j of a variety ω from i is given by

$$q_{ij}(\omega) := \frac{\alpha}{\gamma + \eta M_j} + \frac{\eta}{\gamma} \frac{\mathbb{P}_j}{\gamma + \eta M_j} - \frac{1}{\gamma} p_{ij}(\omega),$$

where $\alpha, \gamma, \eta > 0$ and $\mathbb{P}_j := \sum_{k \in \mathcal{C}} \int_{\omega \in \Omega_{kj}} p_{kj}(\omega) \, d\omega$. This demand has a choke-price function given by

$$\mathcal{P}_j^{\max}\left(\mathbb{P}_j, M_j\right) := \frac{\alpha\gamma + \eta\mathbb{P}_j}{\gamma + \eta M_j}. \tag{1}$$

A specific value of $\mathcal{P}_j^{\max}$ is denoted by $p_j^{\max}$, which summarizes j's competitive conditions and cannot be influenced by any firm unilaterally. The demand can be reexpressed in terms of this choke-price value by

$$q\left[p_j^{\max}, p_{ij}\left(\omega\right)\right] := \frac{p_j^{\max} - p_{ij}\left(\omega\right)}{\gamma}. \tag{2}$$

This implies that ω's demand schedule in j is fully determined conditional on values $p_j^{\max}$ and $p_{ij}\left(\omega\right)$. As a result, $p_j^{\max}$ acts as a single sufficient statistic for market conditions, making its composition irrelevant from the firm's point of view: different combinations of M_j and $\mathbb{P}_j$ that generate the same $p_j^{\max}$ are considered equivalent.

For a given $p_j^{\max}$, an active firm from i with marginal cost c sets the following optimal price in j,

$$p\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} + c\tau_{ij}}{2}, \tag{3}$$

with markups

$$m\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p\left(p_j^{\max}, c; \tau_{ij}\right)}{c\tau_{ij}}. \tag{4}$$

Moreover, its optimal profit in j is $\pi\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{\left(p_j^{\max} - c\tau_{ij}\right)^2}{4\gamma}$.

The mass of firms from i that are active in j is $M_{ij} = M_i^E G_i\left(c_{ij}^*\right)$, where c_{ij}^* is the marginal cost that makes a firm from i indifferent between serving j or not and is given by

$$c^*\left(p_j^{\max}; \tau_{ij}\right) := \frac{p_j^{\max}}{\tau_{ij}}. \tag{5}$$

Finally, free entry implies the following condition for each country i:

$$\sum_{k\in\mathcal{C}} \int_{\underline{c}_i}^{c_{ik}^*} \pi\left(p_k^{\max}, c; \tau_{ik}\right) \mathrm{d}G_i\left(c\right) = F_i. \tag{6}$$

2.1 Equilibrium

The results of the paper follow naturally by interpreting the framework as a large aggregative economy in the sense of Acemoglu and Jensen (2010; 2015). The distinctive features of these economies are the existence of a continuum of firms and a single aggregate summarizing market conditions. These properties allow us to express the equilibrium through two systems of equations that identify $\left(p_k^{\max *}, M_k^{E*}\right)_{k\in\mathcal{C}}$. Once those values are identified, any other equilibrium variable can be pinned down.

The first system of equations comprises the free-entry condition in each country, evaluated at the optimal marginal cost cutoffs. Specifically, substituting (5) into (6), the optimal expected profit of a firm from i in j is

$$\pi_{ij}^{\text{expect}}\left(p_j^{\max};\tau_{ij}\right) := \int_{\underline{c}_i}^{c^*\left(p_j^{\max};\tau_{ij}\right)} \pi\left(p_j^{\max}, c; \tau_{ij}\right) \mathrm{d}G_i\left(c\right),$$

so that the first system of equations is

$$\pi_i^{\text{expect}}\left[\left(p_k^{\max *};\tau_{ik}\right)_{k\in\mathcal{C}}\right] = F_i \text{ for each } i \in \mathcal{C}, \tag{FE-MO}$$

where $\pi_i^{\text{expect}}\left[\left(p_k^{\max *};\tau_{ik}\right)_{k\in\mathcal{C}}\right] := \sum_{k\in\mathcal{C}} \pi_{ik}^{\text{expect}}\left(p_k^{\max *};\tau_{ik}\right)$.

The second system of equations reflects the equilibrium at the market stage (i.e., after firms decide whether to pay the entry cost). This requires that the equilibrium choke price is self-generated by the firms' optimal decisions. Formally, considering that $\mathcal{P}_i^{\max}$ evaluated at the optimal variables defines a function $\mathcal{P}_i^{\max *}\left[p_i^{\max}, \left(M_k^E\right)_{k\in\mathcal{C}}; \left(\tau_{ki}\right)_{k\in\mathcal{C}}\right]$,[5] the second system is

$$p_i^{\max *} = \mathcal{P}_i^{\max *}\left[p_i^{\max *}, \left(M_k^{E*}\right)_{k\in\mathcal{C}}; \left(\tau_{ki}\right)_{k\in\mathcal{C}}\right] \text{ for each } i \in \mathcal{C}. \tag{MS-MO}$$

In sum, $\left(p_k^{\max *}, M_k^{E*}\right)_{k\in\mathcal{C}}$ is pinned down by solving the systems (FE-MO) and (MS-MO); after this, any optimal variable can be identified.

[5] This follows by using (1), which entails that $\mathcal{P}_j^{\max}$ depends on $\mathbb{P}_j$ and M_j. Likewise, $\mathbb{P}_j$ depends in equilibrium on optimal prices and the marginal cost cutoffs, which are respectively given by (3) and (5), while M_{ij} additionally depends on M_i^E.

2.2 Pro-Competitive Effects

The analysis focuses on whether a trade shock creates pro-competitive effects in the home market of a country labeled H. Pro-competitive effects are defined as reductions in markups and the marginal-cost cutoff of H's domestic firms. These variables are given in equilibrium by (4) and (5), and so their behavior is completely determined by $p_H^{\max *}$. This implies that the analysis of pro-competitive effects is equivalent to investigating how $p_H^{\max *}$ is affected.

To identify the impact on $p_H^{\max *}$, I exploit that the equilibrium conditions are separable: $(p_k^{\max *})_{k\in\mathcal{C}}$ is exclusively identified by the system (FE-MO), without need to solve for (MS-MO) or to know $\left(M_k^{E*}\right)_{k\in\mathcal{C}}$. This implies that pinning down $p_H^{\max *}$ only requires identifying $(p_k^{\max *})_{k\in\mathcal{C}}$ through the system (FE-MO).

I specifically investigate whether exposure of domestic firms to tougher import competition or better export opportunities creates pro-competitive effects. With this goal, I consider reductions in import and export trade costs in a small country. Following Demidova and Rodríguez-Clare (2009; 2013) and Melitz (2018), H is a small country when changes in H's domestic market do not impact the home market of H's trading partners. Formally, this means that $\left(p_j^{\max *}, M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$ is not affected by shocks in H. The small-country assumption allows me to isolate each of the mechanisms I am interested in. This is in contrast to what occurs in particular with a reduction in H's import trade costs under two large countries—it would simultaneously change $\left(p_j^{\max *}\right)_{j\in\mathcal{C}\setminus\{H\}}$ and hence affect H's export conditions.[6]

To simplify matters, suppose a scenario with set of countries $\mathcal{C} := \{H, F\}$, where H is a small country and F a composite country that represents the rest of the world. Since F's choke price acts as a parameter under shocks to a small country, I denote it by $\overline{p}_F^{\max *}$. The value $p_H^{\max *}$ is identified through the system (FE-MO), which given the small-country

[6] In Appendix B, I show that the small-country assumption can be formalized as foreign countries having a continuum of trading partners, including H. This implies that shocks in H are negligible for the foreign firms' expected profits, and hence do not impact the mass of Chinese firms willing to pay the entry cost. This results in the trading partners' domestic conditions remain unaffected (i.e., $\left(p_j^{\max *}, M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$ does not vary in equilibrium).

assumption collapses to H's free-entry condition:

$$\pi_{HH}^{\text{expect}}\left(p_H^{\max *}\right)+\pi_{HF}^{\text{expect}}\left(\overline{p}_F^{\max *};\tau_{HF}\right)=F_H. \tag{7}$$

The equation establishes that only shocks affecting H's expected profit are capable of changing (7) and hence of generating pro-competitive effects. This has two consequences.

First, decreases in export trade costs are pro-competitive. Formally, reductions in τ_{HF} lower $p_H^{\max *}$, thereby decreasing the markups and marginal-cost cutoff of domestic firms in H. Intuitively, this captures that better export opportunities increase the expected profits of firms from H, thus inducing a greater mass of firms from H to pay the entry cost. Ultimately, a subset of these firms survive and serve the domestic market, increasing competition in H.

Second, variations in import trade costs have no impact on $p_H^{\max *}$. This can be noticed since changes in τ_{FH} do not affect (7)—they only affect (MS-MO). The result reflects that tougher import competition is exactly offset by reductions in the mass of incumbents from H, leaving the competitive environment in H unaffected. Thus, quantities, prices, markups, and the marginal-cost cutoff of H's domestic firms are not impacted; only M_H^{E*} is affected.[7]

2.3 Implications for Large Countries

Consider now a set of large countries $\mathcal{C} := \{H, F\}$ and a reduction in H's import trade costs. The focus is on the competitive effects in H's home market when there is two-way trade in the industry, i.e. when both countries simultaneously export and import. This case is particularly relevant since a "Metzler paradox" arises, meaning that reductions in H's import trade costs decrease competition in H, and hence the markups and survival marginal cost cutoff of H's domestic firms increase. Based on the results of my paper, I

[7]Notice that my results only focus on the consumer benefits of pro-competitive effects, but welfare could still change even if the choke price does not vary. In the particular case considered by Melitz and Ottaviano (2008), which assumes a Pareto distribution, the choke price is also a sufficient statistic for welfare. Nonetheless, this is not a general property of the linear demand. More generally, a demand can depend on a single sufficient statistic, and yet welfare depend on more than one aggregate. Thus, the composition of the choke price could still matter for utility, turning microeconomic aspects relevant for welfare, even if the choke price does not change.

show that this outcome can be rationalized as exclusively capturing effects of worse export conditions in H.

Competitive effects in H are identified by the impact on $p_H^{\max *}$, which requires solving the system (FE-MO). Unlike the case of a small country, this system now comprises the free-entry conditions of H and F, and depends on both $p_H^{\max}$ and $p_F^{\max}$. Intuitively, it determines that a reduction in H's import trade costs triggers two effects: it exposes H's domestic firms to tougher import competition as in the small-country case, but it also impacts F's home market. The latter arises since better export opportunities in F to sell in a large country have a non-trivial impact on F's expected profits, which induces entry of domestic firms in F and hence increases domestic competition in F (i.e., $p_F^{\max}$ decreases). I define the effects taking place in H through this mechanism as H's export-conditions channel.

The emergence of the export-conditions channel does not affect the conclusions regarding tougher import competition or better export opportunities. It only implies that competitive outcomes in H under two-way trade are influenced by changes in H's export conditions. In particular, if changes in H's export conditions are null or irrelevant for H, a reduction in H's import trade costs still entails a zero impact on H's competitive effects.

To show this, consider an industry with one-way trade, where some firms from F export to H, but firms from H only serve home. This occurs if, for instance, trade costs in the industry are asymmetric and τ_{HF} is significantly high. In this scenario, no firm from H exports and hence changes in the conditions to serve F are irrelevant from H's point of view. Due to this, the export-conditions channel is shut, and so the total impact on H's home market of a reduction in τ_{FH} exclusively reflects that H's firms face tougher import competition. Thus, as in the small-country case, $p_H^{\max *}$ does not vary in equilibrium. This can be formally shown through the system (FE-MO) given by

$$\pi_{HH}^{\text{expect}}\left(p_H^{\max *}\right) = F_H, \tag{8}$$

$$\pi_{FF}^{\text{expect}}\left(p_F^{\max *}\right) + \pi_{FH}^{\text{expect}}\left(p_H^{\max *}; \tau_{FH}\right) = F_F, \tag{9}$$

determining that $p_H^{\max *}$ is identified by (8), and hence independently of τ_{FH}.

On the contrary, the usual case with two-way trade activates the export-conditions channel. Melitz and Ottaviano (2008) show that a reduction in τ_{FH} in this setting decreases competition in H under a Pareto distribution. I establish in this paper that, actually, this outcome always emerges when demands summarize market conditions through a single aggregate and independently of the productivity distribution assumed.

To illustrate this, the system (FE-MO) under two-way trade is

$$\pi_{HH}^{\text{expect}}\left(p_H^{\max *}\right) + \pi_{HF}^{\text{expect}}\left(p_F^{\max *}; \tau_{HF}\right) = F_H, \tag{10}$$

$$\pi_{FF}^{\text{expect}}\left(p_F^{\max *}\right) + \pi_{FH}^{\text{expect}}\left(p_H^{\max *}; \tau_{FH}\right) = F_F. \tag{11}$$

Considering infinitesimal variations for simplicity, then $\frac{\mathrm{d}p_H^{\max *}}{\mathrm{d}\tau_{FH}} < 0$ and $\frac{\mathrm{d}p_F^{\max *}}{\mathrm{d}\tau_{FH}} > 0$. This implies that $\mathrm{d}\tau_{FH} < 0$ decreases competition in H (i.e., $p_H^{\max *}$ increases) and increases competition in F (i.e., $p_F^{\max *}$ decreases).

Relative to one-way trade, H's competitive environment is impacted differently since changes in $p_F^{\max *}$ affect H's expected profits. In particular, the decrease in $p_F^{\max *}$ reflects a tougher competitive environment in F's home market. From H's point of view, increased competition in F represents worse export conditions, which decrease competition in H once worse export conditions trigger effects similar to lower export opportunities.

The intuition can be shown more clearly by utilizing differential calculus. Denote the implicit solution $p_H^{\max *}$ to (10) by $p_H^{\max *}\left(p_F^{\max *}\right)$ and the implicit solution $p_F^{\max *}$ to (11) by $p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)$. The former function reflects the impact on H's competitive environment due to changes in H's expected export profits through $p_F^{\max *}$. As for $p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)$, it captures in particular the impact of better export opportunities to sell in H (i.e., a decrease in τ_{FH}) on F's expected profits and hence on F's domestic conditions, $p_F^{\max *}$.

