Restricting Entry without Aggressive Pricing

Martín Alfaro[*] David Lander[†]
University of Alberta Peking University

July 2021

Abstract

This paper studies a strategic-investment model under endogenous entry of followers. We extend the standard setting à la Etro by incorporating multiple heterogeneous leaders and demand-enhancing investments directly affecting competition. Our findings indicate that all leaders simultaneously restrict entry without harming each other. Moreover, while entry accommodation never arises, a wide range of strategies is consistent with aggressive behavior, including quality upgrades exclusively targeted to high-valuation consumers. By using the tools of aggregative games, we provide conditions over demand primitives to identify when a leader over-invests and whether it (i) raises or lowers its price, (ii) increases or decreases its revenue, and (iii) supplies greater or lower quantities. We illustrate the results by completely characterizing outcomes under the Logit and CES demands.

Keywords: demand-enhancing investments, leaders, endogenous entry, free entry, aggregative games.

JEL codes: L11, L13, L22, D43.

[*]University of Alberta, Department of Economics. 9-08 HM Tory Building, Edmonton, AB T6G 2H4, Canada; email: malfaro@ualberta.ca. Link to personal website. Declarations: Not applicable.

[†]Peking University HSBC Business School, Shenzhen, China 518055; email: lander@phbs.pku.edu.cn. Link to personal website. Declarations: Not applicable.

1 Introduction

Scenarios where leading and following firms compete constitute one of the core topics in Industrial Organization. Broadly speaking, they can be separated into two approaches: Stackelberg-type models, where a leader directly commits to some price/quantity, and models of strategic investments, where a leader affects the nature of market competition through investments.

While these approaches have traditionally focused on cases with only one potential entrant, recent studies have analyzed the leader's behavior in industries with free entry of followers (e.g., Etro, 2006, 2008; Anderson et al., 2020). Their main conclusion is that accommodating entry is never optimal, since any attempt to soften competition is undermined by entry of additional followers. Consequently, the leader always behaves aggressively with the goal of limiting entry.

The scenarios considered in those papers are based on several simplifying assumptions. While they make it possible to clearly contrast outcomes relative to setups with one follower, they simultaneously limit the scope of industries that can be captured. In particular, there are two assumptions worth mentioning.

First, these studies consider industries with several followers, but only one leader. Yet there are numerous examples of industries with multiple leaders, such as Coca-Cola and PepsiCo in carbonated beverages, Adidas and Nike in sports apparel, or McDonald's and Burger King in the fast-food industry. The existence of only one leader determines that restricting entry is always beneficial for this firm. However, the existence of several leaders behaving more aggressively could result in a Pareto-dominated situation, where each leader ends up with lower profits. This is in fact the conclusion obtained by Gilbert and Vives (1986) under a homogeneous-good industry with one potential entrant.

Second, these studies model investments as only impacting a firm's own profits. While this is reasonable as a first approximation to model cost-reducing investments, it becomes narrow in scope for demand-enhancing investments. It determines that a firm cannot directly affect competition (and hence the rivals' profits) by improving features of its good. Thus, it leaves out industries where competition occurs primarily through innovation, such as the cell phone industry with Apple and Samsung. These firms create a tougher competitive environment by constantly launching overhauled versions of their cell phones, even when their prices barely change.

This second point has strong implications for the outcomes that can be captured. In particular, it establishes that aggressive pricing is the only possible strategy to limit entry, which basically turns investments into commitment devices to reduce a firm's own price. The fact that the same strategy is predicted irrespective of the investment considered could result in counterintuitive outcomes for some industries and types of investments. For instance, it determines that a leader always downgrades "quality" under demand-enhancing investments that increase a consumer's willingness to pay, as Etro (2006) concludes.

In this paper, we revisit models of strategic investments under free entry of followers. Our framework departs from extant models in two respects. First, we focus on demand-enhancing investments that strengthen competition, and so directly affect the demand of *all* firms. Intuitively, this captures that firms compete for the same set of customers, which can be attracted by lowering prices or improving non-price features of goods. In addition, we account for multiple leaders. Moreover, we allow them to be heterogeneous to capture that each possibly has an asymmetric influence in industry conditions. Thus, for instance, we can accommodate cases like Coca-Cola and PepsiCo, where the former has traditionally been the firm with the greatest influence in the carbonated beverages industry.

Our results indicate that each leader strategically uses its investment to restrict entry of the least-profitable firms, without having other leaders as a target. Moreover, while the deployment of this strategy requires increasing competition, leaders do not inflict mutual damage; only potential entrants are affected. Consequently, leaders do not end up trapped in a Pareto-inferior situation and each earns greater profits.

We also show that strengthening competition can be achieved via a wide array of strategies, with radically different implications for market outcomes. Given the richness of possibilities, we identify conditions in terms of model primitives to characterize outcomes. Depending on the features of the industry analyzed, we show that a leader could downgrade quality and engage in aggressive pricing as in the standard endogenous-entry models. In other cases, a leader could restrict entry by upgrading quality and charging higher prices. In fact, we show that aggressive behavior is consistent with overhauling varieties to exclusively attract the highest-valuation consumers and charge them high prices. This could reduce total quantities and revenues of a leader; however, by making it more difficult for followers to attract the most lucrative consumers, market profitability reduces and entry is effectively limited.

In Section 3, we begin the analysis by describing the model setup. Our framework considers an industry with a horizontally differentiated good, where multiple heterogeneous leaders compete with an unbounded pool of followers governed by free-entry rules. Joint with prices, each firm makes investment decisions that enhance their own demand and involve sunk fixed costs, as in Sutton (1991; 1998).[1] These investments could either increase or decrease the price elasticity of demand, and hence lower or raise a firm's price.[2] When in particular they increase prices, we refer to them as investments in "quality."

As for demand, we suppose it depends on a firm's own price and investments, along with a real-valued function that defines the competitive environment. Such a function depends on all firms' prices and investments, reflecting that a firm can strengthen competition by either lowering its price or overhauling its good. Demands with such a functional form encompass augmented versions of the three most common cases utilized in the literature: the CES, Logit, and linear demand. Additionally, it covers other standard cases such as the translog and demands derived from an additively separable indirect utility. At a formal level, it determines that the game is aggregative.[3]

In Section 4, we isolate the leaders' strategic motives to invest by utilizing the standard two-stage approach by Fudenberg and Tirole (1984; 1991). This requires comparing the outcomes between a simultaneous- and sequential-move game. In the latter, leaders make investments choices prior to the followers' entry choices and the market stage. As for the simultaneous-move games, it constitutes a non-strategic benchmark where investments decisions are unobserved, and hence cannot be used strategically.

In Section 5, we establish the main results emerging from the comparison. First, we show each leader always chooses its investment to strengthen competition and limit entry. Furthermore, even when they all behave more aggressively simultaneously, leaders do not inflict mutual damage and each garners greater profits. The results hold irrespective of whether prices are strategic substitutes or complements, and independently of the nature of demand-enhancing investments (i.e., price decreasing or increasing). Moreover, they could entail deploying an under- or over-investment strategy relative to the simultaneous-move

[1] In Appendix E, we also extend our characterization of results for the case in which demand-enhancing investments affect a firm's marginal cost. The conclusions are the same as in the baseline model.

[2] Investments that increase the price elasticity of demand can arise when they make the good more appealing for low-valuation or price-sensitive consumers. This could change the customers' composition and induce a firm to reduce its price.

[3] Basically, the fact that the game is aggregative implies that optimal payoffs and strategies can be expressed as functions of a single sufficient statistic. For a survey on aggregative games, see Jensen (2018).

game, with different implications regarding prices, revenues, and quantities. Nonetheless, regardless of the situation considered, accommodating entry is never optimal by a similar reason as in Etro's (2006; 2008): positive profits of followers induce additional entry, thereby undermining any attempt to weaken competition.

Second, we delve into the implications of strategic behavior for market outcomes. Unlike the standard endogenous-entry models, the strategy deployed by each leader is now setup-specific and can result in different market outcomes. Thus, a leader could under- or over-invest to increase competition since investing triggers two effects on the competitive environment. First, it directly toughens competition by making a firm's variety more appealing. Second, it concurrently affects a firm's incentive to choose its price. Thus, depending on whether the firm is induced to decrease or increase its price, this effect could make competition tougher or softer.

In particular, over-investing always arises when demand-enhancing investments decrease prices, since in that case both the direct and indirect effects work in the same direction. Additionally, it also takes place when investments do not raise a leader's price to such an extent that it decreases competition; remarkably, this outcome arises even under pronounced price effects that could end up reducing both the quantity and revenue of a leader.

Etro (2006) characterizes the results when a leader under-invests. This strategy generates similar outcomes in our framework, and therefore we focus on the implications of the over-investment case in Section 6. In contrast to the under-investment case, which always predicts aggressive pricing, over-investing is compatible with a multiplicity of outcomes. Due to this, we provide conditions in terms of demand primitives to identify when a leader over-invests and whether it (i) raises or lowers its price, (ii) increases or decreases its revenue, and (iii) supplies greater or lower quantities.

The results enable us to identify specific types of strategic behavior. In addition to aggressive pricing, two strategies to restrict entry are worth noting. The first involves a leader upgrading quality and charging higher prices, while simultaneously increasing its quantities sold and revenues. The second entails a pronounced upgrade in quality targeted to the group of consumers with the greatest willingness to pay. This provides a leader with incentives to substantially increase its price, thereby reducing its total quantities and revenues. Yet, this strategy would effectively reduce the market's profitability and hence restrict entry, by making it more difficult for followers to attract the most lucrative consumers.

Finally, in Section 7, we apply our characterization of results. This aims to answer the question: what kind of strategic behavior are we predicting under standard demands? We illustrate this by analyzing two of the most common demands in applied work: quality-augmented variants of the Logit and CES. Under these demands, each leader always over-invests to limit entry, sets higher prices, and increases its sales. Moreover, each leader increases its quantities under the Logit demand, whereas a leader does it under the CES when its market share is not disproportionately large.

Also, while the multiplicity of outcomes in the general case precludes any conclusion regarding welfare, the Logit and CES have specific implications on this matter. They can be easily derived using the results of Anderson et al. (2020), given the particular way in which non-price features are embedded in these demands. Specifically, we show that consumer surplus does not vary since the opposing effects impacting it completely offset each other; on the contrary, total industry profits increase. Consequently, consumers are always better off when profits are passed back to them.

2 Related Literature and Contributions

Our paper is related to a vast literature studying strategic moves by leaders to gain a better market position. The results we find are relevant for two strands of the literature: studies with strategic use of investments and Stackelberg-type models under endogenous entry.

Regarding the literature on strategic investments, Fudenberg and Tirole (1984) constitutes the classic reference under restricted entry.[4] This paper highlights the indeterminacy of results depending on whether competition is in strategic substitutes or complements, and the possibility of entry accommodation. Likewise, Etro (2006) is the first study to characterize these models under endogenous entry. He provides the important insight that accommodation is never profitable under free entry.[5] Such a conclusion is derived in a framework with one leader, along with cost-reducing investments and investments in quality that only affect a firm's own profit directly.

[4]The literature on leaders sinking investments to affect market outcomes is initiated by Spence (1977) and Dixit (1980). Fudenberg and Tirole (1984) provide a taxonomy of strategies for these models to identify the key factors leading to a disparity of outcomes. For instance, Shapiro (1989) classifies different articles and identifies 17 setups that can be explained through it. See also Gilbert (1989) and Vives (2001).

[5]For an up-to-date survey of free-entry models, see Polo (2018). For outcomes under Cournot competition, see Etro (2006) and Alfaro (2020).

Our contribution in this respect is twofold. First, we characterize models of endogenous entry under the presence of multiple (heterogeneous) leaders. This enables us to encompass industries dominated by a few firms, and account for possible asymmetries regarding a leader's relevance for industry outcomes.

Second, we obtain results under more general demand-enhancing investments than those utilized in the literature. In particular, investments in quality à la Etro arise as a special case of our framework. They refer to investments that increase demand and each consumer's willingness to pay, but do not directly affect competition. This case entails that only one outcome is possible: the leader downgrades quality to reduce its price.

In contrast, when demand-enhancing investments directly affect competition, the strategy to strengthen competition allows for various possibilities regarding a leader's investment, price, quantity, and revenue. In this context, our characterization of outcomes based on demand primitives becomes relevant: it makes it possible to know what type of demand is more suitable for capturing a specific type of strategic behavior.

Regarding Stackelberg games under price-leadership and endogenous entry, the closest articles to ours are Etro (2008) and Anderson et al. (2020).[6] In these games, firms do not make investment decisions and the leader directly commits to some level of prices. Their main conclusion is that the leader never accommodates entry, and necessarily restricts entry by adopting an aggressive pricing strategy. However, as indicated by Tirole (1988) and Vives (2001), Stackelberg-type games should be in principle interpreted as reduced-form models. This is because price/quantity choices do not generally entail sunk investments, and hence would not be irrevocable decisions.[7] Our results under this interpretation indicate that a price-leadership model replicates the outcomes of strategic-investments games when they entail aggressive pricing. However, it also leaves out a wide range of cases arising under investment games, since it cannot replicate outcomes where the leader charges a higher price.

Finally, our paper is related to an emerging literature utilizing the tools of Aggregative Games to analyze oligopoly models. This has been recently put forth in the Industrial-Organization literature by Acemoglu and Jensen (2013), Nocke and Schutz (2018), and Anderson et al. (2020), among others. Unlike these papers, we consider firms that choose

[6] For a survey of Stackelberg games, see Julien (2018). Also, Kokovin et al. (2017) study a Stackelberg model with free entry, and Tesoriere (2017) provides general results for this setup.

[7] Thus, for instance, these authors reinterpret quantities in Stackelberg as capacity choices, and profits as reduced-form functions that subsume the Nash equilibrium at the market stage.

investments in addition to prices. Furthermore, we consider demand-enhancing investments that affect all firms' profits, which creates more complex strategic interactions relative to investments only impacting a firm's own profit.

To the best of our knowledge, the only paper using an aggregative-games approach to incorporate demand-enhancing investments that affect all firms is Motta and Tarantino (2017). Their goal is to study the effects of horizontal mergers with a fixed number of firms that compete à la Bertrand. However, they only consider functional forms that turn investment and price choices isomorphic to a uni-dimensional firm's problem (e.g., demands that depend only on a firm's quality-price ratio). This allows them to apply the results by Anderson et al. (2020), which exclusively deal with single-choice frameworks.

Instead, we show that the aggregative-games approach can be applied without imposing such restrictions on investments. This can make a big difference regarding the scope of results that could be obtained using the tools of aggregative games. For instance, if we had exclusively focused on standard demands (e.g., quality-augmented variants of the Logit and CES), we would have concluded that the leader's strategy is unambiguous regarding investments, prices, revenues, and quantities.

3 Model Setup

We consider an industry in isolation that is composed of horizontally differentiated varieties. Each firm produces a unique variety i from the set of all conceivable varieties $\overline{\Omega}$. This set is partitioned into subsets $\mathscr{L}$ and $\mathcal{F}$, where a firm $i \in \mathscr{L}$ is referred to as a leader and a firm $i \in \mathcal{F}$ as a follower.

3.1 Supply Side

Firm i has constant marginal costs given by c_i. This parameter is common knowledge, and we suppose a symmetric marginal cost $c_{\mathcal{F}}$ for each $i \in \mathcal{F}$. Moreover, any leader has a lower marginal cost than a follower (i.e., $c_i < c_{\mathcal{F}}$ for each $i \in \mathscr{L}$) and followers have to pay a fixed sunk cost F to produce. We show in Appendix D that similar results hold by allowing for a subset of heterogeneous followers.

An active firm i decides on prices $p_i \in P$ and investments $z_i \in Z$, where P and Z are non-negative compact intervals with $0 \in Z$. We denote a strategy for i by $\mathbf{x}_i := (p_i, z_i)$. Moreover, investments z_i entail sunk expenditures $f_z(z_i)$, where f_z is a smooth convex function with $f_z(0) = 0$. Finally, the subset of varieties served in the market is denoted by Ω, and a profile of strategies for active firms is $\mathbf{x} := (\mathbf{x}_i)_{i \in \Omega}$.

3.2 Demand Side

We suppose a demand system that depends on a firm's own strategy and a function aggregating the strategies of all firms. Demands with such a property encompass several standard cases, including the Multinomial Logit, the CES, and a linear demand.[8] This makes it possible to use the tools of Aggregative Games, which are especially suitable for models with firm heterogeneity and multidimensional strategies. Under such games, a firm's profits function and its derivatives can be described by the firm's own strategy and a scalar function capturing the strategies of all firms.

We first define the demand system and then explain its properties.

Definition DEM. *The **demand** of a variety i, denoted q_i, is a smooth real-valued function $q(\mathbf{x}_i, \mathbb{A})$ where $\frac{\partial q(\mathbf{x}_i, \mathbb{A})}{\partial \mathbb{A}} < 0$, $\frac{\partial q(\mathbf{x}_i, \mathbb{A})}{\partial p_i} < 0$ and $\frac{\partial q(\mathbf{x}_i, \mathbb{A})}{\partial z_i} > 0$. In addition, we refer to $\mathcal{A}$ as an aggregator and a specific value in its range, $\mathbb{A}$, as an aggregate. Formally, $\mathcal{A}$ is a smooth function such that $\mathcal{A}(\mathbf{x}) := H\left[\sum_{i' \in \Omega} h(\mathbf{x}_{i'})\right]$ where $H' > 0$, $\frac{\partial h(\mathbf{x}_{i'})}{\partial p_{i'}} < 0$, and $\frac{\partial h(\mathbf{x}_{i'})}{\partial z_{i'}} > 0$.*

In Definition DEM, we have distinguished between $\mathcal{A}$ and $\mathbb{A}$, following Acemoglu and Jensen (2013). Using their terminology, $\mathcal{A}$ is referred to as an *aggregator*. This function takes all firms' strategies as inputs and produces a real number, $\mathbb{A}$, as an output. Examples of aggregators include the price index in a CES demand and the sum of all prices in a linear demand. As for $\mathbb{A}$, it is referred to as *an aggregate*, which is a value in the range of $\mathcal{A}$ and acts as a statistic that summarizes market conditions. Intuitively, an aggregate constitutes a measure of how tough the competitive environment is. This follows because decreases in prices and greater investments increase $\mathbb{A}$ which, in turn, reduce the demand and profits of any firm.

Some remarks about demands as in Definition DEM are in order. First, the definition allows for cases where prices are strategic complements or substitutes, as we show formally

[8] See Appendix B for a description of these and other demands covered. For instance, we also cover demands that derive from discrete-continuous choices (Nocke and Schutz, 2018) and from an additively separable indirect utility (Bertoletti and Etro, 2015).

in Appendix C.[9] Thus, our results are independent of this feature.

Second, investments are demand-enhancing since $\frac{\partial q_i}{\partial z_i} > 0$. Nonetheless, we have not indicated whether they increase or decrease the price elasticity of demand. Thus, we can cover different types of demand-enhancing investments. For instance, we are able to encompass investments that boost both demand and a consumer's willingness to pay, as is the case with product overhauls, after-sales services, and improvements in brand image. It also makes it possible to cover investments that make the good more appealing for price-sensitive and low-valuation consumers, which could change the composition of consumers and hence raise a firm's price elasticity.

To distinguish between these two types of investments, we define "quality" following Sutton (1991; 1998): any variable that increases the quantity demanded, reduces the price elasticity of demand, and requires sunk fixed costs. We extend the results for quality investments that additionally increase marginal costs in Appendix E. All the results are qualitatively the same.

3.2.1 Quality-Augmented CES and Logit

We close our description of demand by providing two examples whose functional forms satisfy all the properties stated in Definition DEM. They correspond to standard quality-augmented variants of the CES and Logit demand. These demands are frequently utilized in several fields of Economics and enable us to illustrate our findings.

The variant of the augmented CES we present is used in, for instance, Baldwin and Harrigan (2011), Feenstra and Romalis (2014), Hottman et al. (2016), and Redding and Weinstein (2020). It is given by

$$q(\mathbf{x}_i, \mathbb{A}) := E \frac{(p_i)^{-\sigma} (z_i)^{\sigma-1}}{\mathbb{A}}, \tag{1}$$

where E is the industry expenditure, $\sigma > 1$, and $\mathbb{A}$ is obtained through the following aggregator:

$$\mathcal{A}(\mathbf{x}) := \sum_{i' \in \Omega} (p_{i'}/z_{i'})^{1-\sigma}.$$

As for the Logit model, the following variant is employed in, for instance, Anderson et al.

[9] There, we show that strategic complements and substitutes can be determined through the sign of $\frac{\partial \varepsilon^p(\mathbf{x}_i,\mathbb{A})}{\partial \mathbb{A}}$, where $\varepsilon^p(\mathbf{x}_i, \mathbb{A})$ is the price elasticity of demand. Our results are consistent with any sign for this term.

(1992), Nocke and Schutz (2018), and Anderson et al. (2020):

$$q\left(\mathbf{x}_i, \mathbb{A}\right) := \frac{\exp\left(\frac{z_i - p_i}{\alpha}\right)}{\mathbb{A}}, \tag{2}$$

where $\alpha > 0$, and $\mathbb{A}$ is an aggregate of the following aggregator:

$$\mathcal{A}\left(\mathbf{x}\right) := \sum_{i' \in \Omega} \exp\left(\frac{z_{i'} - p_{i'}}{\alpha}\right).$$

4 Equilibrium Analysis

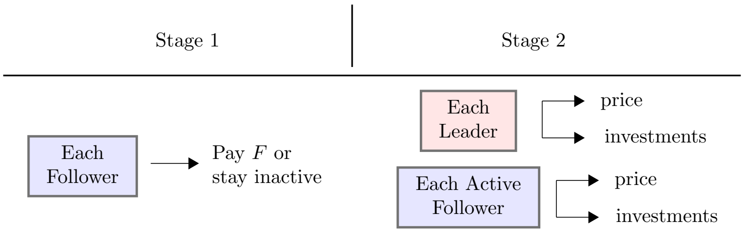
To identify the strategic use of investments by leaders, we compare equilibrium outcomes between a simultaneous- and a sequential-move game. These labels refer to the timing of the leaders' investments choices. Specifically, in the sequential-move scenario, leaders decide on investments prior to both followers' entry decisions and the market stage. Instead, the simultaneous-move case acts as a non-strategic benchmark in which followers do not observe investments, thereby removing each leader's strategic motive when choosing investments.

For each of these scenarios, we suppose that the equilibrium exists, is unique, and interior. Furthermore, we consider that each leader's profit function evaluated at optimal prices is strictly quasi-concave in investments. Finally, we consider free-entry equilibria where some followers are always active, and ignore the integer constraint so that zero profits hold, as in Etro (2006). All the results hold under the alternative assumption made in Anderson et al. (2020) that followers are modeled as a continuum, which implies that zero profits arise as an equilibrium outcome.

4.1 Simultaneous-Move Equilibrium

The timing of the simultaneous-move scenario is presented in Figure 1 and is as follows. First, each follower decides whether to pay the entry cost F or stay inactive. After this, each leader and each follower that paid the fixed cost F decide on prices and investments.

Figure 1. *Simultaneous-Move Scenario - Choices in Each Stage*



Solving the model by backward induction, consider a firm i that decides to serve the market. Thus, it chooses prices and investments by maximizing its gross profits, which are given by

$$\pi_i\left[\mathbf{x}_i, \mathcal{A}\left(\mathbf{x}\right)\right] := q\left[\mathbf{x}_i, \mathcal{A}\left(\mathbf{x}\right)\right]\left(p_i - c_i\right) - f_z\left(z_i\right). \tag{3}$$

Given rivals' strategies, i's optimal decisions are implicitly determined by the following first-order conditions:

$$\frac{\partial \pi_i\left(\mathbf{x}_i, \mathbb{A}\right)}{\partial p_i} + \frac{\partial \pi_i\left(\mathbf{x}_i, \mathbb{A}\right)}{\partial \mathbb{A}}\frac{\partial \mathcal{A}\left(\mathbf{x}\right)}{\partial p_i} = 0, \tag{4}$$
$$\frac{\partial \pi_i\left(\mathbf{x}_i, \mathbb{A}\right)}{\partial z_i} + \frac{\partial \pi_i\left(\mathbf{x}_i, \mathbb{A}\right)}{\partial \mathbb{A}}\frac{\partial \mathcal{A}\left(\mathbf{x}\right)}{\partial z_i} = 0. \tag{z-sim}$$

Instead of characterizing optimal strategies by best-response functions, we exploit that the game analyzed is aggregative in the sense of Cornes and Hartley (2012). Thus, optimal strategies can be characterized in terms of the so-called *backward-response functions* (Acemoglu and Jensen, 2013). They express each firm's optimal strategy as a function of the aggregate, which includes a firm's own strategy in addition to the strategies of all the other firms.[10]

Formally, the implicit solutions of (4) and (z-sim) for i determine the following backward-response functions:

$$\mathbf{x}_i\left(\mathbb{A}\right) := \left[p_i\left(\mathbb{A}\right), z_i\left(\mathbb{A}\right)\right],$$

where the optimal strategy of followers is denoted by $\mathbf{x}_{\mathcal{F}}\left(\mathbb{A}\right) := \left(p_{\mathcal{F}}\left(\mathbb{A}\right), z_{\mathcal{F}}\left(\mathbb{A}\right)\right)$.

To characterize the Nash equilibrium at the market stage, we also exploit the aggregative-game structure of the model. Thus, define *the aggregate backward-response function* by $\mathcal{A}^{\text{sim}}$,

[10] This becomes possible since the terms $\frac{\partial \mathcal{A}(\mathbf{x})}{\partial p_i}$ and $\frac{\partial \mathcal{A}(\mathbf{x})}{\partial z_i}$ are functions of $(\mathbf{x}_i, \mathbb{A})$ exclusively, due to the additive separability of the aggregator. Formally, $\frac{\partial \ln \mathcal{A}(\mathbf{x})}{\partial \ln z_i} = \frac{H'\left[H^{-1}(\mathbb{A})\right]}{\mathbb{A}}\frac{\partial h(\mathbf{x}_i)}{\partial \ln z_i}$ and $\frac{\partial \ln \mathcal{A}(\mathbf{x})}{\partial \ln p_i} = \frac{H'\left[H^{-1}(\mathbb{A})\right]}{\mathbb{A}}\frac{\partial h(\mathbf{x}_i)}{\partial \ln p_i}$, so that the first terms of these expressions are a function of $\mathbb{A}$ while the second ones of $\mathbf{x}_i$.

which corresponds to the function $\mathcal{A}$ evaluated at the backward-response functions. Formally,

$$\mathcal{A}^{\text{sim}}(\mathbb{A}, M) := H\left\{Mh\left[\mathbf{x}_{\mathcal{F}}(\mathbb{A})\right] + \sum_{i\in\mathscr{L}} h\left[\mathbf{x}_i(\mathbb{A})\right]\right\}, \tag{5}$$

where M is the number of followers that are active in the market. Since each firm's decision is expressed as a function of $\mathbb{A}$, there is a Nash equilibrium at the market stage if and only if $\mathbb{A}$ is a fixed point of $\mathcal{A}^{\text{sim}}$. Intuitively, this condition entails that the firms' optimal decisions self-generate the value $\mathbb{A}$ and is given by

$$\mathcal{A}^{\text{sim}}(\mathbb{A}, M) = \mathbb{A}. \tag{NE-sim}$$

As for free entry, a zero-profits condition emerges. For its characterization, denote the optimal profits of followers by $\pi_{\mathcal{F}}(\mathbb{A})$, which corresponds to (3) evaluated at $\mathbf{x}_{\mathcal{F}}(\mathbb{A})$. Then, this condition is defined as:

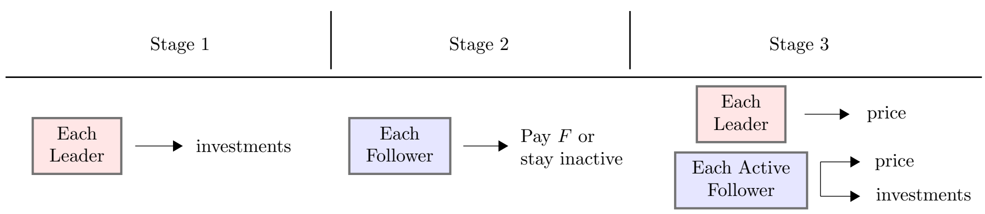
$$\pi_{\mathcal{F}}(\mathbb{A}) = F. \tag{ZP}$$

Overall, the equilibrium outcome for the simultaneous-move scenario can be identified through values M^{sim} and $\mathbb{A}^{\text{sim}}$ that satisfy conditions (NE-sim) and (ZP). Given these values, the equilibrium decisions of any firm can be determined. This includes the investments z_i^{sim} for each $i \in \mathscr{L}$, which can be characterized through (z-sim).

4.2 Sequential-Move Equilibrium

The timing of the sequential-move scenario is presented in Figure 2. It is the same as in the simultaneous-move case, except that leaders make their investments decisions at the beginning of the game. Thus, first, each leader makes an investment choice. After this, each follower observes these investments and decides whether to pay F or stay inactive. Finally, each leader chooses prices, while the followers that paid the fixed cost F decide on both prices and investments.

Figure 2. *Sequential-Move Scenario - Choices in Each Stage*



4.2.1 Market Stage

Employing a backward-induction procedure, we begin by characterizing the optimal decisions at the market stage. Unlike the simultaneous-move scenario, this stage defines a class of subgames for each vector of leaders' investments, $\mathbf{z}^{\mathscr{L}} := (z_i)_{i\in\mathscr{L}}$. Given the structure of the game, the characterization of the market-stage solution is similar to that in the simultaneous-move model.

As for leader i, its optimal price is characterized by the same first-order condition, (4). Therefore, conditional on $\mathbb{A}$, the investments made by any other rival firm in the first stage do not affect (4), and the implicit solution to (4) determines a function $p_i(z_i, \mathbb{A})$.

Regarding followers, the characterization of their decisions is also identical to that in the simultaneous-move scenario. If a follower decides to serve the market, its optimal prices and investments are described by the same vector of strategies as in the simultaneous-move case, $\mathbf{x}_{\mathcal{F}}(\mathbb{A})$. This occurs because, conditional on $\mathbb{A}$, the marginal profits of followers do not depend on $\mathbf{z}^{\mathscr{L}}$.

Given these optimal choices, the condition for a Nash equilibrium at the market stage requires defining an aggregate backward-response function, $\mathcal{A}^{\text{seq}}$, for given $\mathbf{z}^{\mathscr{L}}$. This corresponds to $\mathcal{A}$ evaluated at the optimal decisions:

$$\mathcal{A}^{\text{seq}}\left(\mathbb{A}, M, \mathbf{z}^{\mathscr{L}}\right) := H\left\{Mh\left[\mathbf{x}_{\mathcal{F}}(\mathbb{A})\right] + \sum_{i\in\mathscr{L}} h\left[p_i(z_i, \mathbb{A}), z_i\right]\right\}. \tag{6}$$

Thus, there is a Nash equilibrium at the market stage when $\mathbb{A}$ constitutes a fixed point of $\mathcal{A}^{\text{seq}}$:

$$\mathcal{A}^{\text{seq}}\left(\mathbb{A}, M, \mathbf{z}^{\mathscr{L}}\right) = \mathbb{A}. \tag{NE-seq}$$

As for free entry, a follower's optimal profits are the same as in the simultaneous-move scenario, i.e., $\pi_{\mathcal{F}}(\mathbb{A})$. Consequently, the zero-profits condition is still given by (ZP).

All this establishes that the equilibrium of the subgame for a given $\mathbf{z}^{\mathscr{L}}$ can be identified through values $M\left(\mathbf{z}^{\mathscr{L}}\right)$ and $\mathbb{A}^*\left(\mathbf{z}^{\mathscr{L}}\right)$ such that (ZP) and (NE-seq) hold.

4.2.2 Equilibrium

Before deriving the optimal investments by leaders, we investigate some properties of the subgame for a given $\mathbf{z}^{\mathscr{L}}$. This allows us to find the solution for the leaders' investments more easily.

Recall that active followers' strategies are a function $\mathbf{x}_{\mathcal{F}}(\mathbb{A})$, so that their decisions are independent of $\mathbf{z}^{\mathscr{L}}$ conditional on $\mathbb{A}$. Additionally, leader i's optimal price determines a function $p_i(z_i, \mathbb{A})$, which is also independent of $\mathbf{z}^{\mathscr{L}}_{-i} := (z_{i'})_{i' \in \mathscr{L}\setminus\{i\}}$ conditional on $\mathbb{A}$. Thus, the optimal choices of firms at the market stage require analyzing how $\mathbb{A}$ is identified.

Inspection of (ZP) reveals that it completely identifies $\mathbb{A}^{\text{seq}}$, where $\mathbb{A}^{\text{seq}}$ is the equilibrium value of $\mathbb{A}$ in the sequential-move scenario. As a corollary, since equation (ZP) is not directly affected by $\mathbf{z}^{\mathscr{L}}$, it is determined that $\mathbb{A}^{\text{seq}} = \mathbb{A}^{\text{sim}}$. Hence, the same equilibrium aggregate is pinned down in both scenarios, which we denote by $\mathbb{A}^*$.