Totally differentiating (10) and (11),

$$\begin{aligned} \frac{\mathrm{d}p_H^{\max *}}{\mathrm{d}\tau_{FH}} &= \frac{\partial p_H^{\max *}}{\partial p_F^{\max}} \frac{\mathrm{d}p_F^{\max *}}{\mathrm{d}\tau_{FH}}, \\ \frac{\mathrm{d}p_F^{\max *}}{\mathrm{d}\tau_{FH}} &= \frac{\partial p_F^{\max *}}{\partial \tau_{FH}} + \frac{\partial p_F^{\max *}}{\partial p_H^{\max}} \frac{\mathrm{d}p_H^{\max *}}{\mathrm{d}\tau_{FH}}, \end{aligned} \tag{12}$$

where $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} > 0$ and $\frac{\partial p_i^{\max *}}{\partial p_j^{\max}} < 0$ for $i, j \in \mathcal{C}$. By solving the system (12) and working out the expressions,

$$\underbrace{\mathrm{d}p_H^{\max *}}_{\text{total effect } (>0)} = \underbrace{\kappa \frac{\partial p_H^{\max *}}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}}{\partial \tau_{FH}}}_{H\text{'s export-conditions channel } (<0)} \mathrm{d}\tau_{FH}, \tag{13}$$

where $\mathrm{d}\tau_{FH} < 0$, and $\kappa := \left(1 - \frac{\partial p_H^{\max *}}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}}{\partial p_H^{\max}}\right)^{-1}$ is a multiplier of effects that satisfies $\kappa > 1$ under regularity conditions.

Equation (13) establishes that the impact on H's competitive environment is caused by changes in H's export conditions, i.e. by the impact of τ_{FH} on $p_F^{\max *}$. Specifically, $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}}$ captures the direct impact of τ_{FH} on F's expected profits, which translates into a change in F's domestic conditions, $p_F^{\max *}$. In turn, F's domestic conditions represent H's export conditions. Thus, $\frac{\partial p_H^{\max *}}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}}{\partial \tau_{FH}}$ reflects the impact of variations in H's export conditions on H's expected profits, and hence on its competitive environment, $p_H^{\max *}$. Finally, this triggers indirect effects in equilibrium, which are captured by the multiplier $\kappa > 1$.

3 The Model

In this section, I begin by formalizing a monopolistic-competition setting with firm heterogeneity à la Melitz. Then, I define a demand system that summarizes market conditions through a single aggregate. Finally, I solve for the equilibrium and highlight some of its properties. All the proofs of this paper are relegated to Appendix A.

3.1 Structure of the Model

There is a world economy with a set of countries $\mathcal{C}$, where a variable's subscript ij refers to i as the origin country and j as the destination country. The description of the setup takes countries $i, j \in \mathcal{C}$.

To focus on the effects triggered by product-market mechanisms, I consider an insignificant industry that does not affect factor prices or income in any country. This can be rationalized by assuming a continuum of industries as in Neary (2016), so that any specific industry has a negligible impact on the country's aggregate conditions.

The supply side in i is characterized by a set of firms $\overline{\Omega}_i$ that are ex-ante identical and do not know their productivity. Each of these firms produces a unique variety and has the option of paying a fixed sunk entry cost $F_i > 0$ to receive a productivity draw φ. Productivity is a continuous random variable that has non-negative support $\left[\underline{\varphi}_i, \overline{\varphi}_i\right]$ with $\overline{\varphi}_i \in \mathbb{R}_{++} \cup \{\infty\}$, and a strictly increasing cdf G_i with density g_i. I denote the mass of firms from country i paying the entry cost by M_i^E.

A firm from i that pays the entry cost can choose not to sell in country j or to do so by paying an overhead fixed cost $f_{ij} \geq 0$ (with strict inequality if the demand choke price in j is infinite). Producing in i to serve j entails constant marginal costs $c_i(\varphi, \tau_{ij})$, where τ_{ij} is a trade cost that a firm from i incurs to sell in j. I adopt the convention that trade costs in the domestic market (if any) never vary, and assume c_i is smooth, decreasing in φ, and increasing in τ_{ij}. Moreover, I suppose that markets are segmented, and that every firm that exports also sells domestically in equilibrium.

If firm ω from i decides to serve j, it chooses a price $p_{ij}(\omega) \in \left[\underline{p}_j, \overline{p}_j\right]$, where $\overline{p}_j \in \mathbb{R}_{++} \cup \{\infty\}$ equals the demand's choke price. I incorporate the decision of not serving j by assuming that any unavailable variety in j has a price $\overline{p}_j$. Also, I denote the vector of prices in j of all the varieties from i by $\mathbf{p}_{ij} := (p_{ij}(\omega))_{\omega \in \overline{\Omega}_i}$ and endow it with the pointwise order relation.

Finally, the mass of active firms in i selling to j is $M_{ij} := [1 - G_i(\varphi_{ij})] M_i^E$, where φ_{ij} is the productivity cutoff of a firm from i to break even in country j. Furthermore, $\Omega_{ji} := [0, M_{ji}]$ is the subset of varieties from j that are sold in i, and $M_i := \sum_{j \in \mathcal{C}} M_{ji}$ is the total mass of varieties consumed in i.

Definition 1. ***The market structure is à la Melitz*** *when it is given by the setup described above.*

Notice that a **market structure à la Krugman** arises as a special case of a setting à la Melitz. It emerges when each country has a degenerate productivity distribution and $f_{ij} = 0$ for $i, j \in \mathcal{C}$. Thus, all the results of the paper are valid for this market structure too.

3.2 Demand System

I formalize a demand system that summarizes market conditions through a real number. The assumption is satisfied for a large group of standard demands used in the International-Trade field, including demands that derive from an additively separable utility, the linear demand by Melitz and Ottaviano (2008), and the translog demand by Feenstra (2003).[8] With this goal, I begin by establishing some definitions.

Definition 2. *A* ***price aggregator*** *for country i is a function $\mathcal{P}_i\left[(\mathbf{p}_{ji})_{j\in\mathcal{C}}\right] := \sum_{j\in\mathcal{C}}\int_{\omega\in\overline{\Omega}_j} h_{ji}\left[p_{ji}(\omega)\right]\mathrm{d}\omega$. A* ***price aggregate*** *for country i is a value $\mathbb{P}_i \in \operatorname{range}\mathcal{P}_i$*

A price aggregator takes the vector of prices as an input and provides a real number as an output. I refer to this real number as a price aggregate.[9] Intuitively, a price aggregate represents a statistic that summarizes information related to prices in a country's market. This statistic could be the average or variance of prices, and it even includes the mass of varieties since $M_i := \sum_{j\in\mathcal{C}}\int_{\omega\in\overline{\Omega}_j} \mathbb{1}_{(p_{ji}(\omega)<\overline{p}_i)}\,\mathrm{d}\omega$.

The definitions can be illustrated through the linear demand from Section 2, which has two price aggregators, $M_i\left[(\mathbf{p}_{ji})_{j\in\mathcal{C}}\right]$ and $\mathbb{P}_i\left[(\mathbf{p}_{ji})_{j\in\mathcal{C}}\right]$, with price aggregates being specific values of each. Based on these concepts, I define an aggregator and an aggregate.

Definition 3. *Let $\boldsymbol{\mathcal{P}}_i := \left(\mathcal{P}_i^k\right)_{k=1}^K$ and $\mathbf{P}_i := \left(\mathbb{P}_i^k\right)_{k=1}^K$ with $K<\infty$. Let each $\mathcal{P}_i^k$ be a price aggregator and $\mathbb{P}_i^k$ a price aggregate as in Definition 2, with functions $\left(h_{ji}^k\right)_{j\in\mathcal{C}}$ for*

[8] Specifically, it covers: 1) demands from an additively separable direct utility as in Krugman (1979), which includes Simonovska's (2015) Stone-Geary, the generalized CES as in Jung et al. (2015) and Arkolakis et al. (2019), and Behrens and Murata's (2007) CARA, 2) Melitz and Ottaviano's (2008) linear demand, 3) Feenstra's (2003) translog demand, 4) demands from an additively separable indirect utility as in Bertoletti et al. (2018), including their version of the addilog demand, 4) demands from discrete choice models as in Luce (1959) and McFadden (1973), including the Logit, 5) Nocke and Schutz's (2018) demands from discrete-continuous choices models, and 6) constant expenditure demand systems as in Vives (2001) and Bjornerstedt and Verboven (2016). See Appendix D for how these demands are defined in terms of a scalar. Examples of demands that depend on more than one single aggregate, and hence not covered, are the demand by Kimball (1995), the demands considered in Matsuyama and Ushchev (2017), and the QMOR system by Feenstra (2018).

[9] The aggregator collapses to a real number since there is a continuum of firms, making the price aggregator vanish any aggregate uncertainty. The property arises formally under some appropriate law of large numbers, such that the real number representing the aggregate corresponds to a degenerate random variable that takes a value $\mathbb{P}_i$ with probability one. For further details, see Uhlig (1996) and Acemoglu and Jensen (2010).

*each k. An **aggregator** for country i is a smooth real-valued function $\mathcal{A}_i(\mathbb{P}_i)$, whereas an **aggregate** for country i is a value $\mathbb{A}_i \in \operatorname{range} \mathcal{A}_i$.*[10]

The aggregate can be interpreted as a measure of competition in a country's market, with the aggregator indicating the different compositions that generate this specific level of competition. Taking the linear demand from Section 2 as an example, the aggregator corresponds to the choke-price function $\mathcal{P}_i^{\max}$ defined in (1). Likewise, the aggregate is a specific value in the range of $\mathcal{P}_i^{\max}$, which I referred to as $p_i^{\max}$. Thus, the aggregator indicates the different combinations of prices $\mathbb{P}_i$ and mass of firms M_i that result in a specific value $p_i^{\max}$. Based on these definitions, I define the demand system in the following way.

Assumption DEM. *The **demand** in j of firm ω from i is given by*

$$q_{ij}(\omega) := \max\left\{0, q_j\left[\mathbb{A}_j, p_{ij}(\omega)\right]\right\},$$

where $\mathbb{A}_j$ is as in Definition 3, and $\mathcal{A}_j$ is decreasing in $(\mathbf{p}_{kj})_{k\in\mathcal{C}}$ when it is defined through $\boldsymbol{\mathcal{P}}_k$. Moreover, q_j is a smooth function that is decreasing in $p_{ij}(\omega)$ and in $\mathbb{A}_j$, and has a choke price belonging to $\mathbb{R}_{++} \cup \{\infty\}$.

Some remarks about Assumption DEM are in order. First, it establishes that, conditional on a firm's price, $\mathbb{A}_j$ is all that matters to characterize a firm's demand in j. In other words, the specific combination of prices and masses of firms that gives rise to an aggregate does not provide any valuable information to a firm.[11]

Second, Assumption DEM is defined such that increases in $\mathbb{A}_j$ represent tougher competition. This follows since reductions in prices increase $\mathbb{A}_j$, which in turn reduces a firm's demand in j. Notice that decreases in prices encompass not only reductions in prices of

[10] It can be shown that range $\mathcal{A}_i$ is compact and convex, which enables us to obtain well-defined equilibrium conditions. This occurs because, even when optimal prices are discontinuous due to entry and exit, price aggregators have a compact convex range by Lyapunov's Convexity Theorem. For further details, see Appendix F.

[11] Mathematically, Assumption DEM states that the demand of firm ω from i in j satisfies weak separability of $\left(\mathbb{P}_j^k\right)_{k=1}^K$ from $p_{ij}(\omega)$. Exploiting this feature and based on the differential characterization of weak separability due to Leontief (1947) and Sono (1961), I provide conditions to check whether a demand satisfies Assumption DEM in Appendix E.

active firms, but also entry of firms, which occurs when a firm lowers its price relative to the choke price.

Finally, some demands depending on a single aggregate are commonly presented in the literature by an aggregate capturing greater competition through decreases in $\mathbb{A}_j$. Cases like this can be easily accommodated by Assumption DEM. For instance, greater competition under the linear demand is expressed through decreases in $p_j^{\max}$, in which case setting $\mathbb{A}_j := 1/p_j^{\max}$ would fit Assumption DEM. Ultimately, what matters for defining demand is that there is a monotone relation between $\mathcal{A}_j$ and $(\mathbf{p}_{kj})_{k\in\mathcal{C}}$, and between $q_{ij}(\omega)$ and $\mathbb{A}_j$.

3.3 Equilibrium Conditions

I derive the equilibrium by supposing that it exists, is unique, and interior. For details about existence and uniqueness, see Appendix F.

Consider a firm from i with productivity φ. This firm sets a price equal to the choke price if it does not serve j, while its price if it is active in j is

$$p_{ij} = m_j(\mathbb{A}_j, p_{ij})\, c_i(\varphi, \tau_{ij}), \tag{14}$$

where $m_j(\mathbb{A}_j, p_{ij}) := \frac{\varepsilon_j(\mathbb{A}_j, p_{ij})}{\varepsilon_j(\mathbb{A}_j, p_{ij})-1}$ is the firm's markup in j, with $\varepsilon_j(\mathbb{A}_j, p_{ij}) := -\left.\frac{\partial \ln q_j(\mathbb{A}_j, p)}{\partial \ln p}\right|_{p=p_{ij}}$. Denote the implicit p_{ij} that satisfies (14) by $p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$, and refer to the markup evaluated at this price by $m_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$.[12] Then, optimal prices for each $\varphi \in \left[\underline{\varphi}_i, \overline{\varphi}_i\right]$ are

$$p_{ij}^*(\mathbb{A}_j, \varphi_{ij}, \varphi; \tau_{ij}) := \begin{cases} p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij}) & \text{if } \varphi \geq \varphi_{ij} \\ \overline{p}_j & \text{otherwise} \end{cases}, \tag{PRICE}$$

where φ_{ij} is the productivity cutoff of a firm from i to serve j. Given this pricing rule, the

[12] There is some abuse of notation by denoting optimal prices of active firms by $p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$. Since the demand function for positive quantities is a function $q_j(\cdot)$, and the demand does not depend on i conditional on τ_{ij}, strictly speaking the function should be written as $p_j(\mathbb{A}_j, \varphi; \tau_{ij})$. I use the notation $p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ so that the country of origin can be immediately identified. A similar remark applies to the rest of the optimal variables.

firm's optimal quantity in j is given by

$$q_{ij}^{*}\left(\mathbb{A}_j,\varphi_{ij},\varphi;\tau_{ij}\right):=\begin{cases} q_{ij}\left[\mathbb{A}_j,p_{ij}\left(\mathbb{A}_j,\varphi;\tau_{ij}\right)\right] & \text{if } \varphi\geq\varphi_{ij} \\ 0 & \text{otherwise}\end{cases}. \tag{15}$$

Consequently, its optimal gross profit in j is

$$\pi_{ij}\left(\mathbb{A}_j,\varphi;\tau_{ij}\right):=q_{ij}\left[\mathbb{A}_j,p_{ij}\left(\mathbb{A}_j,\varphi;\tau_{ij}\right)\right]\left[p_{ij}\left(\mathbb{A}_j,\varphi;\tau_{ij}\right)-c_i\left(\varphi,\tau_{ij}\right)\right],$$

and so its equilibrium profit in j becomes

$$\pi_{ij}^{*}\left(\mathbb{A}_j,\varphi_{ij},\varphi;\tau_{ij},f_{ij}\right):=\mathbb{1}_{\left(\varphi\geq\varphi_{ij}\right)}\left[\pi_{ij}\left(\mathbb{A}_j,\varphi;\tau_{ij}\right)-f_{ij}\right].$$

The derivation of optimal profits makes it possible to define the productivity cutoff of a firm from i to sell in j. Formally, this corresponds to the level of productivity that provides zero profits:

$$\pi_{ij}\left(\mathbb{A}_j,\varphi_{ij};\tau_{ij}\right)=f_{ij} \text{ for each } i,j\in\mathcal{C}. \tag{ZCP}$$

I denote the implicit solution φ_{ij} to (ZCP) by $\varphi_{ij}^{*}\left(\mathbb{A}_j;\tau_{ij},f_{ij}\right)$.

Given a mass of incumbents $\mathbf{M}^E:=\left(M_j^E\right)_{j\in\mathcal{C}}$, the equilibrium at the market stage requires that all firms choose prices optimally. This condition can be characterized in a straightforward way by exploiting the existence of a single aggregate. Specifically, all firms' decisions in i are determined exclusively by $\mathbb{A}_i$, which is a value belonging to the range of the aggregator $\mathcal{A}_i$. Thus, there is equilibrium at i's market stage when there exists a value $\mathbb{A}_i$ such that each firm's optimal decision self-generates it. Formally, this means that, for a given $\mathbf{M}^E$, the aggregate $\mathbb{A}_i$ has to be a fixed point of $\mathcal{A}_i^{*}$, which is the aggregator evaluated at the optimal variables:

$$\mathbb{A}_i=\mathcal{A}_i^{*}\left[\mathbb{A}_i,\mathbf{M}^E;\left(\tau_{ji},f_{ji}\right)_{j\in\mathcal{C}}\right] \text{ for each } i\in\mathcal{C}. \tag{MS}$$

The fact that the right-hand side of (MS) is a function of $\left(\mathbb{A}_i,\mathbf{M}^E;\left(\tau_{ji},f_{ji}\right)_{j\in\mathcal{C}}\right)$ follows by the form of $\mathcal{P}_i^k$ in equilibrium. More precisely, $\mathcal{P}_i^k$ takes firms' optimal prices from j as inputs of the function. Likewise, optimal prices are completely characterized by the price decision $p^*_{ji}(\mathbb{A}_i, \varphi_{ji}, \varphi; \tau_{ji})$, the survival productivity cutoff $\varphi^*_{ji}(\mathbb{A}_i; \tau_{ji}, f_{ji})$, and the density of the mass of firms for each productivity level, which for country j is $M^E_j g_j(\varphi)$. Thus, in equilibrium, the coordinate k of $\boldsymbol{\mathcal{P}}^*_i$ can be described by a function $\mathcal{P}^{k*}_i \left[\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j\in\mathcal{C}}\right]$.[13] All this implies that the aggregator can be expressed by a function $\mathcal{A}^*_i \left(\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j\in\mathcal{C}}\right) := \mathcal{A}_i \left[\boldsymbol{\mathcal{P}}^*_i \left(\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j\in\mathcal{C}}\right)\right]$.

Finally, by substituting in $\varphi^*_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij})$, the free-entry conditions can be expressed in the following way:

$$\sum_{j\in\mathcal{C}} \pi^{\text{expect}}_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij}) = F_i \text{ for each } i \in \mathcal{C}, \tag{FE}$$

where $\pi^{\text{expect}}_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij}) := \int^{\overline{\varphi}_i}_{\varphi^*_{ij}(\mathbb{A}_j;\tau_{ij},f_{ij})} \left[\pi_{ij}(\mathbb{A}_j, \varphi; \tau_{ij}) - f_{ij}\right] \mathrm{d}G_i(\varphi)$.

3.4 Equilibrium Properties

The goal in subsequent sections is to analyze whether trade shocks are pro-competitive. Pro-competitive effects are defined as reductions in the prices of active domestic firms and increases in the domestic firms' productivity cutoff. These variables are determined by (PRICE) and (ZCP), and so completely identified in i by $\mathbb{A}^*_i$.

The way I set the equilibrium conditions makes it possible to identify some equilibrium properties, with implications about how $\mathbb{A}^*_i$ can be pinned down. They follow by simple observation, and I state them in the following lemma.

Lemma 1. *Suppose the market structure in each country $i \in \mathcal{C}$ is à la Melitz, with demands as in Assumption DEM. Then:*

- *all the equilibrium values can be determined by identifying $(\mathbb{A}^*_i)_{i\in\mathcal{C}}$ and $\mathbf{M}^{E*}$, which are in turn pinned down by the system comprising (FE) and (MS), and*
- *the system of equations formed by (FE) and (MS) is separable, such that $(\mathbb{A}^*_i)_{i\in\mathcal{C}}$ is pinned down exclusively by (FE) and independently of $\mathbf{M}^{E*}$.*

[13] To see this more clearly, assuming price aggregators defined through the active firms' prices, the coordinate k of $\boldsymbol{\mathcal{P}}^*_i$ would be given by $\sum_{j\in\mathcal{C}} M_{ji} \int^{\overline{\varphi}_j}_{\varphi^*_{ji}(\mathbb{A}_i;\tau_{ji},f_{ji})} h^k_{ji}\left[p_{ji}(\mathbb{A}_i, \varphi; \tau_{ji})\right] \frac{\mathrm{d}G_j(\varphi)}{1-G_j(\varphi^*_{ji})}$, which by using that $\frac{M_{ji}}{1-G_j(\varphi^*_{ji})} = M^E_j$ becomes $\sum_{j\in\mathcal{C}} M^E_j \int^{\overline{\varphi}_j}_{\varphi^*_{ji}(\mathbb{A}_i;\tau_{ji},f_{ji})} h^k_{ji}\left[p_{ji}(\mathbb{A}_i, \varphi; \tau_{ji})\right] \mathrm{d}G_j(\varphi)$.

The lemma establishes that $(\mathbb{A}_i^*)_{i\in\mathcal{C}}$ is completely determined by the system (FE), and so independently of the system (MS) and $\mathbf{M}^{E*}$. As a corollary, it is only necessary to solve the system (FE) to study pro-competitive effects; solving for (MS) or knowing $\mathbf{M}^{E*}$ is not.

An interpretation of what conditions (FE) and (MS) accomplish can be provided. The aggregate $\mathbb{A}_i^*$ represents a measure of competition in country i. As such, (FE) pins down the **level** of competition in each country that is consistent with zero expected profits. Furthermore, by definition, $\mathbb{A}_i$ is a value that belongs to the range of the aggregator $\mathcal{A}_i$. Thus, (MS) determines the **composition** of the aggregator in i that generates the level of competition $\mathbb{A}_i^*$. Likewise, the composition of the aggregator is given by the masses of firms and their prices, which are equivalently expressed in equilibrium through the masses of incumbents, prices, and the productivity cutoffs. However, $\mathbb{A}_i^*$ identifies the equilibrium prices and survival productivity cutoffs, leaving only the mass of incumbents to be determined. In sum, (FE) identifies each country's aggregate, while (MS) pins down the masses of incumbents consistent with the equilibrium aggregates.

3.5 Assumptions for Comparative Statics

When a trade shock does not impact the aggregate, the results can be characterized without further assumptions. As I show below, this applies in particular when I study tougher import competition in a small country. On the contrary, further assumptions are needed when a trade shock predicts a non-zero impact on competitive effects. This is relevant in particular for the analysis of export-related channels.

The following assumption is really mild and all that is needed to obtain pro-competitive effects. It establishes that a more competitive environment makes demand more elastic.

Assumption 1. $\frac{\partial \varepsilon_i(\mathbb{A}_i, p)}{\partial \mathbb{A}_i} > 0$ *for any* $(\mathbb{A}_i, p)$.

If we are only interested in pro-competitive effects, no other assumption is needed. Instead, the following assumption is required if the goal is to characterize the impact on the masses of incumbents and the mechanism behind some results.

Assumption 2. $\frac{\partial \mathcal{A}_i^*\left(\mathbb{A}_i, \mathbf{M}^E\right)}{\partial \mathbb{A}_i} < 1$ *for any* $\left(\mathbb{A}_i, \mathbf{M}^E\right)$.

When evaluated locally at $\mathbb{A}_i^*$, Assumption 2 constitutes an "almost" if-and-only-if condition for uniqueness of equilibrium at the market stage (i.e., after entry decisions of incumbents have been made). The assumption can additionally be interpreted as a global stability condition for the market stage.

4 A Small Country

In this section, I analyze reductions in import and export trade costs in a small country H. The study of this case is relevant on its own, since it characterizes outcomes in small economies. Nonetheless, its main relevance for the paper is that trade shocks in a small country directly isolate the import-competition and export-opportunities channel. These channels are respectively defined as the pro-competitive effects exclusively caused by exposure of domestic firms to tougher import competition and better business opportunities to sell abroad. As I show in the next subsection, this contrasts with a reduction in import trade costs in a large country, which simultaneously changes a country's export conditions and hence does not isolate the import-competition channel.

The formal concept of small country I utilize follows Demidova and Rodríguez-Clare (2009; 2013) and Melitz (2018). These authors extend the notion of a small country in perfect competition to the case of monopolistic competition with heterogeneous firms. The first pair does it for a CES demand, while the latter for demands that derive from an additively separable utility function. Their definition establishes that changes in H have no impact on the domestic conditions of any foreign country. Thus, $\left(\mathbb{A}_j^*, M_j^{E*}\right)_{j\in\mathcal{C}\backslash\{H\}}$ is not impacted by shocks in H.

Demidova and Rodríguez-Clare (2013) show that this definition arises as an equilibrium outcome under a CES demand when H's share of the world population is negligible. I provide an alternative rationalization in Appendix B for any demand satisfying Assumption DEM. This considers that each country has a continuum of trading partners, including H. The rationalization identifies the relevant equilibrium conditions for solving the model under demands as in DEM. Furthermore, it provides an intuitive explanation to why $\left(\mathbb{A}_j^*, M_j^{E*}\right)_{j\in\mathcal{C}\backslash\{H\}}$ is not affected: a shock in H's market has a negligible impact on each trading partner's expected profits.

This interpretation becomes important for the analysis of trade between two large countries. It basically determines that better export opportunities to sell in a small country do not induce entry of domestic firms in foreign countries, and so domestic competition in the foreign countries is unaffected. Notice that this does not preclude changes in the mass of firms of a trading partner serving the small country—given a mass of incumbents, extensive-margin adjustments still occur through variations in the productivity cutoffs.

Incorporating that H is a small country, the systems (FE) and (MS) collapse to one equation each:

$$\pi_{HH}^{\text{expect}}\left(\mathbb{A}_H^*\right) + \sum_{j\in\mathcal{C}\backslash\{H\}} \pi_{Hj}^{\text{expect}}\left(\mathbb{A}_j^*; \tau_{Hj}, f_{Hj}\right) = F_H, \tag{16}$$

$$\mathbb{A}_H^* = \mathcal{A}_H^*\left[\mathbb{A}_H^*, M_H^{E*}, \left(M_j^{E*}\right)_{j\in\mathcal{C}\backslash\{H\}}; \left(\tau_{jH}, f_{jH}\right)_{j\in\mathcal{C}\backslash\{H\}}\right], \tag{17}$$

implying in particular that (16) completely identifies $\mathbb{A}_H^*$.

Simple inspection shows that a shock to $\left(\tau_{jH}, f_{jH}\right)_{j\in\mathcal{C}\backslash\{H\}}$ does not affect (16). Consequently, tougher import competition does not impact H's aggregate—it only decreases the mass of domestic incumbents. The following proposition formalizes this.

Proposition 1: ***Tougher Import Competition in a Small Country***

Consider a small country H and suppose there is either a small or large reduction in τ_{jH} or f_{jH} for each country $j \in \mathcal{C}\backslash\{H\}$ assumed large. Then,

- *$\mathbb{A}_H^*$ remains the same (i.e., competition in H does not vary),*
- *φ_{HH}^* does not vary,*
- *$p_{HH}^*(\varphi)$ and $q_{HH}^*(\varphi)$ for each $\varphi \geq \varphi_{HH}^*$ do not vary, and*
- *M_H^{E*} and M_{HH}^* decrease.*

The mechanism behind the result is that there are two opposing effects on H's competitive environment. First, competition in H increases due to tougher import competition. Second, tougher import competition reduces expected profits in H, and so a lower mass of firms from H pays the entry cost; this eventually leads to a reduction in the mass of active domestic firms, which decreases competition in H. Overall, these opposing effects are perfectly offset and determine a null impact on H's competitive environment.

In Appendix C.1, I prove that the import-competition channel is also inactive under multidimensional firm heterogeneity, i.e. when a firm gets draws of both productivity and demand appeal after paying the entry cost. Appendix C.2 additionally shows that tougher import competition does not affect any country-specific decision of a domestic firm. This includes variables such as quality or number of varieties for multiproduct firms. Thus, any choice by a domestic firm is the same before and after a shock to import trade costs.[14]

On the contrary, better export opportunities, which are captured by a negative shock to $(\tau_{Hj}, f_{Hj})_{j\in\mathcal{C}\backslash\{H\}}$, impact (16) and are pro-competitive. The following proposition formally states this.

Proposition 2: ***Better Export Opportunities in a Small Country***

Consider a small country H and suppose there is a small or large reduction in τ_{HF} or f_{HF} where $F \in \mathcal{C}\backslash\{H\}$ is a large country. Then:

- *$\mathbb{A}_H^*$ increases (i.e., competition in H increases),*
- *φ_{HH}^* increases,*
- *$p_{HH}^*(\varphi)$ decreases for each $\varphi \geq \varphi_{HH}^*$ if Assumption 1 holds for H, and*
- *M_H^{E*} increases if Assumption 2 holds for H.*

Under regularity conditions, the result arises since better export opportunities to serve a large country represent non-negligible increases in H's expected profits. This induces a greater mass of firms from H to pay the entry cost, with a subset of them ultimately surviving and serving the domestic market. Thus, competition in H increases, making the markups of domestic firms decrease and the domestic productivity cutoff increase.

5 Large Countries

The small-country case reveals that the export-opportunities channel is active, whereas the import-competition channel is not. In this section, I investigate the consequences of this result for the usual case of two large countries, $\mathcal{C} := \{H, F\}$, and two-way trade (i.e.,

[14] In Appendix C.3, I also show that zero competitive effects of tougher import competition hold under the nested CES and Logit demands with varieties partitioned by country of origin (i.e., into domestic and foreign) or by varieties produced by the same multiproduct firm.

when both countries simultaneously import and export). The goal is to show that this scenario always entails a Metzler paradox, meaning that a reduction in import trade costs decreases competition. Furthermore, I provide a rationalization to this outcome based on competitive effects exclusively reflecting worse export conditions.