The fact that the same aggregate holds in each equilibrium follows because variations in z_i trigger changes in the number of followers M to satisfy (NE-seq); in turn, the magnitude of these effects are completely offset and leave the aggregate unaltered. The mechanism behind this is that variations in a leader's investments initially have an impact on the followers' profits. More precisely, if changes in a leader's investments strengthen competition, profits of followers become lower and induce exit; on the contrary, if variations in a leader's investments soften competition, a follower's profits increase and induce entry. Each case entails that, overall, there are opposing effects of the same magnitude on the competitive environment.

From now on, when it is necessary to indicate that the derivative of a function takes $\mathbb{A}^*$ as given, we use $(\cdot;\mathbb{A}^*)$ as an argument rather than $(\cdot,\mathbb{A})$. Moreover, we define the solution to (NE-seq) by $M\left(\mathbf{z}^{\mathscr{L}};\mathbb{A}^*\right)$. This ensures that, once that $\mathbb{A}^*$ is identified through (ZP), any variation in z_i does not affect $\mathbb{A}^*$; instead, it triggers changes in M to make (NE-seq) hold.

Incorporating $M\left(\mathbf{z}^{\mathscr{L}};\mathbb{A}^*\right)$, the relevant aggregator at the first stage can be expressed as a function $\mathcal{A}\left[p_i(z_i;\mathbb{A}^*), z_i; \mathbf{z}^{\mathscr{L}}_{-i}, \mathbb{A}^*\right]$. Due to this, leader i's demand becomes a function $q\left[p_i(z_i;\mathbb{A}^*), z_i;\mathbb{A}^*\right]$. Thus, the problem of a leader i at the first stage is given by

$$\max_{z_i} \pi_i\left[p_i(z_i;\mathbb{A}^*), z_i;\mathbb{A}^*\right], \tag{7}$$

and its first-order condition is

$$\frac{\partial \pi_i\left[p_i(z_i;\mathbb{A}^*), z_i;\mathbb{A}^*\right]}{\partial z_i} + \frac{\partial \pi_i\left[p_i(z_i;\mathbb{A}^*), z_i;\mathbb{A}^*\right]}{\partial p_i}\frac{\partial p_i(z_i;\mathbb{A}^*)}{\partial z_i} = 0. \tag{z-seq}$$

This characterizes the solution z_i^{seq} for each leader i and enables us to find the solution of the whole game.

5 Results

We proceed to characterize the outcomes in the sequential-move equilibrium relative to the simultaneous-move equilibrium. This allows us to identify the strategic motives of leaders to choose investments and their consequences on outcomes.

Our findings indicate that each leader always increases competition and restricts entry of followers, whereby each garners greater profits. On the contrary, accommodating entry is never optimal. Additionally, we show that this can entail under- or over-investing relative to the simultaneous-move game, depending on the nature of investments. All proofs of this paper are relegated to Appendix A.

5.1 General Outcomes

The following proposition identifies the robust conclusions of the model, i.e., those that hold without adding any further assumptions.

Proposition 5.1. *Relative to the simultaneous-move equilibrium, in the sequential-move equilibrium:*

- *each leader behaves more aggressively,*
- *each leader has greater profit, and*
- *the number of active followers is lower.*

The proposition indicates that each leader i uses its investments to strengthen competition. Formally, this means that they choose investments to increase $h(\mathbf{x}_i)$, which is the term defining how i's strategy affects the competitive environment, $\mathcal{A}$. In this way, each leader reduces the followers' profits and induces their exit, which ultimately leaves the competitive environment unaffected.

Since h could become greater due to increases in investments and/or a reduction in prices, a leader could end up under- or over-investing in equilibrium. This depends on both the nature of investments (i.e., whether they decrease or increase a leader's own price) and the magnitude of the effects of prices and investments on the competitive environment. Nonetheless, irrespective of the specific strategy chosen, leaders always find it optimal to limit entry. On the contrary, accommodating strategies to soften competition are doomed to fail: under free entry, they end up attracting entry of followers and sabotaging a leader's attempt to keep profits high.

Furthermore, the proposition states that, also independently of the strategy chosen, each leader earns greater profits in the sequential-move scenario. This outcome is not trivial, due to the existence of multiple heterogeneous leaders. In such a setting, we might imagine that more aggressive behavior by leaders could create tougher competition to the extent that they reduce the profits of some leaders. The proposition establishes that this is not the case. This follows because increases in investments by leaders have exit of followers as a counterpart, thereby leaving the competitive conditions unaffected in equilibrium. As a result, leaders do not inflict mutual damage on each other.

5.2 Investments

While we have established that each leader makes competition tougher to restrict entry, we have yet to characterize how this is accomplished via investments. In particular, this could entail an under- or over-investment strategy relative to the simultaneous-move game. The reason is that an increase in investments triggers two effects on the competitive environment.

First, there is a direct effect that strengthens competition by making the good more appealing. Additionally, there is an indirect effect that acts through a leader's own price. The sign of this effect depends on the nature of investments, which could provide a leader with an incentive to either reduce or raise its own price, and hence have opposing effects on the competitive environment. Consequently, different combinations of these two effects determine that investing could potentially toughen or soften competition.

For the case where investments induce a leader to reduce its price, there is only one possible outcome: strengthening competition requires over-investing, since both the direct and indirect effects act in the same direction.

As for investments that induce a leader to increase its price, the direct and indirect effects have opposing effects. Therefore, two possibilities arise. Over-investment occurs if the direct effect of investments dominates the impact through prices. On the contrary, under-investment arises when investing increases prices to such an extent that competition is reduced overall.

Based on this analysis, the condition for over-investing can be stated through an upper bound on $\frac{\partial \ln p_i(z_i;\mathbb{A}^*)}{\partial \ln z_i}$, which is the impact of leader i's investments on its own price. Formally,

$$\frac{\partial \ln p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)}{\partial \ln z_i} < -\frac{\partial \ln h\left[p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right), z_i^{\text{sim}}\right]}{\partial \ln z_i}\left(\frac{\partial \ln h\left[p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right), z_i^{\text{sim}}\right]}{\partial \ln p_i}\right)^{-1}, \quad (8)$$

where the right-hand side is positive. In words, (8) indicates that the impact of i's investment on its price evaluated at the simultaneous-move equilibrium has to be negative or, alternatively, positive but not disproportionately large. Making use of this, we establish the following proposition.

Proposition 5.2. *Relative to the simultaneous-move equilibrium, in the sequential-move equilibrium:*

- *if* (8) *holds, then leader* i *over-invests,*
- *otherwise, if the inequality in* (8) *is reversed, leader* i *under-invests.*

6 Over-Investment

One important conclusion of the previous section is that the strategy to strengthen competition is setup-specific. Such a feature determines that the model is capable of generating a variety of results in terms of investments, prices, quantities, and revenues. This motivates providing a characterization of outcomes by identifying conditions in terms of model primitives.

A scenario with under-investment determines qualitative results akin to Etro (2006). Specifically, under-investing only arises when investments increase consumer willingness to pay, in which case a leader always engages in aggressive pricing. This is shown formally in Appendix A.3. Due to this, we focus on the over-investment case.

6.1 Over-Investing and Prices

We begin by translating the condition for over-investment, (8), in terms of model primitives. Since this condition is stated as an upper bound on $\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i}$, it is necessary to describe the effect on prices to obtain conclusions for investments. Due to this, we simultaneously characterize both variables.

Specifically, the impact of leader i's investment on its price for a given $\mathbb{A}$ is

$$\frac{\partial \ln p_i\left(z_i;\mathbb{A}\right)}{\partial \ln z_i} = \frac{\frac{\partial \ln \varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]}{\partial \ln z_i}}{1-\varepsilon^p\left[p_i\left(z_i;\mathbb{A}\right),z_i;\mathbb{A}\right]-\frac{\partial \ln \varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]}{\partial \ln p_i}}, \tag{9}$$

where $\varepsilon^p(\mathbf{x}_i, \mathbb{A}) := -\frac{\mathrm{d} \ln q[\mathbf{x}_i, \mathcal{A}(\mathbf{x})]}{\mathrm{d} \ln p_i}$ is the price elasticity of i's demand. Moreover,

$$\operatorname{sgn}\left(\frac{\partial p_i(z_i; \mathbb{A})}{\partial z_i}\right) = \operatorname{sgn}\left(-\frac{\partial \varepsilon^p[p_i(z_i; \mathbb{A}), z_i; \mathbb{A}]}{\partial z_i}\right), \tag{10}$$

so that the sign of $\frac{\partial p_i(z_i;\mathbb{A})}{\partial z_i}$ can be easily identified when the sign of $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}$ is the same for any $(\mathbf{x}_i, \mathbb{A})$. It simply requires determining whether investments make demand more or less price inelastic for a given aggregate value.

Based on this, the following assumption ensures that the condition for over-investment, (8), is satisfied.

Assumption 6.1. *Either* $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} > 0$ *or* $\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1-\varepsilon^p(\mathbf{x}_i,\mathbb{A})-\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} < -\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i}\right)^{-1}$ *holds for any* $(\mathbf{x}_i, \mathbb{A})$.

To provide an interpretation of Assumption 6.1, recall that greater investments impact the competitive environment through two channels: a direct one that makes competition tougher and an indirect one via prices that could decrease or increase competition. Assumption 6.1 ensures that the indirect effect either reinforces or does not revert the sign of the direct effect, so that (8) holds.

Specifically, $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} > 0$ ensures that greater investments provide an incentive for i to decrease its price, thereby reinforcing the strengthening effect on competition. If $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} < 0$, greater investments by i provide it with an incentive to raises its price. For this case, the second condition in Assumption 6.1 establishes that the effect via prices is not disproportionately large, and so the direct effect dominates.

By making use of this assumption, we obtain the following result.

Proposition 6.1. *Suppose that Assumption 6.1 holds. Then, relative to the simultaneous-move equilibrium, in the sequential-move scenario:*

- *(8) is satisfied, and so leader i over-invests,*
- *leader i charges a higher price if* $\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i} < 0$ *for any* $(\mathbf{x}_i, \mathbb{A})$*, and a lower price if* $\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i} > 0$ *for any* $(\mathbf{x}_i, \mathbb{A})$.

The proposition has implications for the augmented CES and Logit, (1) and (2). These demands satisfy Assumption 6.1 and $\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i} < 0$. Thus, each leader over-invests and charges higher prices in the sequential-move equilibrium.

The fact that this outcome arises under these demands is relevant since they are frequently used in several fields of Economics, particularly in empirical work. Furthermore, the deployment of such a strategy establishes a contrast relative to models with no direct effects of investments on competition. In that case, a leader always under-invests and sets lower prices.

6.2 Quantities, Revenues, and Market Share

Assumption 6.1 is relatively mild when it is assessed in terms of the restrictions that it imposes on other outcomes: it ensures that a leader over-invests, but its impact on prices, quantities, and revenues is not necessarily determinate.

If investments increase a firm's price elasticity, all the results can actually be identified: a leader always charges lower prices, obtains greater revenues, and sells more quantities. As a consequence, they generate similar outcomes to Etro (2006).

On the contrary, investments that decrease a firm's price elasticity are compatible with various effects on a leader's prices, quantities, and revenues. Thus, further assumptions are required to characterize this case. In particular, while we have already established conditions to identify the impact on prices, we have yet to do so for the remaining variables of interest.

6.2.1 Revenues and Market Share

The propositions we establish below encompass results for investments that decrease or increase a leader's price. Nonetheless, since the results for the former are determinate, our explanations focus on the scenario where investments increase a leader's price.

The total impact of greater investments on a leader's revenue can be decomposed into their impact on prices and quantities. On the one hand, revenue increases due to the greater prices and the direct effect of investments on quantities. On the other hand, revenue decreases given the reductions in quantities caused by higher prices.

To ensure that a leader's revenue is greater when it over-invests, we state a condition that restricts the positive effect of investments on a firm's own price, $\frac{\partial p_i(z_i;\mathbb{A}^*)}{\partial z_i}$. Such a condition allows for pronounced effects via prices that decrease a leader's equilibrium quantities, although it precludes that their magnitude is so significant that revenue reduces.

For its statement, we respectively define i's price and investments elasticities of demand ignoring the effect on the aggregate by ξ_i^p and ξ_i^z. They are given by functions $\xi^p(\mathbf{x}_i;\mathbb{A}) := \frac{\partial \ln q[\mathbf{x}_i,\mathcal{A}(\mathbf{x})]}{\partial \ln p_i}$ and $\xi^z(\mathbf{x}_i;\mathbb{A}) := \frac{\partial \ln q[\mathbf{x}_i,\mathcal{A}(\mathbf{x})]}{\partial \ln z_i}$.

Assumption 6.2. *For any* $(\mathbf{x}_i, \mathbb{A})$, *we suppose that*

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1 - \varepsilon^p(\mathbf{x}_i, \mathbb{A}) - \frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} < \frac{\xi^z(\mathbf{x}_i, \mathbb{A})}{\xi^p(\mathbf{x}_i, \mathbb{A}) - 1} \tag{11}$$

when $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} < 0$.

The following proposition establishes the main results we can derive using it.

Proposition 6.2. *Suppose Assumption 6.1 holds, so that leader i over-invests. Relative to the simultaneous-move equilibrium, in the sequential-move scenario:*

- *if either* $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} > 0$ *or Assumption 6.2 holds, leader i has a greater revenue,*
- *if Assumption 6.2 with the inequality in (11) reversed holds, leader i has a lower revenue.*

The proposition establishes that a leader obtains greater revenue when Assumption 6.2 holds. In particular, this assumption is always satisfied under the quality-augmented versions of the CES and Logit. Furthermore, since it can be shown that the revenues of followers as a group always decrease, it triggers a reallocation of (expenditure-based) market share from followers to leaders.[11]

This result is, nonetheless, silent about whether each leader increases its market share when Assumption 6.2 holds. Moreover, general results in this regard are not possible under heterogeneity of leaders, since these firms have different gains in terms of revenues. This can be appreciated through a scenario where the least-productive leader slightly increases its revenue, while the rest of leaders substantially increase them. If this is the case, the industry expenditures may increase to such an extent that the least-productive leader's market share becomes lower, while the rest of leaders gain market share.

Nevertheless, we can strengthen the result for the CES case, which is a demand satisfying constant industry expenditures. Under this demand, each leader ends up with both a greater revenue and market share in the sequential-move scenario.

6.2.2 Quantities

The results obtained for revenues and prices do not allow us to infer what occurs with quantities. For instance, Assumption 6.2 ensures that a leader's revenue is greater when prices are higher, but this allows for either lower or greater quantities. Ultimately, whether a leader sells more or less in such a scenario depends on how marked the increase in price is.

[11] We formally show that this holds in the proof of Proposition 6.2.

Due to this, more stringent conditions need to be placed on the effect of investments on a firm's own price to ensure that a leader sells greater quantities. This is captured by the following condition.

Assumption 6.3. *For any* $(\mathbf{x}_i, \mathbb{A})$, *we suppose that*

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1 - \varepsilon^p(\mathbf{x}_i, \mathbb{A}) - \frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} < \frac{\xi^z(\mathbf{x}_i, \mathbb{A})}{\xi^p(\mathbf{x}_i, \mathbb{A})} \tag{12}$$

when $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} < 0$.

Based on this, we can establish the following result.

Proposition 6.3. *Suppose Assumption 6.1 holds, so that leader i over-invests. Relative to the simultaneous-move equilibrium, in the sequential-move scenario:*

- *if either $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} > 0$ or Assumption 6.3 holds, then leader i sells more quantities,*
- *if Assumption 6.3 with the inequality in (12) reversed holds, then leader i sells less quantities.*

Assumption 6.3 implies Assumption 6.2 in equilibrium.[12] In words, this means that a leader has higher revenues if it sells greater quantities, irrespective of whether it charges a higher or a lower price. In terms of the demands used as examples, Assumption 6.3 is always satisfied for the quality-augmented version of Logit, (2). As for the CES variant, (1), it holds as long as a leader's market share is not disproportionately large.

Additionally, when a leader charges a higher price and has a lower revenue in equilibrium, it necessarily sells less quantities. Such an outcome rationalizes scenarios where a small group of high-valuation consumers defines the profitability of the market. In this case, restricting entry could call for targeting these consumers by substantially upgrading quality. This, in turn, would make it optimal to charge extremely high prices, so that the total revenues and quantities sold reduce. In the end, such a strategy effectively reduces the followers' profits, since it makes it more difficult for followers to attract the most lucrative consumers.

6.3 Welfare

To analyze the welfare implications of strategic investments, we suppose that demands are integrable (i.e., they can be derived from a representative consumer). Moreover, we consider

[12] This follows because $\xi^p(\mathbf{x}_i, \mathbb{A}) > 1$ in any interior equilibrium, and so $\frac{\xi^z(\mathbf{x}_i,\mathbb{A})}{\xi^p(\mathbf{x}_i,\mathbb{A})} < \frac{\xi^z(\mathbf{x}_i,\mathbb{A})}{\xi^p(\mathbf{x}_i,\mathbb{A})-1}$.

demand-enhancing investments that have a direct impact on utility. Thus, they can be thought of as improving (objective or perceived) non-price features of a variety.

The first conclusion we can obtain is regarding producer surplus, which is always greater in the sequential-move equilibrium. This arises because each leader earns greater profits by Proposition 5.1, whereas exit of followers has no impact on it since they always garner zero profits.

Regarding consumer surplus, it is not possible to derive conclusions without additional structure on the model. This is because consumer welfare is subject to opposing effects. First, there are changes in prices and non-price features of each variety, with at least one of these channels predicting improvements in a consumer's well being. Second, the crowding-out of followers reduces the number of available varieties, and this always has a negative impact on welfare given that the good is differentiated.

6.3.1 Welfare under the CES and Logit

Welfare results are ambiguous in the general case since we cannot predict what occurs with consumer surplus. Nevertheless, there are some demands where the impact on it can be identified. Specifically, this occurs when both the demand and indirect utility that generates it depend on the same aggregate. Such a feature is exhibited by two demands that are ubiquitous in empirical work: the CES and Multinomial Logit.

Under these demands, consumer surplus does not vary since the aggregate is the same in the simultaneous- and sequential-move equilibrium. The result holds irrespective of how the non-price characteristic is embedded into the demands, and hence regardless of whether there is under- or over-investing.[13]

To provide some intuition about this outcome, consider the particular case of the quality-adjusted variants, (1) and (2). In these cases, there are opposing effects on consumer surplus. On the one hand, potential competition by followers makes leaders overhaul the non-price feature of their varieties, which is welfare-improving. On the other hand, welfare decreases due to both a lower number of varieties and each leader's higher price. Overall, these effects perfectly offset under both demands, which means the consumer's well-being is unchanged.

[13] A linear demand is an example of a system that depends on one aggregate, but the indirect utility from which it is derived is a function of two aggregates. In those cases, the same demand's aggregate is consistent with different levels of prices, investments, and number of varieties. Thus, consumer surplus depends on the weight that consumers give to each component. See Anderson et al. (2020) for further details.

To formalize this, we base on Nocke and Schutz (2018) and Anderson et al. (2020). These studies characterize an indirect utility function with two properties: it depends on one aggregate and defines a demand as in Definition DEM which depends on the same aggregate. In particular, we consider an extension of their results by embedding an endogenous non-price feature into the utility function, as in our framework.

Following Anderson et al. (2020), consider an economy with a representative consumer. She offers one unit of labor inelastically, which is the only production factor. Moreover, we unify the welfare analysis by supposing she is the owner of firms. Thus, profits are passed back to her, and her income Y consists of wages and profits.

There is also a horizontally differentiated industry and a homogeneous outside good. The latter is taken as the numéraire and produced under perfect competition, thereby pinning down wages. As a result, the differentiated industry exclusively impacts the rest of the economy through income effects triggered by variations in profits.

Furthermore, the upper-tier utility function between the differentiated and homogeneous good is quasi-linear. Specifically, the representative consumer has an indirect utility function V given by

$$V(Y, \mathbf{x}) := Y + \alpha \ln \left[\sum_{i \in \Omega} h(\mathbf{x}_i) \right], \tag{13}$$

where $\alpha \in \mathbb{R}_{++}$.

Assuming that income is high enough to have a positive consumption of both goods, we can apply Roy's identity. Thus, the demand of variety i is given by

$$q(\mathbf{x}) = \alpha \frac{-\frac{\partial h(\mathbf{x}_i)}{\partial p_i}}{\sum_{i \in \Omega} h(\mathbf{x}_i)}.$$

To express the indirect utility and demand of i in terms of an aggregator, let $\mathcal{A}(\mathbf{x}) := \sum_{i \in \Omega} h(\mathbf{x}_i)$. Therefore,

$$V[Y, \mathcal{A}(\mathbf{x})] := Y + \alpha \ln [\mathcal{A}(\mathbf{x})], \tag{14}$$

$$q[\mathbf{x}_i, \mathcal{A}(\mathbf{x})] := \alpha \frac{-\frac{\partial h(\mathbf{x}_i)}{\partial p_i}}{\mathcal{A}(\mathbf{x})}. \tag{15}$$

By defining h appropriately, it is possible to encompass different cases. Specifically, through $h(\mathbf{x}_i) := \exp\left(\frac{z_i - p_i}{\alpha}\right)$ and $\alpha := 1$, we can encompass the quality-augmented Logit (2). In addition, a CES demand as in (1) corresponds to the case $h(\mathbf{x}_i) := (p_i/z_i)^{1-\sigma}$ and $\alpha :=$

$(\sigma - 1)^{-1}$.

When the indirect utility function is given by (14), agents are better off in the sequential-move equilibrium relative to the simultaneous-move scenario. The reason is as follows. First, consumer surplus of the differentiated good does not vary. This is because the same $\mathbb{A}^*$ holds in the simultaneous- and sequential-move equilibrium, which is now also a sufficient statistic for consumer welfare. Second, each leader obtains greater profits, whereas followers always earn zero profits; therefore, there are greater total industry profits in the sequential-move equilibrium. Since agents have a quasi-linear utility function and profits are passed back to them, their income increases and they can consume more of the outside good.

7 An Application: Quality-Variants of Logit and CES

Given the multiplicity of outcomes, it is pertinent to know the strategic behavior captured by specific demands. In particular, we consider two demands that are especially relevant in applied work: the quality-augmented CES and Multinomial Logit, given by (1) and (2). We begin by indicating the assumptions that these demands satisfy, which allows us to characterize their outcomes subsequently.

Proposition 7.1. *Suppose demand is the quality-augmented CES or Multinomial Logit, given by* (1) *and* (2). *Then,*

- *each satisfies* $\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i} < 0$ *for any* $(\mathbf{x}_i, \mathbb{A})$, *Assumption 6.1, and Assumption 6.2,*
- *Assumption 6.3 is satisfied by the Logit; it also holds under the CES as long as leader i's market share is not disproportionately large, and*
- *each can be generated by an indirect utility function as in* (14).

The result allows us to apply Propositions 6.1, 6.2, and 6.3, along with the results regarding welfare from Section 6.3.1. Thus, strategic investments by leaders generate the following outcomes relative to the simultaneous-move game:

[1] Each leader strengthens competition to limit entry of followers and garners greater profit.

[2] Each leader deploys an over-investment strategy and charges a higher price.

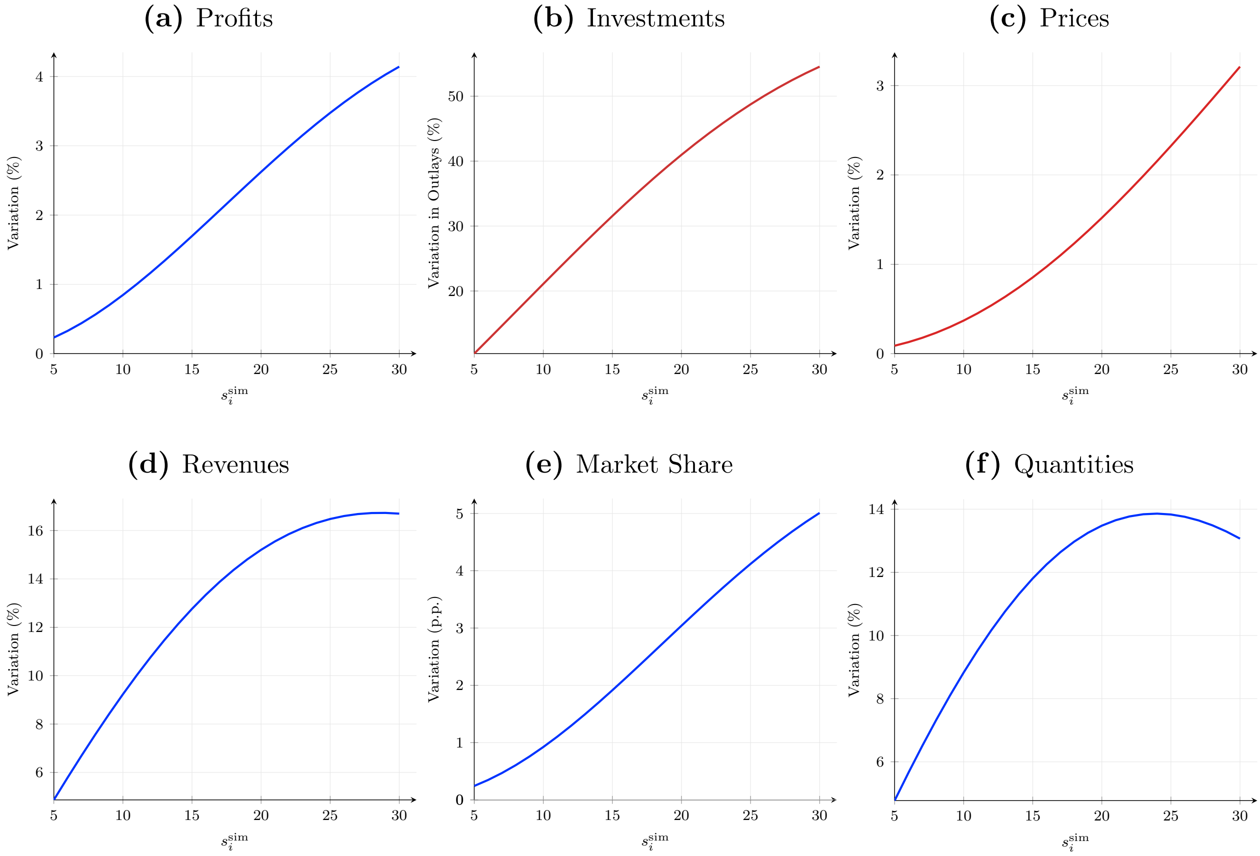
[3] Each leader increases its revenue. Furthermore, leaders as a group accrue a greater market share, and additionally each leader increases its market share in the CES case.

[4] Each leader sells more quantities in the Logit case, whereas in the CES this occurs as long as each leader's market share is not disproportionately large.

[5] The outcome is welfare-improving.

To illustrate these results, we resort to some visual aids based on the quality-augmented CES. Figure 3 depicts the outcomes in the sequential-move scenario relative to the simultaneous-move case. The graphs exhibit the variables of one specific leader, with results expressed as functions of this leader's market share in the simultaneous-move scenario, s_i^{sim}. It exploits that we can recover the behavior of this leader once we have a value for s_i^{sim}.

Figure 3. *Quality-Augmented CES: Outcomes in the Sequential-Move Scenario relative to the Simultaneous-Move Case*



Note: s_i^{sim} and s_i^{seq} refer to the market share of leader i in the simultaneous- and sequential-move equilibrium, respectively. The graph is based on the quality-augmented CES, (1), with investment outlays given by $f_i^z(z_i) := f^z(z_i)^\beta$ with $f^z, \beta > 0$ and parameters $(\sigma, \beta) := (3, 4)$. For details, see Appendix A.4.1.

Figure 3a captures that leaders do not inflict mutual damage, allowing each to garner a greater profit. This result goes beyond the CES case, and holds for any demand as in Definition DEM.

Furthermore, our framework captures Etro's (2006) insight that entry accommodation never arises under endogenous entry. However, the description in Figure 3 of how a leader limits entry starkly differs from that study.

Specifically, Etro obtains results for investments that increase demand and a consumer's willingness to pay, but do not directly affect the competitive environment. Through the lens of our model, this arises as a special case where the demand is as in Definition DEM, satisfies $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} < 0$, and the effect of investments on the aggregator is negligible (i.e., $\frac{\partial h}{\partial z_{i'}} \to 0$ for each firm i'). When that is the case, a leader always downgrades quality and sets lower prices.[14] The intuition for this is that the competitive environment is only affected by prices. Thus, each leader strengthens competition by using its investments as a commitment device to reduce its price.

A similar outcome arises under price-leadership models with free entry, as in Etro (2008) and Anderson et al. (2020). In that case, leaders do not make investment decisions but instead directly commit to some level of prices. Thus, a leader is only able to increase competition by engaging in aggressive pricing.

In contrast, Figures 3b and 3c highlight through red curves that a leader restricts entry by upgrading quality and charging higher prices. Furthermore, although the graphs depict the choices of one specific leader, they hold for each leader that is active in the market.

Our framework allows for this possibility since firms can attract customers through their choices on both price and non-price attributes of the good. Thus, it captures natural scenarios in which a leader increases the appeal of its variety to attract consumers, which can strengthen competition despite any concurrent increase in price.

Additionally, Figure 3d reflects that each leader obtains greater revenues. In the particular case of the CES demand, which exhibits constant expenditures, this also implies that each active leader in the market also increases its domestic market share (Figure 3e).

Finally, Figure 3f depicts the case of a leader that increases its quantities. Nonetheless, it is also possible that a leader decreases its quantities if it has a large market share.[15]

[14]Formally, $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} < 0$ implies $\frac{\partial p_i(z_i;\mathbb{A})}{\partial z_i} > 0$ by (10). In addition, $\frac{\partial h}{\partial z_{i'}} \to 0$ implies that the right-hand side of (8) tends to zero. Therefore, we can apply Proposition 5.2 to the case where the inequality in (8) is reversed, so that there is under-investing. Furthermore, since investments increase a consumer's willingness to pay, lower investments imply that each leader charges a lower price.

[15]The threshold level of market share depends on the elasticity of substitution, σ, where a greater value of this parameter increases the critical market share. In our example with $\sigma := 3$, quantities increase if the leader's market share is lower than 60% in the simultaneous-move scenario.

8 Conclusions

We have revisited a recent literature on the strategic behavior of leaders under endogenous entry. Existing studies on the topic have obtained conclusions under several simplifying assumptions. This ended up characterizing leaders as always pursuing the same strategy to restrict entry and thus generating the same outcomes, irrespective of the industry considered. To address this, our analysis relaxed two of these limiting assumptions.

First, we dispensed with assuming there is only one leader, and instead considered the potential for multiple heterogeneous leaders. This adds a layer of complexity to the model, since a leader's strategy now imposes an externality not only on followers but also other leaders. However, it also makes it possible to cover more realistic settings, where several firms with an asymmetric capacity to influence industry conditions act as leaders. There are indeed many examples of industries where leaders coexist, such as Apple and Samsung in the cell-phone industry, Coca-Cola and PepsiCo in the beverage industry, Adidas and Nike in sports apparel, or Starbucks and Dunkin' Donuts in the American coffee market.

Second, the literature on endogenous entry has obtained results for cost-reducing investments and demand-enhancing investments à la Etro. These investments do not have a direct impact on the competitive environment and instead only directly affect a firm's own profits. This has strong consequences for what kind of outcomes may emerge. In particular, it entails that a leader always deploys an aggressive-pricing strategy, so that investments act as a commitment device to reduce a firm's own price. In the particular case where industries compete primarily through product improvements, this implies that a leader always downgrades quality, since cutting prices is the only way to restrict entry.

On the contrary, our framework has incorporated the existence of multiple leaders and demand-enhancing investments that affect competition. This makes the analysis particularly suitable for industries where firms compete for the same set of customers through both prices and non-price features of goods. Our results indicate that a leader always restricts entry and, despite all leaders simultaneously behaving more aggressively, they do not inflict mutual damage upon each other. Consequently, each leader garners a greater profit.

Our model also rationalizes a disparity of strategies to increase competition, with radically different consequences for market outcomes. From an intuitive point of view, some of the possible strategies deployed are difficult to distinguish from an accommodating strategy. For instance, we have identified that a leader could restrict entry through quality improvements, while simultaneously charging such an extremely high price that it reduces its own quantities and revenues. A corollary of this is that knowing the features of the industry analyzed is as crucial as in models with barriers to entry: it is still the only way in which we can infer the consequences of a leader's more aggressive behavior.

Attending to the disparity of possible outcomes, we have also characterized conditions to identify outcomes through demand primitives. They identify whether a leader over-invests relative to a non-strategic benchmark, along with its effects on its price, revenue, and quantity. Applying these findings, we have characterized the outcomes emerging under two standard demands used in applied work: quality-variants of the Logit and CES. Under these demands, each leader over-invests in quality, sets higher prices, and increases its revenues. Moreover, a leader's quantity increases under the Logit, whereas this occurs for the CES when a leader's market share is not disproportionately large. Finally, the outcome is also welfare-improving, even when welfare is ambiguous in the general case.

References

Acemoglu, D. and M. K. Jensen (2013). Aggregate Comparative Statics. *Games and Economic Behavior 81*, 27–49.

Alfaro, M. (2020). On strategic investments by leader firms under endogenous entry and quantity competition. *Economics Bulletin 40*(4), 3231–3240.