For the analysis, I consider a reduction in import trade costs in H. This allows me to simultaneously study reductions in import trade costs (effects in H) and in export trade costs (effects in F). I illustrate the explanations through a decrease in τ_{FH}, but the results I derive are also valid for a reduction in f_{FH}.

Unlike what occurs in a small country, a reduction in import trade costs in a large country not only represents tougher import competition, but also changes in its trading partner's competitive environment. The latter occurs because, given the size of H, a reduction in H's import trade costs implies non-negligible better export opportunities for F, which increases F's expected profits in a non-trivial way. This induces a greater mass of firms from F to pay the entry cost, with a subset of them eventually surviving and serving the domestic market. Thus, competition in F increases. In sum, H is exposed to tougher import competition *and* faces changes in its export conditions. I refer to the competitive effects in H generated by the latter as H's export-conditions channel.

The export-opportunities and export-conditions channel operate through variations in expected profits, and actually have similar implications. The distinction is only incorporated because different variables trigger each channel: considering H, the export-opportunities channel affects H through reductions in its export trade costs (i.e., τ_{HF}), while the export-conditions channel acts through changes in the foreign country's competitive conditions (i.e., $\mathbb{A}_F$). Notice that the small-country assumption effectively shuts the export-conditions channel, since $\mathbb{A}_F$ is assumed fixed following shocks in H.

It is worth remarking that my conclusions regarding the import-competition channel (and the export-opportunities channel) are not affected by the consideration of large countries—it is still true that solely exposing firms to tougher competition results in zero competitive effects. It only implies that a reduction in import trade costs under two large countries does not directly isolate the import-competition channel, since the export-conditions channel also influences the competitive environment. Nonetheless, identical results as in the small-country case emerge when the export-conditions channel is shut.

To see this, suppose a scenario with one-way trade, where firms from F export to H, but firms from H only serve their domestic market. A decrease in τ_{FH} represents tougher import competition for firms from H, like in the small-country case. However, unlike the small-country case, this shock also affects F's domestic market, by providing F with better export opportunities that entail non-trivial effects on its expected profits. However, this aspect is irrelevant for H in the scenario considered: no firm from H sells abroad, and so any change in F's home market does not affect firms from H. Due to this, H's competitive environment is impacted exclusively through the import-competition channel, and hence it remains unaffected.

To establish this, the free-entry condition in H and F under one-way trade is respectively

$$\int_{\varphi_{HH}^{*}(\mathbb{A}_H)}^{\overline{\varphi}_H} \left[\pi_{HH}\left(\mathbb{A}_H,\varphi\right)-f_{HH}\right]\mathrm{d}G_H\left(\varphi\right)=F_H, \tag{18}$$

$$\int_{\varphi_{FF}^{*}(\mathbb{A}_F)}^{\overline{\varphi}_F} \left[\pi_{FF}\left(\mathbb{A}_F,\varphi\right)-f_{FF}\right]\mathrm{d}G_F\left(\varphi\right)+\int_{\varphi_{FH}^{*}(\mathbb{A}_H;\tau_{FH})}^{\overline{\varphi}_F} \left[\pi_{FH}\left(\mathbb{A}_H,\varphi;\tau_{FH}\right)-f_{FH}\right]\mathrm{d}G_F\left(\varphi\right)=F_F. \tag{19}$$

Notice that F in this scenario is exclusively affected by better export opportunities. In fact, a scenario with one-way trade determines identical quantitative results to the small-country case, with H capturing the import-competition channel and F the export-opportunities channel. The following proposition formalizes this.

Proposition 3: *Two Large Countries and One-Way Trade*

Consider a world economy with set of large countries $\mathcal{C} := \{H, F\}$. Moreover, suppose that some firms from F export to H, while firms from H only serve home. If there is a decrease in τ_{FH} or f_{FH}, then:

- *The quantitative effects in country H are identical to a reduction in import trade costs in a small country. This implies in particular that $\mathbb{A}_H^*$ does not vary, and so φ_{HH}^*, $p_{HH}^*(\varphi)$, and $q_{HH}^*(\varphi)$ for each $\varphi \geq \varphi_{HH}^*$ remain unaffected.*
- *The quantitative effects in country F are identical to a reduction in export trade costs in a small country. This implies in particular that $\mathbb{A}_F^*$ increases (i.e., competition in F increases), φ_{FF}^* increases, and $p_{FF}^*(\varphi)$ decreases for each $\varphi \geq \varphi_{FF}^*$ if Assumption 1 holds for F.*

On the contrary, a decrease in τ_{FH} under two-way trade impacts H's competitive environment through the export-conditions channel. This leads to a Metzler paradox, which means that competition in H decreases. Before providing a rationalization to this outcome, I begin by formally stating the result. To ultimately provide an interpretation through differential calculus, I consider infinitesimal variations of τ_{FH} and f_{FH}. The same results hold under large changes of these variables.[15]

Proposition 4: *Two Large Countries and Two-Way Trade*

Consider a world economy with set of large countries $\mathcal{C} := \{H, F\}$ and two-way trade. Suppose that $|J_{FE}| > 0$ and $|J_{MS}| > 0$, where J_{FE} and J_{MS} refer to the Jacobians of the systems (FE) and (MS), respectively. If there is a small decrease in τ_{FH} or f_{FH}, then:

- *$\mathbb{A}_H^*$ decreases (i.e., competition in H decreases), and φ_{HH}^* and M_H^{E*} decrease. Moreover, $p_{HH}^*(\varphi)$ increases for each $\varphi \geq \varphi_{HH}^*$ if Assumption 1 holds for H.*
- *$\mathbb{A}_F^*$ increases (i.e., competition in F increases), and φ_{FF}^* and M_F^{E*} increase. Moreover, $p_{FF}^*(\varphi)$ decreases for each $\varphi \geq \varphi_{FF}^*$ if Assumption 1 holds for F.*

One way to interpret the impact on H's competitive environment is given by comparing Propositions 3 and 4. Both cases consider an exposure of firms in H to tougher competition, but the latter additionally incorporates that H exports. For F, the decrease in H's import trade costs represents better business opportunities to sell in a large country, ultimately increasing competition in F. While this aspect is irrelevant under one-way trade because no firm from H exports, it becomes crucial when there is two-way trade: it determines that firms from H face worse export conditions, and so their expected profits decrease. This leads to a decrease in competition in H, once worse export conditions have similar effects to lower export opportunities.

The same intuition can be more clearly provided by using infinitesimal calculus. To do this, I use that the free-entry condition in H and F under two-way trade is respectively

$$\int_{\varphi_{HH}^*(\mathbb{A}_H)}^{\overline{\varphi}_H} \left[\pi_{HH}(\mathbb{A}_H, \varphi) - f_{HH}\right] \mathrm{d}G_H(\varphi) + \int_{\varphi_{HF}^*(\mathbb{A}_F)}^{\overline{\varphi}_H} \left[\pi_{HF}(\mathbb{A}_F, \varphi) - f_{HF}\right] \mathrm{d}G_H(\varphi) = F_H, \tag{20}$$

$$\int_{\varphi_{FF}^*(\mathbb{A}_F)}^{\overline{\varphi}_F} \left[\pi_{FF}(\mathbb{A}_F, \varphi) - f_{FF}\right] \mathrm{d}G_F(\varphi) + \int_{\varphi_{FH}^*(\mathbb{A}_H;\tau_{FH})}^{\overline{\varphi}_F} \left[\pi_{FH}(\mathbb{A}_H, \varphi; \tau_{FH}) - f_{FH}\right] \mathrm{d}G_F(\varphi) = F_F. \tag{21}$$

[15] Basically, this follows by adding similar regularity conditions holding for Jacobians to those stated in Proposition 4.

Denote the implicit solution $\mathbb{A}_H$ to (20) by $\mathbb{A}_H(\mathbb{A}_F)$. This function determines how changes in H's export conditions (i.e., $\mathbb{A}_F$) affect H's expected profits and hence H's home market (i.e., $\mathbb{A}_H$). Likewise, let the implicit solution $\mathbb{A}_F$ to (21) be $\mathbb{A}_F(\mathbb{A}_H;\tau_{FH})$. This function shows how better export opportunities to sell in H (i.e., decreases in τ_{FH}) impact F's expected profits and hence F's home market (i.e., $\mathbb{A}_F$).

Defining $(\mathbb{A}_H^*, \mathbb{A}_F^*)$ as the pair of values such that both (20) and (21) hold, it can be shown after some algebra that

$$\underbrace{\mathrm{d}\mathbb{A}_H^*}_{\text{total effect }(<0)} = \underbrace{\kappa \frac{\partial \mathbb{A}_H\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F} \frac{\partial \mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial \tau_{FH}}}_{H\text{'s export-conditions channel }(>0)} \mathrm{d}\tau_{FH}, \tag{AGG-H}$$

$$\underbrace{\mathrm{d}\mathbb{A}_F^*}_{\text{total effect }(>0)} = \underbrace{\kappa \frac{\partial \mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial \tau_{FH}}}_{F\text{'s export-opportunities channel }(<0)} \mathrm{d}\tau_{FH} \tag{AGG-F}$$

where $\mathrm{d}\tau_{FH} < 0$, $\frac{\partial \mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial \tau_{FH}} < 0$, $\frac{\partial \mathbb{A}_H\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F} < 0$, and $\frac{\partial \mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial \mathbb{A}_H} < 0$. Moreover, $\kappa := \left(1 - \frac{\partial \mathbb{A}_H\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F}\frac{\partial \mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial \mathbb{A}_H}\right)^{-1} > 1$ is a multiplier of effects capturing all the indirect effects triggered in equilibrium.

Intuitively, since tougher import competition in isolation entails no competitive effects, the impact on H's competitive environment is triggered by τ_{FH} affecting $\mathbb{A}_F$ directly. To see this, we can interpret the terms in (AGG-H). The term $\frac{\partial \mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial \tau_{FH}}\mathrm{d}\tau_{FH}$ captures the impact on F's competitive environment due to the better export opportunities for F. They act by increasing F's expected profits, thereby inducing entry of domestic firms and hence increasing competition in F. Furthermore, the term $\frac{\partial \mathbb{A}_H\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F}$ reflects that changes in H's export conditions (i.e., increases in $\mathbb{A}_F^*$) reduce H's expected profits, thus inducing exit of domestic firms that lowers competition in H. This last effect triggers indirect effects, which are captured through the multiplier κ.

(AGG-H) and (AGG-F) can also be used to rationalize the results when H is a small country. Consider that H is Iceland (a small country) and F is China (a large country). A reduction in Iceland's import trade costs determines that active Chinese exporters reduce their prices and that a greater mass of Chinese firms starts serving Iceland (due to a reduction in China's productivity cutoff to serve Iceland). However, tougher import competition in isolation has a null impact on Iceland's competitive environment. Additionally, using the interpretation of small country given in Appendix B, a shock to a small country H has a negligible impact on F's expected profits, meaning that $\frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \tau_{FH}} \to 0$. Consequently, (AGG-$F$) implies that $\mathbb{A}_F^*$ is not impacted, and hence neither is $\mathbb{A}_H^*$ due to (AGG-H). Intuitively, the result reflects that the size of Iceland's domestic market is insignificant, and hence export opportunities to sell there do not affect China's expected profits. Due to this, the trade shock does not induce entry of domestic firms in China, and the Chinese home market is unaffected.

The case of two large countries, on the contrary, supposes that H is a country like the USA. The consideration of a large country does not modify that tougher import competition in isolation has a null impact on the US' competitive environment. However, better business opportunities in China to sell in the US have a non-negligible impact on Chinese firms' expected profits. This stimulates entry of Chinese firms, which eventually increases competition in the Chinese home market and is captured by $\frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \tau_{FH}} < 0$. Then, by (AGG-$H$), American firms exporting to China face tougher competition, which reduces expected profits in the US. This reduces the mass of American firms paying the entry cost, implying a decrease in the mass of American firms serving home, and ultimately lowering competition in the US.

6 Conclusions

In this paper, I analyzed pro-competitive effects in a monopolistic-competition model with heterogeneity à la Melitz, under standard demands and any productivity distribution. The goal was to investigate the emergence of pro-competitive effects when trade exposes domestic firms to tougher product market competition. Pro-competitive effects were defined as decreases in the domestic firms' average prices, occurring through both reductions in the domestic firms' markups and the exit of the least efficient domestic firms. The analysis was based on the theory of large aggregative economies by Acemoglu and Jensen (2010; 2015), which employs monotone comparative statics and thus obtains results under a minimal set of sufficient conditions.

Considering an industry in isolation, I showed that exposure of domestic firms to tougher import competition results in null competitive effects—it reduces the mass of domestic incumbents to such an extent that it leaves the domestic competitive environment unaffected. Due to this, the domestic firms' quantities, markups, and productivity cutoff are not impacted. On the contrary, better export opportunities are pro-competitive: they decrease the markups of active domestic firms and increase their survival productivity cutoff. The mechanism is an increase in expected profits that induces a greater mass of domestic firms to enter the industry, with a subset of them eventually surviving and serving home.

I showed in particular that these results respectively characterize the impact on a small country of a reduction in import and export trade costs. Furthermore, I considered the case of two large countries. Under one-way trade, where the country studied imports goods but its firms do not export, I showed that reducing import trade costs does not affect the competitive environment. Then, I considered a decrease in import trade costs under two-way trade, and showed that competition decreases. I provided a rationalization to this outcome by showing that competitive effects reflect worse export conditions.

The main implication of my paper concerns the use of a setting with heterogeneity à la Melitz under standard demands. Considering an industry in isolation, the findings indicate that model is suitable for studying pro-competitive effects of policies promoting exporting; on the contrary, it is not for pro-competitive effects of policies that increase import competition in the product market.

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Online Appendix

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A Proofs and Derivations

Conventions: I use the notation $\widehat{x}$ for the natural logarithm of any function or variable x. To avoid cumbersome notation, I occasionally omit the parameters from a function's arguments if they remain fixed during the analysis.

Throughout the proofs, I make use of several standard results from the monotone comparative statics literature. I state the main ones I use in Lemma 2. The reader is referred to Topkis (1998) for further details.[17]

Lemma 2. *Let $f : X \times \Theta_1 \times \Theta_2 \to \mathbb{R}$, with $X := [\underline{x}, \overline{x}]$ and $\Theta_n := \left[\underline{\theta}_n, \overline{\theta}_n\right]$ for $n = 1, 2$. Then:*

- *if f is quasi-supermodular on $X \times \Theta_1 \times \Theta_2$, then $\arg\max_{x \in X} f(x, \theta_1, \theta_2)$ is increasing in θ_1 and θ_2,*
- *if f is log-supermodular on $X \times \Theta_1 \times \Theta_2$, then f is quasi-supermodular on $X \times \Theta_1 \times \Theta_2$, and*
- *f is log-supermodular on $X \times \Theta_1 \times \Theta_2$ iff f is pairwise log-supermodular.*

The following lemmas are used in subsequent proofs. I first state all of them and then provide proofs for them.

Lemma 3. *$\pi_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ is decreasing in $\mathbb{A}_j$ and τ_{ij}, and increasing in φ. Moreover, $\varphi^*_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij})$ is increasing in all its arguments.*

Lemma 4. *$p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ is decreasing in φ and increasing in τ_{ij}.*

Lemma 5. *If Assumption 1 holds for j, then prices $p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ and markups $m_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ are decreasing in $\mathbb{A}_j$.*

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[17]The results follow from Topkis (1978) and Milgrom and Shannon (1994), applied to the specific case of real-valued functions defined over a compact Euclidean domain.

Lemma 6. *$\mathcal{A}_i^*\left[\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j\in\mathcal{C}}\right]$ is increasing in M_j^E for any $j \in \mathcal{C}$.*

Lemma 7. *$\mathcal{A}_i^*\left[\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j\in\mathcal{C}}\right]$ is decreasing in τ_{ji} and f_{ji}, for any $j \in \mathcal{C}$.*

Proof of Lemma 3. Gross profits are $\pi_{ij}(\mathbb{A}_j, p_{ij}, \varphi; \tau_{ij}) := q_j(\mathbb{A}_j, p_{ij})[p_{ij} - c_i(\varphi, \tau_{ij})]$. The domain of prices does not depend on any of the parameters, q_j is decreasing in $\mathbb{A}_j$, and $c_i(\varphi, \tau_{ij})$ is decreasing in φ and increasing in τ_{ij}. Thus, the conditions for a revealed-preference argument can be applied, implying that the optimal gross profits of active firms, $\pi_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$, are necessarily decreasing in $\mathbb{A}_j$ and τ_{ij}, and increasing in φ.

Regarding $\varphi_{ij}^*(\mathbb{A}_j; \tau_{ij}, f_{ij})$, it is defined as the value φ_{ij}^* that satisfies $\lambda\left(\mathbb{A}_j, \varphi_{ij}^*; \tau_{ji}, f_{ij}\right) = 0$ where $\lambda(\cdot) := \pi_{ij}\left(\mathbb{A}_j, \varphi_{ij}^*; \tau_{ji}\right) - f_{ij}$. Moreover, $\lambda(\cdot)$ is increasing in φ, and decreasing in $\mathbb{A}_j$, τ_{ij}, and f_{ij}. Consequently, increases in either $\mathbb{A}_j$, τ_{ij}, or f_{ij} require that φ increases to restore the equality $\lambda(\cdot) = 0$. Thus, $\varphi_{ij}^*(\mathbb{A}_j; \tau_{ij}, f_{ij})$ is increasing in $\mathbb{A}_j$, τ_{ij}. and f_{ij}. ■

Proof of Lemma 4. Let $\beta \in \{-\varphi, \tau_{ij}\}$. Profits are $\pi_{ij}(\mathbb{A}_j, p_{ij}, \varphi; \beta) = q_j(\mathbb{A}_j, p_{ij})[p_{ij} - c_i(\beta)] - f_{ij}$. For any $p > c$, we can apply logs and get $\frac{\partial^2 \widehat{\pi}_{ij}(\mathbb{A}_j, p_{ij}, \varphi; \beta)}{\partial \beta \partial p_{ij}} = \frac{\partial c_i(\beta)}{\partial \beta} \frac{1}{(p_{ij} - c_i(\beta))^2} > 0$. Thus, by Lemma 2, $\pi_{ij}(\mathbb{A}_j, p_{ij}, \varphi; \beta)$ is log-supermodular in (p_{ij}, β). This establishes that prices are decreasing in φ and increasing in τ_{ij}.■

Proof of Lemma 5. By Lemma 2, if we show that $\pi_{ij}(\mathbb{A}_j, p_{ij}, \varphi)$ is log-supermodular in $(p_{ij}, -\mathbb{A}_j)$, or equivalently log-submodular in $(p_{ij}, \mathbb{A}_j)$, then optimal prices are decreasing in $\mathbb{A}_j$. The result follows since $\frac{\partial^2 \widehat{\pi}_{ij}(\mathbb{A}_j, p_{ij}, \varphi)}{\partial \mathbb{A}_j \partial p_{ij}} = \frac{\partial^2 \widehat{q}_j(\mathbb{A}_j, p_{ij})}{\partial \mathbb{A}_j \partial p_{ij}}$ is negative if and only if $\frac{\partial \varepsilon_j(\mathbb{A}_j, \cdot)}{\partial \mathbb{A}_j} > 0$, and the latter holds by Assumption 1 for j. Regarding markups, let $m_{ij} := \frac{p_{ij}}{c_i(\beta)}$ and reexpress gross profits as a function of it: $\pi_{ij}(\mathbb{A}_j, m_{ij}; \beta) = q_j[\mathbb{A}_j, m_{ij} c_i(\beta)](m_{ij} - 1)c_i(\beta)$. Then, $\frac{\partial^2 \widehat{\pi}_{ij}(\mathbb{A}_j, m_{ij})}{\partial m_{ij} \partial \mathbb{A}_j} = \frac{\partial^2 \widehat{q}_j(\mathbb{A}_j, p_{ij})}{\partial \mathbb{A}_j \partial p_{ij}} c_i(\varphi, \tau_{ij})$, which implies that markups are decreasing in $\mathbb{A}_j$ by the same argument as for prices and hence the same condition. ■

Proof of Lemma 6. To show that $\mathcal{A}_i^*$ is increasing in M_j^E, we need to show that $\boldsymbol{\mathcal{P}}_i^*\left(\mathbb{A}_i, \mathbf{M}^E\right)$ is decreasing in M_j^E. Consider an increase in M_j^E for a given $\mathbb{A}_i$. This represents an increase in the mass of firms paying the entry cost, and each of these firms get a productivity draw. We also know that $\mathcal{A}_i$ is decreasing in $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ through $\boldsymbol{\mathcal{P}}_i$, where $\mathbf{p}_{ji}$ comprises the prices of all varieties (including those unavailable). Depending on the productivity draw that a firm gets, it either keeps setting $\overline{p}_i$ and hence not serving the market, or it becomes active and sets $p_{ji}(\mathbb{A}_i^*, \varphi) < \overline{p}_i$. Thus, since $\boldsymbol{\mathcal{P}}_i^*$ is decreasing when a nonzero set of firms increase their prices, the increase in M_j^E decreases $\boldsymbol{\mathcal{P}}_i^*$. ■

Proof of Lemma 7. Let $\beta \in \{\tau_{ji}, f_{ji}\}$. Since $\mathcal{A}_i$ is decreasing in $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ through $\boldsymbol{\mathcal{P}}_i$, the result follows if $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ is increasing in β when $\mathcal{A}_i$ is evaluated at the optimal values. Suppose that β increases. Prices are affected by two different channels. First, $\varphi_{ji}^*(\mathbb{A}_i; \beta_{ji})$ increases when β increases by Lemma 3, so that some of the firms that were setting $p_{ji}(\mathbb{A}_i, \varphi; \tau_{ji})$ now become inactive and set $\overline{p}_i > p_{ji}(\mathbb{A}_i, \varphi; \tau_{ji})$. Second, firms that remain active before and after the change in β set a price $p_{ji}(\mathbb{A}_i, \varphi; \tau_{ji})$, which is increasing in τ_{ji} by Lemma 4 and remains constant if $\beta = f_{ji}$. Thus, $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ is increasing in β when evaluated at the optimal values and the result follows. ■

A.1 Proofs of Section 4

Proof of Proposition 1. Consider a reduction in $\beta_{jH} \in \{\tau_{jH}, f_{jH}\}$ for each $j \in \mathcal{C}\setminus\{H\}$. By Lemma 1, condition (FE) pins down $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$ and is independent of $\mathbf{M}^E$. I show that the system of equations (FE) is not affected by the shock. First, $(\beta_{jH})_{j\in\mathcal{C}\setminus\{H\}}$ does not directly affect condition (FE) for country H. Besides, since H is a small country, $\left(\mathbb{A}_j^*\right)_{j\in\mathcal{C}\setminus\{H\}}$ does not vary. Both facts imply that $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$ does not vary.

Consider firms from H serving any country $k \in \mathcal{C}$. Since $(\mathbb{A}_k^*, \beta_{Hk})_{k\in\mathcal{C}}$ does not vary, neither $p_{Hk}^*(\varphi)$ for $\varphi \geq \varphi_{Hk}^*$ nor $\varphi_{Hk}^*(\mathbb{A}_k^*; \beta_{Hk})$ vary. Moreover, since neither $p_{Hk}^*(\varphi)$ nor $\mathbb{A}_k^*$ vary, $q_{Hk}^*(\varphi)$ does not vary.

Regarding the mass of firms, consider the system of equations given by condition (MS) for each country $j \in \mathcal{C}\setminus\{H\}$. Since H is a small country, $\left(\mathbb{A}_j^*, M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$ does not vary. Moreover, $(\beta_{kj})_{k\in\mathcal{C}, j\in\mathcal{C}\setminus\{H\}}$ does not change since none of these terms are shocked. This implies that, for condition (MS) in H to hold, M_H^{E*} has to adjust. In equilibrium, condition (MS) in H can be expressed as $\mathbb{A}_H^* = \mathcal{A}_H^*\left[\mathbb{A}_H^*, M_H^{E*}, (\beta_{jH})_{j\in\mathcal{C}\setminus\{H\}}\right]$. By Lemma 7, $\mathcal{A}_H^*$ is decreasing in $(\beta_{jH})_{j\in\mathcal{C}}$ and $\mathbb{A}_H^*$ is the same before and after the shock. Thus, condition (MS) in H can only hold as an equality if M_H^{E*} decreases. In addition, since φ_{HH}^* has not changed but M_H^{E*} decreases, then each M_{HH}^* decreases. ■

Proof of Proposition 2. Consider a reduction of $\beta_{HF} \in \{\tau_{HF}, f_{HF}\}$ for some country

$F \neq H$ that is large. By Lemma 1, the system of conditions (FE) pins down $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$. The parameter β_{HF} does not affect directly the condition (FE) of any country $j \in \mathcal{C}\backslash\{H\}$. Moreover, since H is a small country, $\left(\mathbb{A}_j^*, M_j^{E*}\right)_{j\in\mathcal{C}\backslash\{H\}}$ does not vary.

Regarding country H, applying Lemma 1 again, $\mathbb{A}_H^*$ is completely determined by condition (FE) for H, and independently of M_H^{E*}. By Lemma 3, $\pi_{HH}(\mathbb{A}_H, \varphi)$ is decreasing in $\mathbb{A}_H$ and increasing in φ, and $\varphi_{HH}^*(\mathbb{A}_H)$ is increasing in $\mathbb{A}_H$. Moreover, $\pi_{Hj}(\mathbb{A}_j, \varphi; \tau_{Hj})$ is decreasing in τ_{Hj}, and $\varphi_{Hj}^*(\mathbb{A}_j, \tau_{Hj})$ is increasing in τ_{Hj}. By using that profits are increasing in productivity, $\sum_{k\in\mathcal{C}} \pi_{Hj}^{\text{expect}}$ is decreasing in $\mathbb{A}_H^*$ and decreasing in τ_{HF}. From this, it follows that $\mathbb{A}_H^*$ increases to restore zero expected profits when τ_{HF} decreases.

Besides, since $\varphi_{HH}^*(\mathbb{A}_H, \beta_{kH})$ is increasing in $\mathbb{A}_H$, and β_{kH} does not vary, then $\varphi_{HH}^*(\cdot)$ increases by Lemma 3. Moreover, regarding firms with $\varphi \geq \varphi_{kH}^*$, since $\frac{\partial \varepsilon_H(\cdot,\mathbb{A}_H)}{\partial \mathbb{A}_H} > 0$ by Assumption 1, then $m_{HH}^*(\varphi)$ decrease by Lemma 5 and hence $p_{HH}^*(\varphi)$ decrease too.

Concerning M_H^{E*}, since M_j^{E*} for each $j \neq H$ and β_{kH} for any $k \in \mathcal{C}$ do not vary, condition (MS) in H can be expressed as $\mathbb{A}_H^* = \mathcal{A}_H^*\left(\mathbb{A}_H^*, M_H^{E*}\right)$. When Assumption 2 holds, $\mathbb{A}_H^* - \mathcal{A}_H^*\left(\mathbb{A}_H^*, M_H^{E*}\right)$ is increasing in $\mathbb{A}_H^*$. Thus, using Lemma 6 and the fact $\mathbb{A}_H^*$ is greater, M_H^{E*} has to increase to restore the equality of (MS) in H. ■

A.2 Proofs of Section 5

Proof of Proposition 3. Regarding H, the proof follows since the equilibrium condition identifying $\mathbb{A}_H^*$ is given by (18), which does not depend on any trade cost. In particular, it determines that reductions in H's import trade costs cannot affect $\mathbb{A}_H^*$. This implies that prices, quantities, markups, and the survival productivity cutoff of firms from H do not change.

As for country F, equation (19) completely identifies $\mathbb{A}_F^*$. Additionally, $\mathbb{A}_H^*$ is exogenously given by (18). Thus, (19) is exactly the same equation that identifies results in the small country, i.e. (16). Hence, all the results follow verbatim the proof of Proposition 2.

Proof of Proposition 4. Let $\beta_{HF} \in \{\tau_{FH}, f_{FH}\}$. I first show that $\mathbb{A}_H^*$ is increasing in β_{FH}, and $\mathbb{A}_F^*$ decreasing in β_{FH}. Differentiating conditions (20) and (21),

$$\begin{pmatrix} \frac{\partial \pi_{HH}^{\text{expect}}\left(\mathbb{A}_H^*\right)}{\partial \mathbb{A}_H} & \frac{\partial \pi_{HF}^{\text{expect}}\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F} \\ \frac{\partial \pi_{FH}^{\text{expect}}\left(\mathbb{A}_H^*;\beta_{FH}\right)}{\partial \mathbb{A}_H} & \frac{\partial \pi_{FF}^{\text{expect}}\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F} \end{pmatrix} \begin{pmatrix} \frac{\partial \mathbb{A}_H^*}{\partial \beta_{FH}} \\ \frac{\partial \mathbb{A}_F^*}{\partial \beta_{FH}} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\partial \pi_{FH}^{\text{expect}}\left(\mathbb{A}_H^*;\beta_{FH}\right)}{\partial \beta_{FH}} \end{pmatrix}.$$

From this, we get $\frac{\partial \mathbb{A}_H^*}{\partial \beta_{FH}} = \frac{\frac{\partial \pi_{FH}^{\text{expect}}(\mathbb{A}_H^*;\beta_{FH})}{\partial \beta_{FH}} \frac{\partial \pi_{HF}^{\text{expect}}(\mathbb{A}_F^*)}{\partial \mathbb{A}_F}}{|J_{FE}|}$ and $\frac{\partial \mathbb{A}_F^*}{\partial \beta_{FH}} = -\frac{\frac{\partial \pi_{FH}^{\text{expect}}(\mathbb{A}_H^*;\beta_{FH})}{\partial \beta_{FH}} \frac{\partial \pi_{HH}^{\text{expect}}(\mathbb{A}_H^*)}{\partial \mathbb{A}_H}}{|J_{FE}|}$. By Lemma 3, $\pi_{FH}^{\text{expect}}\left(\mathbb{A}_H, \beta_{FH}\right)$ is decreasing in β_{FH} and $\pi_{kl}^{\text{expect}}\left(\mathbb{A}_l, \cdot\right)$ is decreasing in $\mathbb{A}_l$ for $k, l \in \{H, F\}$. Hence, since $|J_{FE}| > 0$, then $\frac{\partial \mathbb{A}_H^*}{\partial \tau_{FH}} > 0$ and $\frac{\partial \mathbb{A}_F^*}{\partial \tau_{FH}} < 0$. Since we are considering a decrease in τ_{FH}, then $\mathbb{A}_H^*$ decreases and $\mathbb{A}_F^*$ increases.