Anderson, S. P., A. De Palma, and J.-F. Thisse (1992). *Discrete choice theory of product differentiation.* MIT press.

Anderson, S. P., N. Erkal, and D. Piccinin (2020). Aggregative games and oligopoly theory: short-run and long-run analysis. *The RAND Journal of Economics 51*(2), 470–495.

Baldwin, R. and J. Harrigan (2011). Zeros, Quality, and Space: Trade Theory and Trade Evidence. *American Economic Journal: Microeconomics 3*(2), 60–88.

Bertoletti, P. and F. Etro (2015). Monopolistic Competition When Income Matters. *The Economic Journal*.

Cornes, R. and R. Hartley (2012). Fully Aggregative Games. *Economics Letters 116*(3), 631–633.

Dixit, A. (1980). The role of investment in entry-deterrence. *The economic journal 90*(357), 95–106.

Etro, F. (2006). Aggressive Leaders. *The RAND Journal of Economics 37*(1), 146–154.

Etro, F. (2008). Stackelberg Competition with Endogenous Entry. *The Economic Journal 118*(532), 1670–1697.

Feenstra, R. C. and J. Romalis (2014). International Prices and Endogenous Quality. *The Quarterly Journal of Economics 129*(2), 477–527.

Fudenberg, D. and J. Tirole (1984). The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look. *American Economic Review 74*(2), 361–366.

Fudenberg, D. and J. Tirole (1991). *Game Theory.* MIT Press.

Gilbert, R. (1989). Mobility Barriers and the Value of Incumbency. In R. Schmalensee and R. Willig (Eds.), *Handbook of Industrial Organization* (1 ed.), Volume 1, Chapter 08, pp. 475–535. Elsevier.

Gilbert, R. and X. Vives (1986). Entry deterrence and the free rider problem. *The Review of Economic Studies 53*(1), 71–83.

Hottman, C., S. J. Redding, and D. E. Weinstein (2016). Quantifying the Sources of Firm Heterogeneity. *The Quarterly Journal of Economics*.

Jensen, M. K. (2018). Aggregative games. In *Handbook of Game Theory and Industrial Organization, Volume I*, Chapters, Chapter 4, pp. 66–92. Edward Elgar Publishing.

Julien, L. A. (2018). Stackelberg games. In *Handbook of Game Theory and Industrial Organization, Volume I.* Edward Elgar Publishing.

Kokovin, S., M. Parenti, J.-F. Thisse, and P. Ushchev (2017). On the dilution of market power.

Motta, M. and E. Tarantino (2017). The effect of horizontal mergers, when firms compete in prices and investments. *Working Paper Series 17.*

Nocke, V. and N. Schutz (2018). Multiproduct-firm oligopoly: An aggregative games approach. *Econometrica 86*(2), 523–557.

Polo, M. (2018). Entry games and free entry equilibria. In *Handbook of Game Theory and Industrial Organization, Volume I.* Edward Elgar Publishing.

Redding, S. J. and D. E. Weinstein (2020). Measuring aggregate price indices with taste shocks: Theory and evidence for ces preferences. *The Quarterly Journal of Economics 135*(1), 503–560.

Shapiro, C. (1989). Theories of Oligopoly Behavior. In R. Schmalensee and R. Willig (Eds.), *Handbook of Industrial Organization* (1 ed.), Volume 1, Chapter 06, pp. 329–414. Elsevier.

Spence, A. M. (1977). Entry, capacity, investment and oligopolistic pricing. *The Bell Journal of Economics*, 534–544.

Sutton, J. (1991). *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration*. MIT press.

Sutton, J. (1998). *Technology and Market Structure: Theory and History*. Theory and History. MIT Press.

Tesoriere, A. (2017). Stackelberg equilibrium with multiple firms and setup costs. *Journal of Mathematical Economics 73*, 86 – 102.

Tirole, J. (1988). *The Theory of Industrial Organization*. Cambridge, Mass.

Vives, X. (2001). *Oligopoly Pricing: Old Ideas and New Tools* (1 ed.), Volume 1. The MIT Press.

Online Appendix - not for publication

The structure of the appendices is as follows. In Appendix A we provide proofs for all the propositions included in the main body of the paper. In Appendix B, we present some examples of demand systems that can be expressed through an additively separable aggregator. In Appendix C, we formally prove a claim stated in the main body of the paper, which indicated that strategic complementarity and substitutability of prices is given by whether a greater aggregate increases or decreases the price elasticity of demand. In Appendix D, we show that all the results of this paper follow verbatim if we incorporate a subset of followers that are heterogeneous. Finally, in Appendix E we extend the results of our model to a scenario where investments require incurring sunk costs and, additionally, affect a firm's own marginal cost.

A Proofs

Throughout the proofs, for i's variable x we use the notation x_i^{sim} and x_i^{seq} for its value in the simultaneous- and sequential-move equilibrium, respectively. Also, we occasionally streamline notation by omitting arguments of functions when it is clear from the context.

A.1 Lemmas

Before proving the propositions of the main part of the paper, we begin by establishing some lemmas.

Lemma 1. $\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i} = \frac{\frac{\partial \ln \varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]}{\partial \ln z_i}}{1-\varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]-\frac{\partial \ln \varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]}{\partial \ln p_i}}$

Proof of Lemma 1. Define the markup of leader i by m_i, which is a given by function $m(\mathbf{x}_i, \mathbb{A}) := \frac{\varepsilon^p(\mathbf{x}_i,\mathbb{A})}{\varepsilon^p(\mathbf{x}_i,\mathbb{A})-1}$. Also, directly denote the price elasticity of i's demand by ε_i^p. The first-order condition for prices determines that $\ln p_i = \ln m(\mathbf{x}_i, \mathbb{A}) + \ln c_i$ and, by totally differentiating for a given $\mathbb{A}$,

$$\mathrm{d} \ln p_i = \frac{\partial \ln m_i}{\partial \ln \varepsilon_i^p} \frac{\partial \ln \varepsilon_i^p}{\partial \ln p_i} \mathrm{d} \ln p_i + \frac{\partial \ln m_i}{\partial \ln \varepsilon_i^p} \frac{\partial \ln \varepsilon_i^p}{\partial \ln z_i} \mathrm{d} \ln z_i.$$

Since $\frac{\partial \ln m_i}{\partial \ln \varepsilon_i^p} = 1 - m_i$ and working out the expression, we obtain that

$$\frac{\partial \ln p_i}{\partial \ln z_i} = \frac{(1-m_i) \frac{\partial \ln \varepsilon_i^p}{\partial \ln z_i}}{1-(1-m_i) \frac{\partial \ln \varepsilon_i^p}{\partial \ln p_i}}. \tag{A1}$$

By dividing numerator and denominator of the right-hand side in (A1) by $(1-m_i)$ and using that $(1-m_i)^{-1} = 1 - \varepsilon_i^p$, the result follows. ■

Lemma 2. $sgn\left\{\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i}\right\} = sgn\left\{-\frac{\partial \ln \varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]}{\partial \ln z_i}\right\}$.

Proof of Lemma 2. Leader i's gross optimal profits are given by

$$\pi_i [\mathbf{x}_i, \mathcal{A}(\mathbf{x}_i)] := q [\mathbf{x}_i, \mathcal{A}(\mathbf{x}_i)] (p_i - c_i) - f_z (z_i).$$

Leader i chooses prices by taking into account their effect on the aggregate. Thus, after some algebraic manipulation, i's marginal profits of prices are:

$$D_p \pi_i (\mathbf{x}_i, \mathbb{A}) =: \frac{\mathrm{d}\pi_i (\mathbf{x}_i, \mathbb{A})}{\mathrm{d}p_i} := \kappa_i \left[-\varepsilon^p (\mathbf{x}_i, \mathbb{A}) + \frac{p_i}{p_i - c_i}\right],$$

where κ_i is given by a function $\kappa(\mathbf{x}_i, \mathbb{A}) := \frac{q(\mathbf{x}_i,\mathbb{A})}{p_i}(p_i - c_i)$ and satisfies $\kappa_i > 0$ in the relevant range where $p_i - c_i > 0$.

Suppose the equilibrium aggregate $\mathbb{A}^*$, which is not affected by z_i in the sequential-move scenario. Then, the lemma is proved by showing how i's marginal profits of price are affected by its investments. Formally,

$$\frac{\partial D_p \pi_i(\mathbf{x}_i; \mathbb{A}^*)}{\partial z_i} = \frac{\partial \kappa(\mathbf{x}_i; \mathbb{A}^*)}{\partial z_i}\left[-\varepsilon^p(\mathbf{x}_i; \mathbb{A}^*) + \frac{p_i}{p_i - c_i}\right] - \kappa_i\left(\frac{\partial \varepsilon^p(\mathbf{x}_i; \mathbb{A}^*)}{\partial z_i}\right). \tag{A2}$$

For the range of optimal prices, the first-order condition of prices determines that $p_i = \frac{\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)}{\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)-1}c_i$, which implies that $\frac{p_i}{p_i-c_i} = \varepsilon^p(\mathbf{x}_i; \mathbb{A}^*)$. Thus, by restricting the domain of prices to their optimal values, then $-\varepsilon^p(\mathbf{x}_i; \mathbb{A}^*) + \frac{p_i}{p_i-c_i} = 0$, which implies that the first term on the right-hand side of (A2) is zero. Thus, since $\kappa_i > 0$,

$$\operatorname{sgn}\left(\frac{\partial D_p \pi_i(\mathbf{x}_i; \mathbb{A}^*)}{\partial z_i}\right) = \operatorname{sgn}\left(-\frac{\partial \varepsilon^p(\mathbf{x}_i; \mathbb{A}^*)}{\partial z_i}\right), \tag{A3}$$

and the result follows. ■

Lemma 3.

$$sgn\left\{-\left[\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i} + \frac{\partial \ln h(p_i^*, z_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(p_i^*, z_i)}{\partial \ln p_i}\right)^{-1}\right]\right\} = sgn\left\{\frac{\mathrm{d}\ln h[p_i(z_i;\mathbb{A}), z_i]}{\mathrm{d}\ln z_i}\right\}, \tag{A4}$$

$$sgn\left\{\frac{\mathrm{d}\ln \mathcal{A}\left[p_i(z_i;\mathbb{A}), z_i; \mathbf{z}_{-i}^{\mathscr{L}}, \mathbb{A}\right]}{\mathrm{d}\ln z_i}\right\} = sgn\left\{\frac{\mathrm{d}\ln h[p_i(z_i;\mathbb{A}), z_i]}{\mathrm{d}\ln z_i}\right\}, \tag{A5}$$

where $p_i^* := p_i(z_i, \mathbb{A})$.

Proof of Lemma 3. By definition, $\frac{\mathrm{d}\ln h[p_i(z_i;\mathbb{A}),z_i]}{\mathrm{d}\ln z_i} > 0$ iff $\frac{\partial \ln h(p_i^*,z_i)}{\partial \ln p_i}\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i} + \frac{\partial \ln h(p_i^*,z_i)}{\partial \ln z_i} > 0$. Besides, since $\frac{\partial \ln h(p_i^*,z_i)}{\partial \ln p_i} < 0$, this holds iff $\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i} < -\frac{\partial \ln h(p_i^*,z_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(p_i^*,z_i)}{\partial \ln p_i}\right)^{-1}$. This proves (A4).

As for (A5), by definition,

$$\frac{\mathrm{d}\ln \mathcal{A}\left[p_i(z_i;\mathbb{A}), z_i; \mathbf{z}_{-i}^{\mathscr{L}}, \mathbb{A}\right]}{\mathrm{d}\ln z_i} = \frac{\partial \ln \mathcal{A}\left(p_i^*, z_i; \mathbf{z}_{-i}^{\mathscr{L}}, \mathbb{A}\right)}{\partial \ln z_i} + \frac{\partial \ln \mathcal{A}\left(p_i^*, z_i; \mathbf{z}_{-i}^{\mathscr{L}}, \mathbb{A}\right)}{\partial \ln p_i}\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i},$$

and, since $\frac{\partial \ln \mathcal{A}(\mathbf{x})}{\partial \ln z_i} = \frac{H'[H^{-1}(\mathbb{A})]}{\mathbb{A}}\frac{\partial h(\mathbf{x}_i)}{\partial \ln z_i}$ and $\frac{\partial \ln \mathcal{A}(\mathbf{x})}{\partial \ln p_i} = \frac{H'[H^{-1}(\mathbb{A})]}{\mathbb{A}}\frac{\partial h(\mathbf{x}_i)}{\partial \ln p_i}$, then

$$\frac{\mathrm{d}\ln \mathcal{A}\left[p_i(z_i;\mathbb{A}), z_i; \mathbf{z}_{-i}^{\mathscr{L}}, \mathbb{A}\right]}{\mathrm{d}\ln z_i} = \frac{H'\left[H^{-1}(\mathbb{A})\right]}{\mathbb{A}}\left[\frac{\mathrm{d}\ln h[p_i(z_i;\mathbb{A}), z_i]}{\mathrm{d}\ln z_i}\right].$$

Therefore, by using that $H' > 0$, the result follows. ■

Lemma 4. *Let* $p_i^{sim} := p_i\left(z_i^{sim}; \mathbb{A}^*\right)$ *where* $\mathbb{A}^*$ *is the equilibrium aggregate in both scenarios. Then:*

Case i) *If* $\frac{\partial \ln p_i\left(z_i^{sim};\mathbb{A}^*\right)}{\partial \ln z_i} > -\frac{\partial \ln h\left(p_i^{sim},z_i^{sim}\right)}{\partial \ln z_i}\left(\frac{\partial \ln h\left(p_i^{sim},z_i^{sim}\right)}{\partial \ln p_i}\right)^{-1}$ *then* $z_i^{sim} > z_i^{seq}$,

Case ii) *If* $\frac{\partial \ln p_i\left(z_i^{sim};\mathbb{A}^*\right)}{\partial \ln z_i} < -\frac{\partial \ln h\left(p_i^{sim},z_i^{sim}\right)}{\partial \ln z_i}\left(\frac{\partial \ln h\left(p_i^{sim},z_i^{sim}\right)}{\partial \ln p_i}\right)^{-1}$ *then* $z_i^{seq} > z_i^{sim}$.

Proof of Lemma 4. Consider leader i. The marginal profits of its investments in the simultaneous- and sequential-move case are respectively given by

$$\gamma_i^{\text{sim}}(\mathbf{x}_i; \mathbb{A}) := \frac{\partial \pi_i(\mathbf{x}_i, \mathbb{A})}{\partial z_i} + \frac{\partial \pi_i(\mathbf{x}_i, \mathbb{A})}{\partial \mathbb{A}}\frac{\partial \mathcal{A}(\mathbf{x})}{\partial z_i}, \tag{A6}$$

$$\gamma_i^{\text{seq}}(\mathbf{x}_i; \mathbb{A}) := \frac{\partial \pi_i(\mathbf{x}_i, \mathbb{A})}{\partial z_i} + \frac{\partial \pi_i(\mathbf{x}_i, \mathbb{A})}{\partial p_i}\frac{\partial p_i(z_i; \mathbb{A})}{\partial z_i}. \tag{A7}$$

Independently of whether we consider the simultaneous- or sequential-move case, optimal prices are characterized by (4). This implies that $\frac{\partial \pi_i(\mathbf{x}_i,\mathbb{A})}{\partial p_i} = -\frac{\partial \pi_i(\mathbf{x}_i,\mathbb{A})}{\partial \mathbb{A}}\frac{\partial \mathcal{A}(\mathbf{x}_i,\mathbb{A})}{\partial p_i}$ and so we can reexpress (A7) as

$$\gamma_i^{\text{seq}}(\mathbf{x}_i;\mathbb{A}) := \frac{\partial \pi_i(\mathbf{x}_i,\mathbb{A})}{\partial z_i} - \frac{\partial \pi_i(\mathbf{x}_i,\mathbb{A})}{\partial \mathbb{A}}\frac{\partial \mathcal{A}(\mathbf{x}_i,\mathbb{A})}{\partial p_i}\frac{\partial p_i(z_i;\mathbb{A})}{\partial z_i}. \tag{A8}$$

Let

$$\Delta_i(z_i;\mathbb{A}^*) := \gamma_i^{\text{seq}}[p_i(z_i;\mathbb{A}^*),z_i;\mathbb{A}^*] - \gamma_i^{\text{sim}}[p_i(z_i;\mathbb{A}^*),z_i;\mathbb{A}^*]. \tag{A9}$$

Then, using (A6) and (A8),

$$\Delta_i(z_i;\mathbb{A}^*) = -\frac{\partial \pi_i(p_i^*,z_i;\mathbb{A}^*)}{\partial \mathbb{A}}\left[\frac{\partial \mathcal{A}(p_i^*,z_i;\mathbf{z}_{-i}^{\mathscr{L}},\mathbb{A}^*)}{\partial p_i}\frac{\partial p_i(z_i;\mathbb{A}^*)}{\partial z_i} + \frac{\partial \mathcal{A}(p_i^*,z_i;\mathbf{z}_{-i}^{\mathscr{L}},\mathbb{A}^*)}{\partial z_i}\right],$$

where $p_i^* := p_i(z_i;\mathbb{A}^*)$. Since $\frac{\partial \pi_i(\cdot)}{\partial \mathbb{A}} < 0$, this determines that, for each $(z_i,\mathbb{A}^*)$,

$$\text{sgn}\{\Delta_i(z_i;\mathbb{A}^*)\} = \text{sgn}\left\{\frac{\mathrm{d}\mathcal{A}\left[p_i(z_i;\mathbb{A}^*),z_i;\mathbf{z}_{-i}^{\mathscr{L}},\mathbb{A}^*\right]}{\mathrm{d}z_i}\right\} = \text{sgn}\left\{\frac{\mathrm{d}h[p_i(z_i;\mathbb{A}^*),z_i]}{\mathrm{d}z_i}\right\}, \tag{A10}$$

where the second equality follows by Lemma 3.

Define $\Delta_i^{\text{sim}} := \Delta_i(z_i^{\text{sim}};\mathbb{A}^*)$. Leader i's profits evaluated at optimal prices are a function of z_i solely. Moreover, they are strictly quasi-concave, so that γ_i^{sim} and γ_i^{seq} are single peaked. Thus, if $\Delta_i^{\text{sim}} > 0$ then leader i over-invests, while if $\Delta_i^{\text{sim}} < 0$ then i under-invests.

We consider the two cases stated in the information of the lemma separately. Case i) establishes that $\frac{\partial \ln p_i(z_i^{\text{sim}};\mathbb{A})}{\partial \ln z_i} > -\frac{\partial \ln h(p_i^{\text{sim}},z_i^{\text{sim}})}{\partial \ln z_i}\left(\frac{\partial \ln h(p_i^{\text{sim}},z_i^{\text{sim}})}{\partial \ln p_i}\right)^{-1}$, which can be rearranged as $\frac{\mathrm{d}\ln h[p_i(z_i^{\text{sim}};\mathbb{A}^*),z_i^{\text{sim}}]}{\mathrm{d}\ln z_i} < 0$. By Lemma 3 and (A10), this implies that $\Delta_i^{\text{sim}} < 0$ and so $z_i^{\text{sim}} > z_i^{\text{seq}}$.

Likewise, Case ii) establishes that $\frac{\mathrm{d}\ln h[p_i(z_i^{\text{sim}};\mathbb{A}^*),z_i^{\text{sim}}]}{\mathrm{d}\ln z_i} > 0$ and, by Lemma 3 and (A10), then $\Delta_i^{\text{sim}} > 0$ and so $z_i^{\text{seq}} > z_i^{\text{sim}}$. ■

A.2 Proofs of Propositions

Proof of Proposition 5.1. Regarding profits, the same $\mathbb{A}^*$ holds under both scenarios. Besides, each leader chooses investments from the same choice set in each scenario. Thus, by a revealed-preference argument, profits are greater in the sequential-move case. This follows because the leader can always have at least the profits of the simultaneous-move scenario by choosing the investments of that case.

Next, we prove that leader i strengthens competition through its choice of investments. Formally, this means that $\Delta h_i > 0$ where

$$\Delta h_i := h[p_i(z_i^{\text{seq}};\mathbb{A}^*),z_i^{\text{seq}}] - h\left[p_i(z_i^{\text{sim}};\mathbb{A}^*),z_i^{\text{sim}}\right]. \tag{A11}$$

Thus, $\Delta h_i = \int_{z_i^{\text{sim}}}^{z_i^{\text{seq}}} \Delta_i(z;\mathbb{A}^*)\,\mathrm{d}z$, where we have used (A10).[16] Next, we consider the two cases in Lemma 4 separately.

Case i) is such that $\frac{\partial \ln p_i(z_i^{\text{sim}};\mathbb{A}^*)}{\partial \ln z_i} > -\frac{\partial \ln h(p_i^{\text{sim}},z_i^{\text{sim}})}{\partial \ln z_i}\left(\frac{\partial \ln h(p_i^{\text{sim}},z_i^{\text{sim}})}{\partial \ln p_i}\right)^{-1}$, where $p_i^{\text{sim}} :=$

[16]Throughout the proofs, we utilize that the functions are smooth and variables belong to a real compact interval, so that we can always apply the Fundamental Theorem of Calculus.

$p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)$. For this case, we have already proved that $\Delta_i^{\text{sim}}<0$ and, so, $z_i^{\text{sim}}>z_i^{\text{seq}}$. Next, we show that $\Delta_i(z)<0$ for any $z\in\left(z_i^{\text{seq}},z_i^{\text{sim}}\right)$. Using the definitions given by (A6) and (A8), we know that $\gamma_i^{\text{sim}}\left(z_i^{\text{sim}},\mathbb{A}^*\right)=0$, $\gamma_i^{\text{seq}}\left(z_i^{\text{seq}},\mathbb{A}^*\right)=0$, and $\gamma_i^{\text{seq}}\left(z_i^{\text{sim}},\mathbb{A}^*\right)<0$. Additionally, by the strict quasi-concavity of profits evaluated at optimal prices and taking $z_i^{\text{sim}}>z>z_i^{\text{seq}}$, then $\gamma_i^{\text{seq}}(z;\mathbb{A}^*)<0$ and $\gamma_i^{\text{sim}}(z;\mathbb{A}^*)>0$. This implies that $\Delta_i(z;\mathbb{A}^*)<0$ for any $z\in\left(z_i^{\text{seq}},z_i^{\text{sim}}\right)$. Thus,

$$\Delta h_i=\int_{z_i^{\text{sim}}}^{z_i^{\text{seq}}}\Delta_i(z;\mathbb{A}^*)\,\mathrm{d}z=-\int_{z_i^{\text{seq}}}^{z_i^{\text{sim}}}\Delta_i(z;\mathbb{A}^*)\,\mathrm{d}z>0.$$

Case ii) indicates that $\frac{\partial\ln p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)}{\partial\ln z_i}<-\frac{\partial\ln h\left(p_i^{\text{sim}},z_i^{\text{sim}}\right)}{\partial\ln z_i}\left(\frac{\partial\ln h\left(p_i^{\text{sim}},z_i^{\text{sim}}\right)}{\partial\ln p_i}\right)^{-1}$, where $p_i^{\text{sim}}:=p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)$. For this case, we have already shown that $\Delta_i^{\text{sim}}>0$ and, so, $z_i^{\text{seq}}>z_i^{\text{sim}}$. Moreover, using the definitions given by (A6) and (A8), then $\gamma_i^{\text{sim}}\left(z_i^{\text{sim}},\mathbb{A}^*\right)=0$, $\gamma_i^{\text{seq}}\left(z_i^{\text{seq}},\mathbb{A}^*\right)=0$, and $\gamma_i^{\text{seq}}\left(z_i^{\text{sim}},\mathbb{A}^*\right)>0$. In addition, by the strict quasi-concavity of profits evaluated at optimal prices and taking z such that $z_i^{\text{seq}}>z>z_i^{\text{sim}}$, then $\gamma_i^{\text{seq}}(z;\mathbb{A}^*)>0$ and $\gamma_i^{\text{sim}}(z;\mathbb{A}^*)<0$. This implies that $\Delta_i(z)>0$ for any $z\in\left(z_i^{\text{sim}},z_i^{\text{seq}}\right)$, so that

$$\Delta h_i=\int_{z_i^{\text{sim}}}^{z_i^{\text{seq}}}\Delta_i(z;\mathbb{A}^*)\,\mathrm{d}z>0.$$

Therefore, irrespective of the case considered, $\Delta h_i>0$ and the result follows.

As for the number of followers, let M^{sim} and M^{seq} be the solution to (5) and (6) for a given $\mathbb{A}^*$, respectively. Thus, (NE-sim) and (NE-seq) imply

$$M^{\text{sim}}h\left[\mathbf{x}_{\mathcal{F}}(\mathbb{A}^*)\right]+\sum_{i\in\mathscr{L}}h\left[p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right),z_i^{\text{sim}}\right]=M^{\text{seq}}h\left[\mathbf{x}_{\mathcal{F}}(\mathbb{A}^*)\right]+\sum_{i\in\mathscr{L}}h\left[p_i\left(z_i^{\text{seq}};\mathbb{A}^*\right),z_i^{\text{seq}}\right],$$

so that

$$M^{\text{seq}}-M^{\text{sim}}=-\frac{\sum_{i\in\mathscr{L}}\Delta h_i}{h\left[\mathbf{x}_{\mathcal{F}}(\mathbb{A}^*)\right]}.\tag{A12}$$

Since we have shown that $\Delta h_i>0$ for any $i\in\mathscr{L}$ and $h>0$, the result follows. ■

Proof of Proposition 5.2. It follows trivially by Lemma 4 once it is noticed that (8) corresponds to Case ii) of that lemma, while the case with reversed inequality in (8) corresponds to Case i). ■

Proof of Proposition 6.1. Regarding investments, Assumption 6.1 implies two possibilities. First, if $\frac{\partial\ln\varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial\ln z_i}>0$, then $\frac{\partial\ln p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)}{\partial\ln z_i}<0$ by Lemma 2. This is encompassed by Case ii) in Lemma 4, and so $z_i^{\text{seq}}>z_i^{\text{sim}}$. Second, suppose that $\frac{\frac{\partial\ln\varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial\ln z_i}}{1-\varepsilon^p(\mathbf{x}_i,\mathbb{A})-\frac{\partial\ln\varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial\ln p_i}}<-\frac{\partial\ln h(\mathbf{x}_i)}{\partial\ln z_i}\left(\frac{\partial\ln h(\mathbf{x}_i)}{\partial\ln p_i}\right)^{-1}$ for any $(\mathbf{x}_i;\mathbb{A})$. Thus, by using Lemma 1, then $\frac{\partial\ln p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)}{\partial\ln z_i}<-\frac{\partial\ln h\left(p_i^{\text{sim}},z_i^{\text{sim}}\right)}{\partial\ln z_i}\left(\frac{\partial\ln h\left(p_i^{\text{sim}},z_i^{\text{sim}}\right)}{\partial\ln p_i}\right)^{-1}$, where $p_i^{\text{sim}}:=p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)$. This is also encompassed by Case ii) in Lemma 4, and hence $z_i^{\text{seq}}>z_i^{\text{sim}}$.

As for prices, $p_i^{\text{seq}}-p_i^{\text{sim}}=\int_{z_i^{\text{sim}}}^{z_i^{\text{seq}}}\frac{\mathrm{d}p_i(z;\mathbb{A}^*)}{\mathrm{d}z}\mathrm{d}z$. Also, by (A3), then $\operatorname{sgn}\left(\frac{\partial p_i(z_i;\mathbb{A}^*)}{\partial z_i}\right)=\operatorname{sgn}\left(-\frac{\partial\varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}^*]}{\partial z_i}\right)$. Thus, if $\frac{\partial\varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i}<0$ for any $(\mathbf{x}_i,\mathbb{A})$ then $p_i^{\text{seq}}>p_i^{\text{sim}}$, while if $\frac{\partial\varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i}>0$ for any $(\mathbf{x}_i,\mathbb{A})$ then $p_i^{\text{seq}}<p_i^{\text{sim}}$. ■

Proof of Proposition 6.2. Since we are supposing that Assumption 6.1 holds, all the results in Proposition 6.1 follow. In particular, $z_i^{\text{seq}}>z_i^{\text{sim}}$. Define i's revenue as $R_i:=p_iq_i$. Its optimal prices in each scenario are a function $p_i(z_i,\mathbb{A})$, which implies that quantities are also a function of the same variables. Thus, given the equilibrium aggregate $\mathbb{A}^*$, the revenue of leader i as a function of its investment is

$$R_i\left(z_i;\mathbb{A}^*\right):=q\left[p_i\left(z_i;\mathbb{A}^*\right),z_i;\mathbb{A}^*\right]p_i\left(z_i;\mathbb{A}^*\right),$$

where the equilibrium revenues in each scenario are $R_i^{\text{sim}}:=R_i\left(z_i^{\text{sim}},\mathbb{A}^*\right)$ and $R_i^{\text{seq}}:=R_i\left(z_i^{\text{seq}},\mathbb{A}^*\right)$. Therefore,

$$R_i\left(z_i^{\text{seq}};\mathbb{A}^*\right)-R_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)=\int_{z_i^{\text{sim}}}^{z_i^{\text{seq}}}\frac{\mathrm{d}R_i\left(z;\mathbb{A}^*\right)}{\mathrm{d}z}\mathrm{d}z.$$

If we show that $\frac{\mathrm{d}R_i(z;\mathbb{A}^*)}{\mathrm{d}z}>0$ for $z\in\left(z_i^{\text{sim}},z_i^{\text{seq}}\right)$, then the result follows. The effect of investments on revenues for a given $\mathbb{A}^*$ are

$$\begin{aligned}\frac{\mathrm{d}\ln R_i\left(z_i;\mathbb{A}^*\right)}{\mathrm{d}\ln z_i}&=\frac{\partial\ln q\left(p_i^*,z_i;\mathbb{A}^*\right)}{\partial\ln z_i}+\frac{\partial\ln q\left(p_i^*,z_i;\mathbb{A}^*\right)}{\partial\ln p_i}\frac{\partial\ln p_i\left(z_i;\mathbb{A}^*\right)}{\partial\ln z_i}+\frac{\partial\ln p_i\left(z_i;\mathbb{A}^*\right)}{\partial\ln z_i},\\&=\xi^z\left(p_i^*,z_i;\mathbb{A}^*\right)-\left[\xi^p\left(p_i^*,z_i;\mathbb{A}^*\right)-1\right]\frac{\partial\ln p_i\left(z_i;\mathbb{A}^*\right)}{\partial\ln z_i},\end{aligned}$$

where $p_i^*:=p_i\left(z_i;\mathbb{A}^*\right)$. Therefore, working out the expression, $\frac{\mathrm{d}\ln R_i(z_i;\mathbb{A}^*)}{\mathrm{d}\ln z_i}>0$ iff $\frac{\partial\ln p_i(z_i;\mathbb{A}^*)}{\partial\ln z_i}<\frac{\xi^z(p_i^*,z_i;\mathbb{A}^*)}{\xi^p(p_i^*,z_i;\mathbb{A}^*)-1}$. If $\frac{\partial\varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i}<0$ and (11) holds, then the latter inequality holds for any $(\mathbf{x}_i,\mathbb{A})$, so that $R_i^{\text{seq}}>R_i^{\text{sim}}$. This inequality can also be used to show that if (11) with reversed inequality holds, then $R_i^{\text{seq}}<R_i^{\text{sim}}$. Finally, if $\frac{\partial\varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i}>0$ then $\frac{\partial\ln p_i(z_i;\mathbb{A}^*)}{\partial\ln z_i}<0$, and moreover $\xi^p\left(p_i^*,z_i;\mathbb{A}^*\right)>1$ in any interior equilibrium, so that $R_i^{\text{seq}}>R_i^{\text{sim}}$.

Next, we prove the claim made after the proposition. This indicates that, if each leader increases its revenue, there is a reallocation of expenditure-based market share from followers towards leaders. Denote by $R_{\mathcal{F}}$ the total revenue of followers as a group, which in equilibrium is

$$R_{\mathcal{F}}\left(\mathbb{A}^*,M\right):=Mq\left[\mathbf{x}_{\mathcal{F}}\left(\mathbb{A}^*\right);\mathbb{A}^*\right]p_{\mathcal{F}}\left(\mathbb{A}^*\right).$$

Since $\mathbb{A}^*$ is the same for both the simultaneous- and sequential-move scenarios, neither $q\left[\mathbf{x}_{\mathcal{F}}\left(\mathbb{A}^*\right);\mathbb{A}^*\right]$ nor $p_{\mathcal{F}}\left(\mathbb{A}^*\right)$ vary. Thus, $R_{\mathcal{F}}$ is only impacted by variations in M. Since $M^{\text{seq}}<M^{\text{sim}}$ by Proposition 5.1, then $R_{\mathcal{F}}$ becomes lower in the sequential-move scenario. Thus, given that additionally $R_i^{\text{seq}}>R_i^{\text{sim}}$ for each $i\in\mathscr{L}$, the result follows. ■

Proof of Proposition 6.3. Given that Assumption 6.1 holds, then $z_i^{\text{seq}}>z_i^{\text{sim}}$. We proceed in a similar fashion to the proof for revenues. Let $q_i\left(z_i;\mathbb{A}^*\right):=q\left[p_i\left(z_i;\mathbb{A}^*\right),z_i;\mathbb{A}^*\right]$, and so

$$q_i\left(z_i^{\text{seq}};\mathbb{A}^*\right)-q_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)=\int_{z_i^{\text{sim}}}^{z_i^{\text{seq}}}\frac{\mathrm{d}q_i\left(z;\mathbb{A}^*\right)}{\mathrm{d}z}\mathrm{d}z.\tag{A13}$$

Furthermore,

$$\begin{aligned}\frac{\mathrm{d}\ln q_i\left(z_i;\mathbb{A}^*\right)}{\mathrm{d}\ln z_i}&=\frac{\partial\ln q\left(p_i^*,z_i;\mathbb{A}^*\right)}{\partial\ln z_i}+\frac{\partial\ln q\left(p_i^*,z_i;\mathbb{A}^*\right)}{\partial\ln p_i}\frac{\partial\ln p_i\left(z_i;\mathbb{A}^*\right)}{\partial\ln z_i},\\&=\xi^z\left(p_i^*,z_i;\mathbb{A}^*\right)-\xi^p\left(p_i^*,z_i;\mathbb{A}^*\right)\frac{\partial\ln p_i\left(z_i;\mathbb{A}^*\right)}{\partial\ln z_i},\end{aligned}$$

where $p_i^*:=p_i\left(z_i;\mathbb{A}^*\right)$. Therefore, $\frac{\mathrm{d}\ln q_i(z_i;\mathbb{A}^*)}{\mathrm{d}\ln z_i}>0$ iff $\frac{\partial\ln p_i(z_i;\mathbb{A}^*)}{\partial\ln z_i}<\frac{\xi^z(p_i^*,z_i;\mathbb{A}^*)}{\xi^p(p_i^*,z_i;\mathbb{A}^*)}$.