Regarding domestic firms in H, by the decrease in $\mathbb{A}_H^*$ and Lemma 3, φ_{HH}^* decreases. Furthermore, by the decrease in $\mathbb{A}_H^*$, Lemma 5, and Assumption 1 in H, then $m_{HH}^*\left(\varphi\right)$ increases for each $\varphi \geq \varphi_{HH}^*$ and so does $p_{HH}^*\left(\varphi\right)$.

Regarding domestic firms in F, by the increase in $\mathbb{A}_F^*$ and Lemma 3, φ_{FF}^* increases. Furthermore, by the increase in $\mathbb{A}_F^*$, Lemma 5, and Assumption 1 in F, then $m_{FF}^*\left(\varphi\right)$ decreases for each $\varphi \geq \varphi_{HH}^*$ and so does $p_{FF}^*\left(\varphi\right)$.

As for M_H^{E*} and M_F^{E*}, they are obtained through the following system of equations given $\mathbb{A}_H^*$ and $\mathbb{A}_F^*$:

$$\mathbb{A}_H^* = \mathcal{A}_H^*\left(M_H^{E*}, M_F^{E*}; \mathbb{A}_H^*, \beta_{FH}\right),$$

$$\mathbb{A}_F^* = \mathcal{A}_F^*\left(M_H^{E*}, M_F^{E*}; \mathbb{A}_F^*\right).$$

Differentiating the system we obtain

$$\begin{pmatrix} \frac{\partial \mathcal{A}_H^*(\cdot)}{\partial M_H^E} & \frac{\partial \mathcal{A}_H^*(\cdot)}{\partial M_F^E} \\ \frac{\partial \mathcal{A}_F^*(\cdot)}{\partial M_H^E} & \frac{\partial \mathcal{A}_F^*(\cdot)}{\partial M_F^E} \end{pmatrix} \begin{pmatrix} \frac{\partial M_H^{E*}}{\partial \tau_{FH}} \\ \frac{\partial M_F^{E*}}{\partial \tau_{FH}} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbb{A}_H^*}{\partial \beta_{FH}}\left(1 - \frac{\partial \mathcal{A}_H^*(\cdot)}{\partial \mathbb{A}_H^*}\right) - \frac{\partial \mathcal{A}_H^*(\cdot)}{\partial \beta_{FH}} \\ \frac{\partial \mathbb{A}_F^*}{\partial \beta_{FH}}\left(1 - \frac{\partial \mathcal{A}_F^*(\cdot)}{\partial \mathbb{A}_F^*}\right) \end{pmatrix}.$$

All the entries in J_{MS} are positive, and $|J_{MS}| > 0$ by assumption. Let $\Delta_1 := \frac{\partial \mathbb{A}_H^*}{\partial \beta_{FH}}\left(1 - \frac{\partial \mathcal{A}_H^*(\cdot)}{\partial \mathbb{A}_H^*}\right) - \frac{\partial \mathcal{A}_H^*(\cdot)}{\partial \beta_{FH}}$ and $\Delta_2 := \frac{\partial \mathbb{A}_F^*}{\partial \beta_{FH}}\left(1 - \frac{\partial \mathcal{A}_F^*(\cdot)}{\partial \mathbb{A}_F^*}\right)$. Given that Assumption 2 holds, then $\Delta_1 > 0$ and $\Delta_2 < 0$. Hence,

$$\frac{\partial M_H^{E*}}{\partial \tau_{FH}} = \frac{\Delta_1 \frac{\partial \mathcal{A}_F^*(\cdot)}{\partial M_F^E} - \Delta_2 \frac{\partial \mathcal{A}_H^*(\cdot)}{\partial M_F^E}}{|J_{MS}|} > 0,$$

$$\frac{\partial M_F^{E*}}{\partial \tau_{FH}} = \frac{\Delta_2 \frac{\partial \mathcal{A}_H^*(\cdot)}{\partial M_H^E} - \Delta_1 \frac{\partial \mathcal{A}_F^*(\cdot)}{\partial M_H^E}}{|J_{MS}|} < 0.$$

Since we are considering a decrease in τ_{FH}, then M_H^{E*} decreases and M_F^{E*} increases.

Derivation of the Results Presented in Section 5. From conditions (20) and (21), we get $\mathbb{A}_H(\mathbb{A}_F)$ and $\mathbb{A}_F(\mathbb{A}_H;\tau_{FH})$, respectively. An equilibrium is a pair $(\mathbb{A}_H^*,\mathbb{A}_F^*)$ such that $\mathbb{A}_H^* = \mathbb{A}_H(\mathbb{A}_F^*)$ and $\mathbb{A}_F^* = \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})$. Differentiating $\mathbb{A}_H(\mathbb{A}_F)$ and $\mathbb{A}_F(\mathbb{A}_H;\tau_{FH})$, and evaluating the expressions at the equilibrium, we get

$$\begin{aligned}\frac{\mathrm{d}\mathbb{A}_H^*}{\mathrm{d}\tau_{FH}} &= \frac{\partial \mathbb{A}_H(\mathbb{A}_F^*)}{\partial \mathbb{A}_F}\frac{\mathrm{d}\mathbb{A}_F^*}{\mathrm{d}\tau_{FH}},\\ \frac{\mathrm{d}\mathbb{A}_F^*}{\mathrm{d}\tau_{FH}} &= \frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \tau_{FH}} + \frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \mathbb{A}_H}\frac{\mathrm{d}\mathbb{A}_H^*}{\mathrm{d}\tau_{FH}}.\end{aligned}$$

Working out the expressions yields

$$\begin{aligned}\frac{\mathrm{d}\mathbb{A}_H^*}{\mathrm{d}\tau_{FH}} &= \frac{\partial \mathbb{A}_H(\mathbb{A}_F^*)}{\partial \mathbb{A}_F}\frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \tau_{FH}}\kappa,\\ \frac{\mathrm{d}\mathbb{A}_F^*}{\mathrm{d}\tau_{FH}} &= \frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \tau_{FH}}\kappa + \frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \mathbb{A}_H}\frac{\partial \mathbb{A}_H(\mathbb{A}_F^*)}{\partial \tau_{FH}}\kappa,\end{aligned}$$

with $\kappa := \left(1 - \frac{\partial \mathbb{A}_H(\mathbb{A}_F^*)}{\partial \mathbb{A}_F}\frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \mathbb{A}_H}\right)^{-1}$.

By (20) and (21), we know that $\frac{\partial \mathbb{A}_i(\mathbb{A}_j^*)}{\partial \mathbb{A}_j} = -\left(\frac{\partial \pi_{ii}^{\text{expect}}(\mathbb{A}_i^*)}{\partial \mathbb{A}_i}\right)^{-1}\frac{\partial \pi_{ij}^{\text{expect}}(\mathbb{A}_j^*;\tau_{ij})}{\partial \mathbb{A}_j}$ for $i \neq j$, and $\frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \tau_{FH}} = -\left(\frac{\partial \pi_{FF}^{\text{expect}}(\mathbb{A}_F^*)}{\partial \mathbb{A}_F}\right)^{-1}\frac{\partial \pi_{FH}^{\text{expect}}(\mathbb{A}_H^*;\tau_{FH})}{\partial \tau_{FH}}$. Now, we determine their signs. By Lemma 3, $\pi_{kl}^{\text{expect}}(\mathbb{A}_l,\cdot)$ is decreasing in $\mathbb{A}_l$ for $k,l \in \{H,F\}$. Hence, $\frac{\partial \mathbb{A}_H(\mathbb{A}_F^*)}{\partial \mathbb{A}_F} < 0$ and $\frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \mathbb{A}_H} < 0$. Moreover, since $|J_{FE}| > 0$, also $\frac{\partial \mathbb{A}_H(\mathbb{A}_F^*)}{\partial \mathbb{A}_F}\frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \mathbb{A}_H} < 1$. Furthermore, neither π_{HH}^{expect} nor π_{HF}^{expect} depend on τ_{FH} directly. By Lemma 3, $\pi_{FH}^{\text{expect}}(\mathbb{A}_H,\tau_{FH})$ is decreasing in τ_{FH} and $\pi_{kl}^{\text{expect}}(\mathbb{A}_l,\cdot)$ is decreasing in $\mathbb{A}_l$ for $k,l \in \{H,F\}$. Both determine that $\frac{\partial \mathbb{A}_F(\mathbb{A}_H^*;\tau_{FH})}{\partial \tau_{FH}} < 0$.

B A Rationalization of the Small-Country Assumption

In the main part of the paper, I have followed Demidova and Rodríguez-Clare (2009, 2013) and Melitz (2018) to define a small country. They consider that H is a small country when, $\left(\mathbb{A}_j^*, M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$ is not affected by changes in H, in notation of my model.

Demidova and Rodríguez-Clare (2013) show that the implications of a small country emerge endogenously under a CES demand when H has a share of the world labor that tends to zero. In this appendix, I provide an alternative rationalization to the concept of small economies based on measure theory. This formalization has identical implications for the CES, and it additionally holds for any demand that summarizes market conditions through a scalar.

To keep matters simple, I consider a framework where every country in the world is a small country. Furthermore, I illustrate the approach by showing that shocks in H do not affect $\left(\mathbb{A}_j^*\right)_{j\in\mathcal{C}\setminus\{H\}}$. A similar procedure can be used to show that $\left(M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$ is not affected.

Formally, proving this requires showing that variations in $(\tau_{kH})_{k\in\mathcal{C}\setminus\{H\}}$ and $(\tau_{Hk})_{k\in\mathcal{C}\setminus\{H\}}$ do not affect $\left(\mathbb{A}_j^*\right)_{j\in\mathcal{C}\setminus\{H\}}$. Since $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$ is entirely determined by (FE) due to Lemma 1. this requires characterizing the system (FE).

It is worth remarking that a world with small economies cannot be captured by simply considering a continuum of countries. If this were the case, (FE) for i would become $\int_{k\in\mathcal{C}} \pi_{ik}^{\text{expect}}\left(\mathbb{A}_k^*;\tau_{ik}\right)\mathrm{d}k = F_i$, which would imply that each country $k\in\mathcal{C}$ is negligible, including i itself. Thus, even shocks to the own domestic market would be inconsequential for the country.

This type of issue can be avoided if we consider countries that, rather, treat their trading partners as negligible. Formally, (FE) for H and each $j\in\mathcal{C}\setminus\{H\}$ would become

$$\pi_{HH}^{\text{expect}}\left(\mathbb{A}_H^*\right) + \int_{k\in\mathcal{C}\setminus\{H\}} \pi_{Hk}^{\text{expect}}\left(\mathbb{A}_k^*;\tau_{Hk}\right)\mathrm{d}k = F_H, \tag{B1}$$

$$\pi_{jj}^{\text{expect}}\left(\mathbb{A}_j^*\right) + \int_{k\in\mathcal{C}\setminus\{j\}} \pi_{jk}^{\text{expect}}\left(\mathbb{A}_k^*;\tau_{jk}\right)\mathrm{d}k = F_j. \tag{B2}$$

Next, I show that changes in H's trade costs do not affect $\left(\mathbb{A}_j^*\right)_{j\in\mathcal{C}\setminus\{H\}}$ under this framework. First, notice that H's import trade costs, i.e., $(\tau_{kH})_{k\in\mathcal{C}\setminus\{H\}}$, only directly affect each (B2). However, since H is negligible, any change in π_{jk}^{expect} with $j\neq k$ has a trivial impact on (B2). Hence, $\left(\mathbb{A}_j^*\right)_{j\in\mathcal{C}\setminus\{H\}}$ is not affected. Second, H's export trade costs, $(\tau_{Hk})_{k\in\mathcal{C}\setminus\{H\}}$, only directly affect (B1). Moreover, any change in $\mathbb{A}_H^*$ has a negligible effect on each (B2). Thus, after a change in $(\tau_{Hk})_{k\in\mathcal{C}\setminus\{H\}}$, the equilibrium is restored through a variation in $\mathbb{A}_H^*$, with $\left(\mathbb{A}_j^*\right)_{j\in\mathcal{C}\setminus\{H\}}$ kept at the same value as in the initial equilibrium.

Applying a similar argument to the system of equations (MS), it can be shown that also $\left(M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$ is not affected by changes in H. Therefore, this approach can be used as an alternative way to justify that $\left(\mathbb{A}_j^*, M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$ is not affected by trade shocks to H, when H is a small country.

C Extensions with an Inactive Import-Competition Channel

In this section, I modify the baseline model and establish some alternative setups in which the import-competition channel is inactive. In particular, I consider three extensions.

First, in Appendix C.1, I demonstrate that the result holds for the case of multidimensional firm heterogeneity. Specifically, I consider a model where firms receive draws of efficiency and variety appeal (quality or taste). In this way, firms are heterogeneous in terms of cost and demand. Then, in Appendix C.2, I extend the setup to account for a vector of firm's country-specific decisions. This vector encompasses standard choice variables considered in the literature, such as quality and number of goods when firms are multiproduct. In Appendix C.3, I consider standard demands with nested structures (i.e., nested versions of the CES and Logit) and two types of partitions for varieties: by country of origin and by varieties belonging to the same multiproduct firm.

To show the results as stark as possible, except for the case of nested demands with a partition by countries, I consider a closed economy and a shock δ that exogenously increases competition. Specifically, δ is a shock that affects (MS) directly, which is equivalent to assuming that δ is a shock to import trade costs for the country under analysis. Thus, it constitutes a simplified way to consider whether the import-competition channel is active or not. In particular, I establish that the import-competition channel is inactive using that, if δ does not affect (FE), then it cannot affect the aggregate of the country.