In case $\frac{\partial\varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i}<0$ and that Assumption 6.3 holds, then the inequality holds for any $(\mathbf{x}_i,\mathbb{A})$, so that $q_i^{\text{seq}} > q_i^{\text{sim}}$. If, instead, $\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i} < 0$ and Assumption 6.2 holds but with reverse inequality of (12), then $q_i^{\text{seq}} < q_i^{\text{sim}}$. Finally, if $\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i} > 0$ then $\frac{\partial \ln p_i(z_i;\mathbb{A}^*)}{\partial \ln z_i} < 0$, and so $q_i^{\text{seq}} > q_i^{\text{sim}}$. ■

A.3 Under-Investment and Aggressive Pricing

In the main body of the paper, we have indicated that the under-investment case entails aggressive pricing as in Etro (2006). Here, we prove this formally.

As in the over-investment case, the condition for leader i to under-invest can also be expressed in terms of demand primitives. This requires that

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1-\varepsilon^p(\mathbf{x}_i,\mathbb{A})-\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} > -\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i}\right)^{-1} \tag{A14}$$

holds for any $(\mathbf{x}_i,\mathbb{A})$. Making use of this, we determine the following.

Proposition A.1. *Suppose that (A14) holds for any $(\mathbf{x}_i,\mathbb{A})$. Then, leader i under-invests and engages in aggressive pricing.*

Proof of Proposition A.1. Regarding investments, we know by Proposition 5.2 that there is under-investment when $\frac{\partial \ln p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)}{\partial \ln z_i} > -\frac{\partial \ln h\left(p_i^{\text{sim}},z_i^{\text{sim}}\right)}{\partial \ln z_i}\left(\frac{\partial \ln h\left(p_i^{\text{sim}},z_i^{\text{sim}}\right)}{\partial \ln p_i}\right)^{-1}$, where $p_i^{\text{sim}} := p_i\left(z_i^{\text{sim}};\mathbb{A}^*\right)$. Moreover, we can apply Lemma 1 to show that (A14) makes that inequality hold. Therefore, $z_i^{\text{seq}} < z_i^{\text{sim}}$.

As for prices, the RHS of (A14) is positive, and so the LHS has to be positive too. By Lemmas 1 and 2, this implies that $\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i} > 0$ and $\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i} < 0$. The fact that i engages in aggressive pricing follows because

$$p_i^{\text{seq}} - p_i^{\text{sim}} = \int_{z_i^{\text{sim}}}^{z_i^{\text{seq}}} \frac{\mathrm{d}p_i(z;\mathbb{A}^*)}{\mathrm{d}z}\mathrm{d}z = -\int_{z_i^{\text{seq}}}^{z_i^{\text{sim}}} \frac{\mathrm{d}p_i(z;\mathbb{A}^*)}{\mathrm{d}z}\mathrm{d}z < 0,$$

and so $p_i^{\text{seq}} < p_i^{\text{sim}}$. ■

A.4 Quality-Augmented CES and Logit

We proceed to prove Proposition 7.1. Since the proof for each demand requires several steps, the proofs are stated as subsections. For the CES, we also show the procedure for the graphs included in Section 7.

A.4.1 Proof for the CES Demand

Demand (1) satisfies that $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} < 0$. Thus, in order to prove Assumption 6.1 and 6.2, it is necessary to show that

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1-\varepsilon^p(\mathbf{x}_i,\mathbb{A})-\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} < \min\left\{-\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i}\right)^{-1}, \frac{\xi^z(\mathbf{x}_i,\mathbb{A})}{\xi^p(\mathbf{x}_i,\mathbb{A})-1}\right\}. \tag{A15}$$

We exploit the property of the CES that elasticities can be expressed in terms of market shares. Formally, the market share of leader i can be expressed by

$$s_i := \frac{p_i q_i}{E} = \frac{(p_i/z_i)^{1-\sigma}}{\mathbb{A}},$$

which determines that the price elasticity of i's demand is given by $\varepsilon_i^p := \sigma - s_i(\sigma - 1)$. Also, the elasticities ignoring the impact of a leader's variable on aggregate conditions are given by $\xi_i^p := \sigma$ and $\xi_i^z := \sigma - 1$. Using that $\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i} = -\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i} = 1 - \sigma$, all these results establish that

$$\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i}\right)^{-1} = \frac{\xi_i^z}{\xi_i^p - 1} = 1.$$

Thus, Assumption 6.2 holds if

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1 - \varepsilon^p(\mathbf{x}_i, \mathbb{A}) - \frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} < 1.$$

Performing the pertinent calculations, $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} = \frac{\partial \ln \varepsilon^p(s_i)}{\partial \ln s_i}\frac{\partial \ln s_i}{\partial \ln z_i}$ and $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i} = \frac{\partial \ln \varepsilon^p(s_i)}{\partial \ln s_i}\frac{\partial \ln s_i}{\partial \ln p_i}$. Since $\frac{\partial \ln \varepsilon^p(s_i)}{\partial \ln s_i} = \frac{s_i(1-\sigma)}{\varepsilon_i^p}$, $\frac{\partial \ln s_i}{\partial \ln z_i} = -\frac{\partial \ln s_i}{\partial \ln p_i} = \sigma - 1$, and $\varepsilon_i^p - 1 = (\sigma - 1)(1 - s_i)$, it can be shown that

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1 - \varepsilon^p(\mathbf{x}_i, \mathbb{A}) - \frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} = \frac{\sigma - \varepsilon_i^p}{\sigma - s_i\varepsilon_i^p}, \tag{A16}$$

which is always lower than 1. Therefore, Assumption 6.2 holds.

As for prices, $\frac{\partial \varepsilon^p(s_i)}{\partial \ln s_i}\frac{\partial \ln s_i}{\partial \ln z_i} = -s_i(\sigma - 1)^2 < 0$, which implies that $\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial z_i} < 0$. Thus, each leader i sets a higher price in the sequential-move equilibrium due to Lemma 2.

As for quantities, next, we show that Assumption 6.3 holds when i's market share is not disproportionately large. Specifically, since

$$\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i}\right)^{-1} = \frac{\xi_i^z}{\xi_i^p - 1} < \frac{\xi_i^z}{\xi_i^p} = \frac{\sigma - 1}{\sigma},$$

this requires us to show that

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1 - \varepsilon^p(\mathbf{x}_i, \mathbb{A}) - \frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} = \frac{\sigma - \varepsilon_i^p}{\sigma - s_i\varepsilon_i^p} < \frac{\sigma - 1}{\sigma},$$

where we have made used of (A16). Working out the expression, the condition for this inequality to hold is

$$s_i\sigma < \sigma - s_i\varepsilon_i^p, \tag{A17}$$

which defines a quadratic function of σ that holds for low values of s_i and is violated for large values. For instance, for $\sigma = 3$, condition (A17) holds for any market share lower than 60% in the simultaneous-move equilibrium. Moreover, the greater σ, the greater the threshold of market share is.

Graphical Illustrations

In this part, we indicate the procedure used for the graphical illustrations presented in Section 7 under a CES demand. The example assumes investments costs given by $f_i^z(z_i) := f^z(z_i)^\beta$.

Denote i's market shares in each equilibrium by s_i^{sim} and s_i^{seq}, with same superscripts for optimal prices and investments. We have established that $s(\mathbf{x}_i, \mathbb{A}) := \frac{(p_i/z_i)^{1-\sigma}}{\mathbb{A}}$. Moreover, since $\mathbb{A}^*$ is the same under both scenarios, we can divide the market-share functions in each equilibrium and determine that

$$\frac{s_i^{\text{seq}}}{s_i^{\text{sim}}} = \frac{\left(p_i^{\text{seq}}/z_i^{\text{seq}}\right)^{1-\sigma}}{\left(p_i^{\text{sim}}/z_i^{\text{sim}}\right)^{1-\sigma}} \tag{A18}$$

$$= \frac{\left(p_i^{\text{seq}}/p_i^{\text{sim}}\right)^{1-\sigma}}{\left(z_i^{\text{seq}}/z_i^{\text{sim}}\right)^{1-\sigma}}. \tag{A19}$$

In turn, optimal investments in each scenario can be expressed by

$$z_i^{\text{sim}} = \left[\frac{(\sigma-1)}{\beta}\frac{Es_i^{\text{sim}}\left(1-s_i^{\text{sim}}\right)}{\varepsilon^p\left(s_i^{\text{sim}}\right)f^z}\right]^{\frac{1}{\beta}},$$

$$z_i^{\text{seq}} = \left[\frac{(\sigma-1)}{\beta}\frac{Es_i^{\text{seq}}\left(1-s_i^{\text{seq}}\right)}{\varepsilon^p\left(s_i^{\text{seq}}\right)f^z}\frac{\sigma}{\sigma-s_i^{\text{seq}}\varepsilon^p\left(s_i^{\text{seq}}\right)}\right]^{\frac{1}{\beta}},$$

which establishes that

$$\frac{z_i^{\text{seq}}}{z_i^{\text{sim}}} = \left[\frac{s_i^{\text{seq}}\left(1-s_i^{\text{seq}}\right)}{s_i^{\text{sim}}\left(1-s_i^{\text{sim}}\right)}\frac{\varepsilon^p\left(s_i^{\text{sim}}\right)}{\varepsilon^p\left(s_i^{\text{seq}}\right)}\frac{\sigma}{\sigma-s_i^{\text{seq}}\varepsilon^p\left(s_i^{\text{seq}}\right)}\right]^{1/\beta}. \tag{A20}$$

Moreover, the quotient of optimal prices in each equilibrium is

$$\frac{p_i^{\text{seq}}}{p_i^{\text{sim}}} = \frac{\varepsilon^p\left(s_i^{\text{seq}}\right)/\left(\varepsilon^p\left(s_i^{\text{seq}}\right)-1\right)}{\varepsilon^p\left(s_i^{\text{sim}}\right)/\left(\varepsilon^p\left(s_i^{\text{sim}}\right)-1\right)}. \tag{A21}$$

Substituting (A20) and (A21) into (A18), we derive an equation that is a function of $\left(s_i^{\text{sim}}, s_i^{\text{seq}}\right)$ for given parameters $\left(\sigma, \frac{\sigma-1}{\beta}\right)$. Supposing some values for these parameters, we can recover s_i^{seq} for a given value of s_i^{sim}.

Furthermore, once we know s_i^{sim} and s_i^{seq}, we can retrieve the increase in prices through (A21). As for investments, outlays are given by $f_i^z(z_i)$, whose variation between scenarios is $\left(z_i^{\text{seq}}/z_i^{\text{sim}}\right)^{\beta}$. Thus, given s_i^{sim} and s_i^{seq}, we can make use of (A20) to compute their increase.

A.4.2 Proof for the Multinomial Logit Demand

This demand is given by (2). The price elasticity of leader i's demand is $\varepsilon_i^p = \frac{p_i}{\alpha}(1-q_i)$, which implies that $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} = \frac{-q(\mathbf{x}_i,\mathbb{A})}{1-q(\mathbf{x}_i,\mathbb{A})}\frac{z_i}{\alpha} < 0$. Moreover, since the demand is expressed in terms of a unit measure of agents, then $q_i < 1$. Thus, if we show that i over-invests, then it charges higher prices in the sequential-move equilibrium.

Next, we prove that Assumptions 6.1 and 6.3 hold, which in turn implies that Assumption 6.2 is satisfied. Formally, this requires proving that

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1-\varepsilon^p(\mathbf{x}_i,\mathbb{A})-\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} < \min\left\{-\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i}\right)^{-1}, \frac{\xi^z(\mathbf{x}_i,\mathbb{A})}{\xi^p(\mathbf{x}_i,\mathbb{A})-1}\right\}. \tag{A22}$$

Given $\xi_i^p = \frac{p_i}{\alpha}$ and $\xi_i^z = \frac{z_i}{\alpha}$, and after some calculations,

$$\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i}\right)^{-1} = \frac{\xi^z(\mathbf{x}_i,\mathbb{A})}{\xi^p(\mathbf{x}_i,\mathbb{A})} = \frac{z_i}{p_i},$$

so that (A22) holds when

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1-\varepsilon^p(\mathbf{x}_i,\mathbb{A})-\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} < \frac{z_i}{p_i}. \tag{A23}$$

Given $m_i := \frac{\varepsilon_i^p}{\varepsilon_i^p - 1} = \frac{\frac{p_i}{\alpha}(1-q_i)}{\frac{p_i}{\alpha}(1-q_i)-1} > 1$, $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i} = 1 + \frac{p_i}{\alpha}\frac{q(\mathbf{x}_i,\mathbb{A})}{1-q(\mathbf{x}_i,\mathbb{A})}$ and $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} = \frac{-q(\mathbf{x}_i,\mathbb{A})}{1-q(\mathbf{x}_i,\mathbb{A})}\frac{z_i}{\alpha}$, then

$$\frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i}}{1 - \varepsilon^p(\mathbf{x}_i,\mathbb{A}) - \frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}} = \frac{[m_i(\mathbf{x}_i,\mathbb{A}) - 1]\left[\frac{q(\mathbf{x}_i,\mathbb{A})}{1-q(\mathbf{x}_i,\mathbb{A})}\frac{z_i}{\alpha}\right]}{1 + [m_i(\mathbf{x}_i,\mathbb{A}) - 1]\left[1 + \frac{p_i}{\alpha}\frac{q(\mathbf{x}_i,\mathbb{A})}{1-q(\mathbf{x}_i,\mathbb{A})}\right]}. \tag{A24}$$

After some algebra and using that $m_i > 1$ when $p_i > c$, it can be shown that, given (A24), then (A23) holds and, so, the result follows.

B Examples of Demands

In this appendix, we present some examples of demands that can be encompassed by Definition DEM. Since there are multiple ways to embed demand-enhancing investments into some of the demand functions, we only present demands defined in terms of prices.

To express them as they are usually presented in the literature, in some cases we define demands that are increasing in $\mathbb{A}$, rather than decreasing as in Definition DEM. This is inconsequential since what matters is that there is a monotone relation between the aggregate and demand. Such a result follows because the aggregator is not uniquely defined, and any monotone transformation defines a new aggregate (Acemoglu and Jensen, 2013). Thus, it is always possible to define a new aggregate by its inverse and express the demand as in Definition DEM.

In the following, any Greek letter represents a positive parameter and E is interpreted as the industry expenditure.