C.1 Multidimensional Heterogeneity

In the baseline model, firms are heterogeneous exclusively in terms of productivity. Next, I show that the import-competition channel is also inactive when there is firm heterogeneity regarding both demand and productivity.

I illustrate this by considering a closed economy and the shock δ to competition. The setup is the same as in the baseline case, but with the following modifications. By paying the entry cost, a firm is assigned a unique variety ω with appeal σ_ω and a productivity draw φ_ω. I denote a firm's type by $\boldsymbol{\theta} := (\sigma, \varphi)$ and assume $\boldsymbol{\theta}$ has support Θ and is distributed by a joint cdf G.

The demand of firm ω is given by

$$q_\omega := \max\left\{0, q\left(\mathbb{A}, p_\omega, \sigma_\omega\right)\right\},$$

and profits of a firm with type $\boldsymbol{\theta}$ is

$$\pi\left(\mathbb{A}, p; \boldsymbol{\theta}\right) := q\left(\mathbb{A}, p, \sigma\right)\left[p - c\left(\varphi\right)\right],$$

which, conditional on (σ, φ), is completely determined by $\mathbb{A}$. This implies that optimal prices and quantities of active firms are respectively functions $p\left(\mathbb{A}, \sigma, \varphi\right)$ and $q\left(\mathbb{A}, \sigma, \varphi\right)$, determining that the optimal profit of a firm with type $\boldsymbol{\theta}$ is

$$\pi\left(\mathbb{A}, \boldsymbol{\theta}\right) := q_\omega\left(\mathbb{A}, \sigma, \varphi\right)\left[p\left(\mathbb{A}, \sigma, \varphi\right) - c\left(\varphi\right)\right].$$

Unlike the baseline case, the entry decision cannot be described by some productivity cutoff. Nonetheless, the entry decision is still determined by $\mathbb{A}$, for a given combination of σ and φ. Formally, defining $\mathcal{E}\left(\mathbb{A}\right) := \{\boldsymbol{\theta} : \pi\left(\mathbb{A}, \boldsymbol{\theta}\right) \geq f\}$, a firm $\boldsymbol{\theta}$ pays the entry cost if $\boldsymbol{\theta} \in \mathcal{E}\left(\mathbb{A}\right)$. Thus, expected profits can be written as

$$\pi^{\text{expect}}\left(\mathbb{A}\right) := \int_{\theta \in \Theta} \mathbb{1}_{(\boldsymbol{\theta} \in \mathcal{E}(\mathbb{A}))}\left[\pi\left(\mathbb{A}, \boldsymbol{\theta}\right) - f\right] \mathrm{d}G\left(\boldsymbol{\theta}\right).$$

This clearly shows that expected profits are completely determined by $\mathbb{A}$. Thus, a shock to (MS) does not affect the equilibrium $\mathbb{A}$.

C.2 Vector of Country-Specific Decisions

In the baseline model, the only decision made by firms at the market stage is regarding prices. Next, I consider the possibility that each firm decides on a vector of country-specific variables.

The framework is as in the baseline model under a closed economy, but incorporating that each firm ω makes a decision on $\mathbf{x}_\omega \in X := \times_{n=1}^{N} [\underline{x}_n, \overline{x}_n] \cup \{\mathbf{x}_0\}$ with $N < \infty$, where $\mathbf{x}_0$ represents inaction (exit of the market). Given a vector of choices, the aggregator is defined in the following way.

Definition 4. *Let* $\boldsymbol{\mathcal{X}} := \left(\mathcal{X}^k\right)_{k=1}^{K}$ *with* $K < \infty$ *such that* $\mathcal{X}^k \left[(\mathbf{x}_\omega)_{\omega\in\Omega}\right] := \int_{\omega\in\overline{\Omega}} u_\omega^k (\mathbf{x}_\omega) \, \mathrm{d}\omega$*,* $\mathbb{X}^k \in \operatorname{range} \mathcal{X}^k$*, and* $\boldsymbol{\mathbb{X}} := \left(\mathbb{X}^k\right)_{k=1}^{K}$*.*[18] *An* ***aggregator*** *is a smooth real-valued function* $\mathcal{A}$ *with* $\boldsymbol{\mathbb{X}} \mapsto \mathcal{A}(\boldsymbol{\mathbb{X}})$*. An* ***aggregate*** *is a value* $\mathbb{A} \in \operatorname{range} \mathcal{A}$*.*

Making use of this definition, I specify the demand function.

Assumption DEM-vec. *The* ***demand of a variety*** ω *is given by,*

$$q(\omega) := \max\{0, q[\mathbb{A}, \mathbf{x}(\omega)]\},$$

where $\mathbb{A}$ *is as in Definition 4 and* q *is a smooth function such that* $q(\mathbb{A}, \mathbf{x}_0) = 0$*.*

Notice that I have not imposed restrictions on how the choice vector is incorporated into demand or the aggregator. This provides a wide scope in terms of the possible functional forms that can be considered. Regarding costs, I assume a general function that might depend on $\mathbf{x}(\omega)$. Formally, $C[q(\omega), \mathbf{x}(\omega)] := q(\omega)\, c(\varphi, \mathbf{x}(\omega)) + f^{\mathbf{x}}[\mathbf{x}(\omega)]$, allowing for the possibility that some of the choices entail fixed costs, affect marginal costs, or both.

The optimization problem of a firm with productivity φ is

$$\max_{\mathbf{x}_\omega} \pi(\mathbf{x}_\omega, \mathbb{A}, \varphi).$$

The system of first-order conditions characterizing the optimal decisions of a φ-type active firm is

$$\frac{\partial \pi(\mathbf{x}_\omega, \mathbb{A}, \varphi)}{\partial \mathbf{x}_\omega} = 0. \tag{C1}$$

[18] I also suppose that the absolute value of u_ω^k and all its derivatives are dominated by integrable positive functions.

Let $\mathbf{x}(\mathbb{A},\varphi)$ be the implicit solution to the system (C1). Incorporating that firms also decide whether to serve the market, the optimal vector of decisions for each $\varphi \in \left[\underline{\varphi},\overline{\varphi}\right]$ is

$$\mathbf{x}^*(\mathbb{A},\varphi^*,\varphi) := \begin{cases} \mathbf{x}(\mathbb{A},\varphi) & \text{if } \varphi \geq \varphi^* \\ \mathbf{x}_0 & \text{otherwise,} \end{cases} \tag{$\mathbf{x}$-vec}$$

where φ^* represents a firm's survival productivity cutoff.

If a φ-type firm serves the market, its optimal gross profits are $\pi(\mathbb{A},\varphi) := \pi[\mathbb{A},\mathbf{x}(\mathbb{A},\varphi),\varphi]$. Therefore, the survival productivity cutoff, $\varphi^*(\mathbb{A})$, is the implicit solution to the following equation

$$\pi(\mathbb{A},\varphi^*) = f.$$

All this determines that a firm's decisions and its profits are completely characterized by the aggregate. Thus, the equilibrium is determined just like in the baseline case, with the zero expected profits condition identifying the equilibrium aggregate $\mathbb{A}^*$. Formally,

$$\pi^{\text{expect}}(\mathbb{A}^*) = F, \tag{FE-vec}$$

where $\pi^{\text{expect}}(\mathbb{A}) := \int_{\varphi^*(\mathbb{A})}^{\overline{\varphi}} [\pi(\mathbb{A},\varphi) - f]\,\mathrm{d}G(\varphi)$. Therefore, since (FE-vec) pins down $\mathbb{A}^*$, a shock to (MS) does not affect $\varphi^*(\mathbb{A}^*)$ or the vector of optimal decisions $\mathbf{x}^*(\varphi) := \mathbf{x}^*[\mathbb{A}^*,\varphi^*(\mathbb{A}^*),\varphi]$.

C.3 Country-Specific Aggregators and Nested Demand Structures

Next, I show conditions under which the import-competition channel is inactive when demand systems have nested structures. In particular, I consider scenarios where varieties are partitioned by country of origin or by varieties produced by the same multiproduct firm. In the latter case, the result holds without additional assumptions. In the case of groups defined by country of origin, it requires putting some structure to the demand, which covers the nested versions of the CES and Logit as special cases.

Before studying these cases, I define demands with nested structures in a general way. Let the set of total varieties be partitioned into L groups, with group l defining a subset of varieties Ω^l. To deal with both a continuum and a discrete number of nests in a unified framework, I endow the subsets of nests with a measure ρ that is either the Lebesgue or the counting measure. The demand of a firm producing a variety ω and belonging to nest l is defined by

$$q^l(\omega) := q^l\left(\mathbb{P}, \mathbb{P}^l, p_\omega\right), \tag{DEM-NEST}$$

with $\mathcal{P}\left[\left(\mathbb{P}^l\right)_0^L\right] := \int_0^L U^l\left(\mathbb{P}^l\right) \mathrm{d}\rho(l)$ and $\mathcal{P}^l\left[(p_{\omega'})_{\omega'\in\Omega^l}\right] := \int_{\omega'\in\Omega^l} u^l\left(p\left(\omega'\right)\right) \mathrm{d}\omega'$, where U^l and u^l are monotone functions, $\mathbb{P} \in \operatorname{range}\mathcal{P}$, and $\mathbb{P}^l \in \operatorname{range}\mathcal{P}^l$.

Next, I divide the analysis for each type of nested structure considered.

C.3.1 Nested Demands with Groups Defined by Own-Firm's Varieties

Suppose the case of multiproduct firms, with group l given by all the varieties produced by firm l. Incorporating this, the framework is actually a particular case of the model presented in Appendix C.2, with a firm's decisions given by the prices of all its varieties. Formally, in terms of a demand as in Assumption DEM-vec, a firm l chooses $\mathbf{x}^l := (p(\omega))_{\omega\in\Omega^l}$, with $\mathcal{P}^l$ and $\mathcal{P}$ respectively replacing $\mathcal{X}^k$ and $\mathcal{A}$. Given this, the import-competition channel is inactive by the results in Appendix C.2.

C.3.2 Nested Demands with Groups Defined by Country of Origin

To focus on the import-competition channel, I consider the case of a country H that is small. Additionally, I suppose that the demand in country $j \in \mathcal{C}\backslash\{H\}$ of a variety ω produced in $k \in \mathcal{C}$ is as in Assumption DEM. In this way, only the demand in H has a nested structure. In particular, suppose that the demands in H are as in (DEM-NEST) with two groups partitioning the varieties according to its origin, i.e., domestic or foreign. Then, the demands in H for a variety produced in H and for a variety produced in $j \in \mathcal{F} := \mathcal{C}\backslash\{H\}$ can be respectively expressed by

$$q_{HH}(\omega) := q_H\left[\mathbb{P}_H, \mathbb{P}_{HH}, p_{HH}(\omega)\right],$$

$$q_{jH}(\omega) := q_j\left[\mathbb{P}_H, \mathbb{P}_{FH}, p_{jH}(\omega)\right],$$

with $\mathcal{P}_H\left[\mathbb{P}_{FH}, \mathbb{P}_{HH}\right] := U^F\left(\mathbb{P}_{FH}\right) + U^H\left(\mathbb{P}_{HH}\right)$ where $\mathbb{P}_{kH} \in \operatorname{range}\mathcal{P}_{kH}$ for

$k \in \{H,F\}$, $\mathcal{P}_{FH}\left[(\mathbf{p}_{jH})_{j\in\mathcal{F}}\right] := \sum_{j\in\mathcal{F}}\int_{\omega\in\Omega_{jH}} u_{jH}\left[p_{jH}(\omega)\right]\mathrm{d}\omega$, and $\mathcal{P}_{HH}(\mathbf{p}_{HH}) := \int_{\omega\in\Omega_{HH}} u_{HH}\left[p_{HH}(\omega)\right]\mathrm{d}\omega$.

Next, I add some structure to the demand in H. Specifically, let $i \in \mathcal{C}$ and $k \in \{H,F\}$, and suppose that each function q_i is weakly separable in $(\mathbb{P}_H, \mathbb{P}_{kH})$ from $p_{iH}(\omega)$. Two demands consistent with this property are the nested CES and the nested Logit (see Appendix D). Incorporating this, the demand of a variety ω produced domestically and produced in $j \in \mathcal{F}$ can be respectively expressed by

$$q_{HH}(\omega) := q_H\left[\mathbb{A}_{HH}, p_{HH}(\omega)\right], \tag{DEM-H}$$

$$q_{jH}(\omega) := q_j\left[\mathbb{A}_{FH}, p_{jH}(\omega)\right], \tag{DEM-F}$$

where $\mathcal{A}_{kH}$ is a smooth real-valued function $(\mathbb{P}_H, \mathbb{P}_{kH}) \mapsto \mathcal{A}_{kH}(\mathbb{P}_H, \mathbb{P}_{kH})$, and $\mathbb{A}_{kH} \in \operatorname{range}\mathcal{A}_{kH}$ with $k \in \{H,F\}$ is a country-specific aggregate. The demand function determines that $\mathbb{A}_{HH}$ completely characterizes the profits and decisions in H of any domestic firm. Specifically, the aggregate determines their domestic optimal prices, quantities, markups, and survival productivity cutoff.

Consider a variation in $(\tau_{jH}, f_{jH})_{j\in\mathcal{F}}$. If we can show that $\mathbb{A}^*_{HH}$ is determined by a system of equations that is independent of these parameters, then changes in import competition do not affect any of the variables mentioned. To show this, take $j \in \mathcal{F}$. The free-entry conditions are

$$\pi^{\text{expect}}_{HH}(\mathbb{A}^*_{HH}) + \sum_{k\in\mathcal{F}} \pi^{\text{expect}}_{Hk}(\mathbb{A}^*_k; \tau_{Hk}) = F_H, \tag{C2}$$

$$\sum_{k\in\mathcal{F}} \pi^{\text{expect}}_{jk}(\mathbb{A}^*_k; \tau_{jk}, f_{jk}) = F_j, \tag{C3}$$

where (C3) holds for each $j \in \mathcal{F}$. Equation (C3) incorporates the fact that H is a small country, so that π^{expect}_{jH} has a negligible impact on j's expected profits. This establishes that (C3) for any $j \in \mathcal{F}$ is not affected by either τ_{jH} or f_{jH}, and so each $\mathbb{A}^*_j$ is not affected. Since each $\mathbb{A}^*_j$ summarizes the export conditions of H and none of them vary, the shocks under consideration do not affect (C2). Therefore, since $\mathbb{A}^*_{HH}$ is pinned down by (C2), which is not affected directly by τ_{jH} or f_{jH}, then $\mathbb{A}^*_{HH}$ does not vary and the result follows.

D Demand Systems: Examples

Next, I provide some examples of demands consistent with Assumption DEM. While some of these demands have particular cases that overlap with other families considered, it is illustrative to show separately how they can be explicitly rewritten in terms of an aggregate.

In order to define demands, it is important to keep in mind two considerations indicated in Section 3.2. First, the aggregator is not uniquely defined, and any monotone transformation defines a new aggregator. Thus, there are infinite ways to express the demand, depending on the aggregator that we use. Furthermore, although Assumption DEM defines a demand with $\mathbb{A}$ impacting it negatively, what matters for the definition is that the demand is monotone in $\mathbb{A}$. Therefore, to establish a direct connection with the literature, in some cases I express the demand through an aggregate $\mathbb{A}$ that has a positive impact on demand.

Consider the demand q_ω of a variety ω, with total measure of varieties sold M and price p_ω. Also, let E be an exogenous demand shifter (e.g., income) and suppose that any Greek letter refers to a positive parameter.

- **Demands from an additively separable direct utility** as in Krugman (1979). Given utility $U\left[(q_{\omega'})_{\omega'\in\Omega}\right] := \int_{\omega'\in\Omega} u\left(q_{\omega'}\right) \mathrm{d}\omega'$ with u monotone, let $g := (u')^{-1}$. Then, $q_\omega := g\left(\mathbb{A}p_\omega\right)$ where $\mathbb{A}$ is the marginal utility of income. Next, I present some special cases of this demand, where $\mathbb{A}$ is redefined appropriately to present the demands in its usual form and, hence, it is not necessarily the marginal utility of income.
  - **Demands derived from an exponential utility** as in Behrens and Murata (2007): $q_\omega := \mathbb{A} - \ln p_\omega^{1/\alpha}$ where $\mathbb{A} := \frac{E}{\mathbb{P}_1} + \frac{\ln \mathbb{P}_1}{\alpha} + \frac{\mathbb{P}_2}{a}$ with $\mathbb{P}_1 := \int_{\omega'\in\Omega} p_{\omega'} \mathrm{d}\omega'$ and $\mathbb{P}_2 := \int_{\omega'\in\Omega} \ln\left(\frac{p_{\omega'}}{\mathbb{P}_1}\right)\frac{p_{\omega'}}{\mathbb{P}_1}\mathrm{d}\omega'$.
  - **Generalized CES** as in Jung et al. (2015) and Arkolakis et al. (2019)**:** $q_\omega := \frac{p_\omega^{-\sigma}}{\mathbb{A}} - \alpha$ with $\mathbb{A} := \frac{\mathbb{P}_2}{E+\alpha\mathbb{P}_1}$, $\mathbb{P}_1 := \int_{\omega'\in\Omega} p_{\omega'}\mathrm{d}\omega'$ and $\mathbb{P}_2 := \int_{\omega'\in\Omega} \left(p_{\omega'}\right)^{1-\sigma}\mathrm{d}\omega'$.
  - **Stone-Geary** (Generalized CES with $\sigma \to 1$) as in Simonovska (2015): $q_\omega :=$

$\frac{1}{\mathbb{A}p_\omega} - \alpha$ with $\mathbb{A} := \frac{M}{E+\alpha\mathbb{P}}$ and $\mathbb{P} := \int_{\omega'\in\Omega} p_{\omega'} \mathrm{d}\omega'$.

- **Melitz and Ottaviano's (2008) linear demand** $q_\omega := \frac{\mathbb{A}-p_\omega}{\gamma}$ with $\mathbb{A} =: p^{\max} := \frac{\alpha+\eta\mathbb{P}}{\gamma+\eta M}$ and $\mathbb{P} := \int_{\omega'\in\Omega} p_{\omega'} \mathrm{d}\omega'$.
- **Feenstra's (2003) translog demand** $q_\omega := \frac{E}{p_\omega}\left[\mathbb{A} - \ln\left(p_\omega\right)\right]$ where $\mathbb{A} := \frac{1+\gamma\mathbb{P}}{M}$ and $\mathbb{P} := \int_{\omega'\in\Omega} \ln p_{\omega'} \mathrm{d}\omega'$.
- **Demands from a discrete choice model** (Luce, 1959; McFadden, 1973): $q_\omega := \frac{h_\omega(p_\omega)}{\mathbb{A}}$ with $\mathbb{A} := H\left(\int_{\omega'\in\Omega} h_{\omega'}\left(p_{\omega'}\right) \mathrm{d}\omega'\right)$. It includes as special cases:
  - Multinomial Logit demand: $h_\omega\left(p_\omega\right) := \exp\left(\alpha - \beta p_\omega\right)$.
  - Multiplicative Competitive Interaction demand: $h_\omega\left(p_\omega\right) := \alpha\left(p_\omega\right)^{-\beta}$.
- **Demands from discrete-continuous choices model** as in Nocke and Schutz (2018): $q_\omega := \frac{\partial h_\omega(p_\omega)/\partial p_\omega}{\mathbb{A}}$ with $\mathbb{A} := H\left(\int_{\omega'\in\Omega} h_{\omega'}\left(p_{\omega'}\right) \mathrm{d}\omega'\right)$. It includes the Logit and the CES without income effects as special cases.
- **Constant expenditure demands** (Vives, 2001): they are isomorphic to demands from a discrete choice model but accounting for income effects. Formally, $q_\omega := \frac{E}{p_\omega}\frac{h_\omega(p_\omega)}{\mathbb{A}}$ with $\mathbb{A} := H\left(\int_{\omega'\in\Omega} h_{\omega'}\left(p_{\omega'}\right) \mathrm{d}\omega'\right)$. It includes:
  - CES: $h\left(p_\omega\right) := \alpha\left(p_\omega\right)^{-\beta}$.
  - Exponential demand: $h\left(p_\omega\right) := \exp\left(\alpha - \beta p_\omega\right)$.
- **Demands from an additively separable indirect utility** as in Bertoletti and Etro (2015): given an indirect utility $V\left[\left(p_{\omega'}\right)_{\omega'\in\Omega}, E\right] := \int_{\omega'\in\Omega} v_{\omega'}\left(\frac{p_{\omega'}}{E}\right) \mathrm{d}\omega'$, demands are $q_\omega := \frac{v'_{\omega'}\left(\frac{p_\omega}{E}\right)}{\mathbb{A}}$ with $\mathbb{A} := \int_{\omega'\in\Omega} v'_{\omega'}\left(\frac{p_{\omega'}}{E}\right)\frac{p_{\omega'}}{E} \mathrm{d}\omega'$.

In Appendix C, I consider an extension to nested demands with groups of varieties defined by their origin (domestic or foreign). There, I show that the import-competition channel is inactive when the demand satisfies weak separability with respect to the aggregators. Following the definition of nested demands used there, I prove next that the cases of Nested CES and Nested Logit satisfy the assumption.

- **Nested CES**

$$q^l\left(\omega\right) := \alpha\mathbb{P}^\sigma\left(\mathbb{P}^l\right)^{\varepsilon-\sigma} p_\omega^{-\varepsilon},$$

where $\alpha > 0$ is a demand shifter, $\left(\mathbb{P}^l\right)^{1-\varepsilon} := \int_{\omega'\in\Omega^l} \left(p_{\omega'}\right)^{1-\varepsilon} \mathrm{d}\omega'$ and $\mathbb{P}^{1-\sigma} :=$

$\int_0^L \left(\mathbb{P}^l\right)^{1-\sigma} \mathrm{d}l$. Thus, $\mathbb{P}$ and $\mathbb{P}^l$ are weakly separable from p_ω.

- **Nested Logit**

$$q^l(\omega) := \alpha \exp\left(-\frac{p_\omega}{\lambda^l}\right) \frac{\left(\mathbb{P}^l\right)^{\lambda^l - 1}}{\mathbb{P}},$$

where $\alpha > 0$ is a demand shifter, $\mathbb{P}^l := \displaystyle\int_{\omega' \in \Omega^l} \exp\left(-\frac{p_{\omega'}}{\lambda^l}\right) \mathrm{d}\omega'$, and $\mathbb{P} := \int_0^L \left(\mathbb{P}^l\right)^{\lambda^l} \mathrm{d}l$. Thus, $\mathbb{P}$ and $\mathbb{P}^l$ are weakly separable from p_ω.

E Differential Characterization of the Demand System

Assumption DEM requires verifying that there exists an aggregate $\mathbb{A}$ such that the demand can be expressed as $q_\omega := \max\{0, q(\mathbb{A}, p_\omega)\}$. Mathematically, this is equivalent to weak separability of $\left(\mathbb{P}^k\right)_{k=1}^K$ from p_ω in q_ω. Results from the separability literature allows us to identify this through a differential characterization of weak separability. I begin by stating the differential characterization of weak separability generically.

Lemma 8. *(Leontief, 1947; Sono, 1961). Let $f : X \to \mathbb{R}$ with $X \subseteq \mathbb{R}_+^N$ with $N < \infty$. Consider a partition of the N variables into R groups $\left\{I^1, I^2, ..., I^R\right\}$ so that $X := \times_{r=1}^R X^r$. Denote generic elements by $\mathbf{x} \in X$ and $\mathbf{x}^r \in X^r$. We say that each group $r = 1, ...R$ is* ***weakly separable*** *from all other variables in f if there exist real-valued functions H and $(h^r)_{r=1}^R$ such that $f(\mathbf{x}) = H\left[h^1(\mathbf{x}^1), ..., h^R\left(\mathbf{x}^R\right)\right]$.*

If $f \in \mathbf{C}^1$, group r is ***weakly separable*** *from the rest of the variables in f if and only if the marginal rate of substitution between any two variables belonging to the group r are independent of any variable which does not belong to r. Formally, $\frac{\partial\left(\frac{\partial f(\mathbf{x})/\partial x_{i'}}{\partial f(\mathbf{x})/\partial x_{i''}}\right)}{\partial x_j} = 0$ for $i', i'' \in r$ and $j \notin r$.*

Making use of this lemma, the following corollary identifies whether a particular demand satisfies Assumption DEM.

Corollary 1. *p_ω is weakly separable from $\left(\mathbb{P}^k\right)_{k=1}^K$ in q_ω if and only if $\frac{\partial\left(\frac{\partial q_\omega/\partial \mathbb{P}^{k'}}{\partial q_\omega/\partial \mathbb{P}^{k''}}\right)}{\partial p_\omega} = 0$ for all $k', k'' = 1, ..., K$.*

F Well-Defined Equilibrium and Uniqueness

In the main text, I assumed that the equilibrium was unique and well-defined. While most of the assumptions needed for this are standard, one feature of the equilibrium deserves some comments: the free-entry conditions pin down the aggregates. These are not variables themselves, but rather values in a function's range that depend on endogenous variables. Nonetheless, I next show this is not an issue since range $\mathcal{A}$ is compact and convex under standard assumptions. To illustrate this as simply as possible, I focus on the case of a closed economy.

First of all, the price domain is $P := \left[\underline{p}, \overline{p}\right]$, with $\underline{p} \in \mathbb{R}_{+}$ and $\overline{p} \in \mathbb{R}_{++} \cup \{\infty\}$, and so compact. Thus, optimal prices exist and are unique under standard Inada conditions and strict quasiconcavity of profits in own prices.

In addition, $\mathcal{P}^{k}(\mathbf{p}) := \int_{\omega \in \overline{\Omega}} h^{k}\left[p(\omega)\right] \mathrm{d}\omega$, where each h^{k} is assumed to be integrable. Also, $\boldsymbol{\mathcal{P}} := \left(\mathcal{P}^{k}\right)_{k=1}^{K}$ with $\boldsymbol{\mathbb{P}} := \left(\mathbb{P}^{k}\right)_{k=1}^{K}$ and $K < \infty$. In the literature, it is usually assumed that price aggregators are expressed in terms of varieties actually sold in the market. This can be formalized by defining $\mathcal{P}^{k}(\mathbf{p}) := \int_{\omega \in \overline{\Omega}} \mathbb{1}_{(p(\omega) < \overline{p})} h^{k}\left[p(\omega)\right] \mathrm{d}\omega$, or simply $\mathcal{P}^{k}(\mathbf{p}) := \int_{\omega \in [0,M]} h^{k}\left[p(\omega)\right] \mathrm{d}\omega$. Thus, given optimal prices, any price aggregator takes the form $\mathcal{P}^{k}(\mathbf{p}) := M^{E} \int_{\varphi^{*}(\mathbb{A})}^{\overline{\varphi}} h^{k}\left[p(\mathbb{A}, \varphi)\right] g(\varphi)\, \mathrm{d}\varphi$. With this characterization, we can apply Lyapunov's Convexity Theorem and conclude that range $\mathcal{P}^{k}$ is compact and convex.[19] Moreover, since $\mathcal{A}$ is continuous on $\times_{k=1}^{K}$range $\mathcal{P}^{k}$, then range $\mathcal{A}$ is compact and convex too. Therefore, range $\mathcal{A} \in \left[\underline{\mathbb{A}}, \overline{\mathbb{A}}\right]$ for some $\underline{\mathbb{A}}, \overline{\mathbb{A}} \in \mathbb{R}_{+}$.

Applying Berge's maximum theorem, if the profits function is continuously differentiable $n+1$ times, then the value function $\pi(\mathbb{A}; \varphi)$ is continuously differentiable n times. Therefore, joint with the smoothness of the costs functions and Lemma 3, $\varphi^{*}(\mathbb{A})$ is a single-valued correspondence that is continuously differentiable. If there exists an integrable function g such that $\left|\frac{\partial \pi}{\partial \mathbb{A}}\right| \leq g$, then we can apply Leibniz rule, and so $\pi^{\text{expect}}(\mathbb{A})$ is continuously differentiable too. Thus, since $\mathbb{A} \in \left[\underline{\mathbb{A}}, \overline{\mathbb{A}}\right]$ and supposing that $\pi^{\text{expect}}\left(\underline{\mathbb{A}}\right) > F$ and $\pi^{\text{expect}}\left(\overline{\mathbb{A}}\right) < F$, an equilibrium $\mathbb{A}^{*}$ exists. By Lemma 3, $\pi(\mathbb{A}, \varphi)$ is decreasing in $\mathbb{A}$,

[19] See, for instance, Aliprantis and Border (2006).

and, also $\varphi^*(\mathbb{A})$ is increasing in $\mathbb{A}$. Thus, $\pi^{\text{expect}}(\mathbb{A})$ is decreasing in $\mathbb{A}$, implying that the equilibrium is unique.

Finally, suppose $\mathbb{A} > \mathcal{A}^*(\mathbb{A}, 0)$ and $\mathbb{A} < \mathcal{A}^*\left(\mathbb{A}, \overline{M}\right)$ for any $\mathbb{A} > \underline{\mathbb{A}}$. Then, by continuity, a solution set $M^E(\mathbb{A}^*)$ exists. To show that it is unique and so a function, we can assume that $\frac{\partial \mathcal{A}^*\left(\mathbb{A}, M^E\right)}{\partial \mathbb{A}} \neq 1$ for any M^E and $\mathbb{A}$, or simply Assumption 2.