- **Demands from a discrete choice model**: $q_i := \frac{h(p_i)}{\mathbb{A}}$ with $\mathcal{A}(\mathbf{x}) := H\left(\sum_{i'} h(p_{i'})\right)$. This covers the case of a standard Multinomial Logit by defining $h(p_i) := \exp(\alpha - \beta p_i)$.
- **Demands from discrete-continuous choices model** as in Nocke and Schutz (2018): $q_i := \frac{\partial h(p_i)/\partial p_i}{\mathbb{A}}$ with $\mathcal{A}(\mathbf{x}) := H\left(\sum_{i'} h(p_{i'})\right)$. It includes the Logit and the CES with normalized expenditure as special cases.
- **Constant expenditure demand systems (Vives, 2001)**: $q_i := \frac{E}{p_i}\frac{h(p_i)}{\mathbb{A}}$ with $\mathcal{A}(\mathbf{x}) := H\left(\sum_{i'} h(p_{i'})\right)$. It includes the CES through $h(p_i) := \alpha (p_i)^{-\beta_i}$ and the exponential demand by $h(p_i) := \exp(\alpha - \beta p_i)$.
- **Linear demand**: $q_i := \alpha - \beta p_i + \mathbb{A}$ with $\mathcal{A}(\mathbf{x}) := \sum_{i'} \gamma p_{i'}$.
- **Translog functional form**: $q_i := \frac{E}{p_i}[\mathbb{A} - \ln(p_i)]$ where $\mathcal{A}(\mathbf{x}) := \sum_{i'} \gamma \ln p_{i'}$.
- **Demands from an additively separable indirect utility**: given an indirect utility $V\left[(p_{i'})_{i' \in i}, E\right] := \sum_{i'} v\left(\frac{p_{i'}}{E}\right)$, the demand of i is given by $q_i := \frac{v'\left(\frac{p_i}{E}\right)}{\mathbb{A}}$ with $\mathcal{A}(\mathbf{x}) := \sum_{i'} v'\left(\frac{p_{i'}}{E}\right)\frac{p_{i'}}{E}$.

C Strategic Complements and Substitutes

In the main body of the paper, we have claimed that our results are independent of whether prices are strategic complements or substitutes. Next, we prove that this is indeed the case. To do this, we show that strategic complementarity of substitutability of prices can be determined through how the aggregate affects the price elasticity of demand. Thus, the claim holds because our results do not make any assumption in this regard.

Consider leader i. Prices are strategic complements (substitutes) when, following increases in prices by rival firms, a firm responds optimally by increasing (decreasing) its price. Formally, this requires characterizing i's best-response price function, which expresses i's optimal price as a function of rivals' prices rather than the aggregate. With this goal, reexpress the aggregator by

$$\mathcal{A}\left(p_i, z_i, h_{-i}\right) := H\left[h_{-i} + h\left(\mathbf{x}_i\right)\right],$$

where $h_{-i} := \sum_{i' \in \Omega \setminus \{i\}} h\left(\mathbf{x}_{i'}\right)$. By expressing $\mathcal{A}$ as a function of h_{-i}, the first-order condition for prices define i's best-response price, $p_i^{\text{br}}\left(z_i, h_{-i}\right)$. Thus, prices are strategic complements when $\frac{\partial p_i^{\text{br}}(z_i,h_{-i})}{\partial h_{-i}} < 0$ and strategic substitutes when $\frac{\partial p_i^{\text{br}}(z_i,h_{-i})}{\partial h_{-i}} > 0$.[17]

Next, we prove that prices are strategic complements or substitutes depending on the sign of $\frac{\partial \varepsilon^p(\mathbf{x}_i,\mathbb{A})}{\partial \mathbb{A}}$. Formally, we show that

$$\operatorname{sgn}\left(\frac{\partial p_i^{\text{br}}\left(z_i, h_{-i}\right)}{\partial h_{-i}}\right) = \operatorname{sgn}\left(\frac{\partial \varepsilon^p\left(\mathbf{x}_i, \mathbb{A}\right)}{\partial \mathbb{A}}\right). \tag{C1}$$

Total profits of leader i can be expressed in terms of h_{-i} by

$$\pi_i\left[\mathbf{x}_i, \mathcal{A}\left(\mathbf{x}_i, h_{-i}\right)\right] := q\left[\mathbf{x}_i, \mathcal{A}\left(\mathbf{x}_i, h_{-i}\right)\right]\left(p_i - c_i\right) - f_z\left(z_i\right) - F.$$

To prove the result, we use that, if $\pi_i\left(\mathbf{x}_i, h_{-i}\right)$ is supermodular in $\left(p_i, h_{-i}\right)$ for the range of optimal prices, then $\frac{\partial p_i^{\text{br}}(z_i,h_{-i})}{\partial h_{-i}} > 0$ (with the opposite sign if it is submodular). Consequently, we prove the result by establishing that

$$\operatorname{sgn}\left(\frac{\partial^2 \pi_i\left(\mathbf{x}_i, h_{-i}\right)}{\partial p_i \partial h_{-i}}\right) = \operatorname{sgn}\left(\frac{\partial \varepsilon^p\left(\mathbf{x}_i, \mathbb{A}\right)}{\partial \mathbb{A}}\right). \tag{C2}$$

Given i's gross optimal profits and after some algebraic manipulation, it can be shown that

$$\frac{\partial \pi_i\left(\mathbf{x}_i, h_{-i}\right)}{\partial p_i} = \kappa\left(\mathbf{x}_i, h_{-i}\right)\left[\frac{\partial \ln q\left(\mathbf{x}_i, h_{-i}\right)}{\partial \ln p_i} + \frac{p_i}{p_i - c_i}\right],$$

where $\kappa\left(\mathbf{x}_i, h_{-i}\right) := \frac{q(\mathbf{x}_i,h_{-i})}{p_i}\left[p_i - c_i\right] > 0$ given that in equilibrium $p_i > c_i$. Since $\varepsilon^p\left(\mathbf{x}_i, h_{-i}\right) = -\frac{\partial \ln q(\mathbf{x}_i,h_{-i})}{\partial \ln p_i}$, then

$$\frac{\partial^2 \pi_i\left(\mathbf{x}_i, h_{-i}\right)}{\partial p_i \partial h_{-i}} = \frac{\partial \kappa}{\partial h_{-i}}\left[-\varepsilon^p\left(\mathbf{x}_i, h_{-i}\right) + \frac{p_i}{p_i - c_i}\right] + \kappa\left(\frac{\partial \varepsilon^p\left(\mathbf{x}_i, h_{-i}\right)}{\partial h_{-i}}\right).$$

For the range of optimal prices, we know that

$$p_i = \frac{\varepsilon^p\left(\mathbf{x}_i, h_{-i}\right)}{\varepsilon^p\left(\mathbf{x}_i, h_{-i}\right) - 1} c_i,$$

which implies that $\frac{p_i}{p_i - c_i} = \varepsilon^p\left(\mathbf{x}_i, h_{-i}\right)$. This determines that $\left[-\varepsilon^p\left(\mathbf{x}_i, h_{-i}\right) + \frac{p_i}{p_i - c_i}\right] = 0$ by restrict-

[17]Notice that, by defining the strategy "inaction" through $\overline{\mathbf{x}} := (\infty, 0)$, strategic complementarity also reflects that exit of rival firms make i increase its prices. This follows because exit requires rivals firms to choose a price ∞ and zero investments, which decreases h_{-i}.

ing the domain of prices to its equilibrium values, and so,

$$\text{sgn}\left(\frac{\partial^2 \pi_i\left(\mathbf{x}_i, h_{-i}\right)}{\partial p_i \partial h_{-i}}\right) = \text{sgn}\left(\frac{\partial \varepsilon^p\left(\mathbf{x}_i, h_{-i}\right)}{\partial h_{-i}}\right).$$

Besides, $\frac{\partial \varepsilon^p(\mathbf{x}_i,h_{-i})}{\partial h_{-i}} = \frac{\partial \varepsilon^p(\mathbf{x}_i,\mathbb{A})}{\partial \mathbb{A}} \frac{\partial \mathcal{A}(\mathbf{x}_i,h_{-i})}{\partial h_{-i}}$ and, since $\frac{\partial \mathcal{A}(\mathbf{x}_i,h_{-i})}{\partial h_{-i}} > 0$, then (C2) holds.

D Heterogeneous Followers

In the baseline model, we have considered that all followers have the same marginal cost, $c_{\mathcal{F}}$. Next, we show that all the results still hold if we incorporate a subset of heterogeneous followers that are active in both the simultaneous- and sequential-move game.

Formally, consider that the set of followers $\mathcal{F}$ can be partitioned into subsets $\underline{\mathcal{F}}$ and $\overline{\mathcal{F}}$. Moreover, each firm $i \in \overline{\mathcal{F}}$ has marginal cost c_i satisfying $c_i < c_{\mathcal{F}}$, and there is a fixed number of firms in $\overline{\mathcal{F}}$. As for a firm $i \in \underline{\mathcal{F}}$, we suppose it still has marginal cost $c_{\mathcal{F}}$. Moreover, all firms in $\overline{\mathcal{F}}$ are active in each scenario and the last entrant belongs to $\underline{\mathcal{F}}$.

The same results hold in this scenario, since (ZP) still pins down the equilibrium aggregate. Additionally, followers in $\overline{\mathcal{F}}$ have optimal prices and investments in each scenario that can be characterized through (4) and (z-sim). Supposing that M refers now to the number of active followers from the set $\underline{\mathcal{F}}$, the setup would only need to take into account that (5) becomes

$$\mathcal{A}^{\text{sim}}\left(\mathbb{A}, M\right) := H\left\{ M h\left[\mathbf{x}_{\mathcal{F}}\left(\mathbb{A}\right)\right] + \sum_{i \in \overline{\mathcal{F}}} h\left[\mathbf{x}_i\left(\mathbb{A}\right)\right] + \sum_{i \in \mathscr{L}} h\left[\mathbf{x}_i\left(\mathbb{A}\right)\right] \right\},$$

while (6) becomes

$$\mathcal{A}^{\text{seq}}\left(\mathbb{A}, M, \mathbf{z}^{\mathscr{L}}\right) := H\left\{ M h\left[\mathbf{x}_{\mathcal{F}}\left(\mathbb{A}\right)\right] + \sum_{i \in \overline{\mathcal{F}}} h\left[\mathbf{x}_i\left(\mathbb{A}\right)\right] + \sum_{i \in \mathscr{L}} h\left[p_i\left(z_i, \mathbb{A}\right), z_i\right] \right\}.$$

E Demand-Enhancing Investments Affecting Marginal Costs

In the main body of the paper, we have followed Sutton (1991; 1998) to model demand-enhancing investments. Thus, we have supposed that they entail sunk fixed costs and have no impact on marginal costs. In this appendix, we extend our results to cover investments that require fixed costs and, additionally, affect a firm's own marginal cost. Formally, we suppose that $c_i\left(z_i\right)$ for each firm i.

Under these types of investments, the results in Propositions 5.1 and 5.2 hold without any modification. Thus, leaders always toughen competition, garner greater profits, and the number of followers is lower. Furthermore, the same condition for under- or over-investing, (8), holds since it is stated in terms $\frac{\partial \ln p_i\left(z_i^{\text{sim}};\mathbb{A}\right)}{\partial \ln z_i}$, irrespective of how this is defined.

On the contrary, the description of outcomes in Section 6 for the over-investment case requires getting a new expression for $\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i}$. This occurs because now investments affect a firm's own marginal costs and, hence, its pricing decisions. Nonetheless, all the propositions hold verbatim once we account for such effect.

To establish this, we begin by stating some lemmas. They are analogous to Lemmas 1 and 2.

Lemma 5.

$$\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i} = \frac{\frac{\partial \ln \varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]}{\partial \ln z_i} - \{\varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]-1\}\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}\ln z_i}}{1-\varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}] - \frac{\partial \ln \varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]}{\partial \ln p_i}}$$

Proof of Lemma 5. Define leader i's markup by m_i, which is a given by function $m(\mathbf{x}_i,\mathbb{A}) := \frac{\varepsilon^p(\mathbf{x}_i,\mathbb{A})}{\varepsilon^p(\mathbf{x}_i,\mathbb{A})-1}$. Also, denote the price elasticity of demand of i by ε_i^p, so that $\frac{\partial \ln m_i}{\partial \ln \varepsilon_i^p} = 1-m_i$. The first-order condition for prices determines that $\ln p_i = \ln m(\mathbf{x}_i,\mathbb{A}) + \ln c_i(z_i)$ and so

$$\begin{aligned}\mathrm{d}\ln p_i &= \frac{\partial \ln m_i}{\partial \ln p_i}\mathrm{d}\ln p_i + \frac{\partial \ln m_i}{\partial \ln z_i}\mathrm{d}\ln z_i + \frac{\mathrm{d}\ln c_i}{\mathrm{d}\ln z_i}\mathrm{d}\ln z_i \\ &= \frac{\partial \ln m_i}{\partial \ln \varepsilon_i^p}\frac{\partial \ln \varepsilon_i^p}{\partial \ln p_i}\mathrm{d}\ln p_i + \frac{\partial \ln m_i}{\partial \ln \varepsilon_i^p}\frac{\partial \ln \varepsilon_i^p}{\partial \ln z_i}\mathrm{d}\ln z_i + \frac{\mathrm{d}\ln c_i}{\mathrm{d}\ln z_i}\mathrm{d}\ln z_i,\end{aligned}$$

which implies

$$\frac{\partial \ln p_i}{\partial \ln z_i} = \frac{(1-m_i)\frac{\partial \ln \varepsilon_i^p}{\partial \ln z_i} + \frac{\mathrm{d}\ln c_i}{\mathrm{d}\ln z_i}}{1-(1-m_i)\frac{\partial \ln \varepsilon_i^p}{\partial \ln p_i}}.$$

By dividing numerator and denominator of the right-hand side by $(1-m_i)$ and using that $(1-m_i)^{-1} = 1-\varepsilon_i^p$, the result follows. ■

Lemma 6.

$$sgn\left\{\frac{\partial \ln p_i(z_i;\mathbb{A})}{\partial \ln z_i}\right\} = sgn\left\{-\left(\frac{\partial \ln \varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]}{\partial \ln z_i} - \{\varepsilon^p[p_i(z_i;\mathbb{A}),z_i;\mathbb{A}]-1\}\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}\ln z_i}\right)\right\}.$$

Proof of Lemma 6. Leader i's gross profits are given by

$$\pi_i[\mathbf{x}_i,\mathcal{A}(\mathbf{x}_i)] := q[\mathbf{x}_i,\mathcal{A}(\mathbf{x}_i)][p_i - c_i(z_i)] - f_z(z_i).$$

Replacing c_i by $c_i(z_i)$, the proof follows verbatim that of Lemma 2, with the difference that

$$D_p\pi_i(\mathbf{x}_i,\mathbb{A}) =: \frac{\mathrm{d}\pi_i(\mathbf{x}_i,\mathbb{A})}{\mathrm{d}p_i} := \kappa_i\left[-\varepsilon^p(\mathbf{x}_i,\mathbb{A}) + \frac{p_i}{p_i - c_i(z_i)}\right],$$

$$\frac{\partial D_p\pi_i(\mathbf{x}_i;\mathbb{A}^*)}{\partial z_i} = \frac{\partial \kappa(\mathbf{x}_i;\mathbb{A}^*)}{\partial z_i}\left[-\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*) + \frac{p_i}{p_i-c_i}\right] - \kappa_i\left(\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)}{\partial z_i} - \frac{p_i\frac{\mathrm{d}c_i(z_i)}{\mathrm{d}z_i}}{(p_i-c_i)^2}\right).$$

For the range of optimal prices, the first-order condition of prices has to be satisfied, so that $p_i = \frac{\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)}{\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)-1}c_i$, which implies that $-\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*) + \frac{p_i}{p_i-c_i} = 0$. Therefore, by using that $\kappa_i > 0$,

$$\operatorname{sgn}\left(\frac{\partial D_p\pi_i(\mathbf{x}_i;\mathbb{A}^*)}{\partial z_i}\right) = \operatorname{sgn}\left(-\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)}{\partial z_i} + \frac{p_i}{[p_i-c_i]^2}\frac{\mathrm{d}c_i(z_i)}{\mathrm{d}z_i}\right),$$

For the range of optimal prices, it is also satisfied that $(p_i-c_i)^2 = \left[\frac{\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)}{\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)-1}-1\right]^2[c_i(z_i)]^2$, and then

$$\frac{p_i}{(p_i-c_i)^2} = [\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)-1]\frac{\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)}{c_i(z_i)}.$$

Therefore,

$$\operatorname{sgn}\left(\frac{\partial D_p\pi_i(\mathbf{x}_i;\mathbb{A}^*)}{\partial z_i}\right) = \operatorname{sgn}\left(-\frac{\partial \varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)}{\partial z_i} + [\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)-1]\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}z_i}\right). \tag{E1}$$

Multiplying the right-hand side of (E1) by $\frac{z_i}{\varepsilon^p(\mathbf{x}_i;\mathbb{A}^*)}$, which does not modify its sign, the result follows. ■

By making use of these lemmas, it is possible to establish analogous assumptions to those stated in the main body of the paper. With this goal, define

$$\lambda\left(\mathbf{x}_i;\mathbb{A}\right) := \frac{\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} - \left[\varepsilon^p\left(\mathbf{x}_i;\mathbb{A}\right)-1\right]\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}\ln z_i}}{1-\varepsilon^p\left(\mathbf{x}_i,\mathbb{A}\right)-\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln p_i}}.$$

Assumption E.4. *Either* $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} - \left[\varepsilon^p\left(\mathbf{x}_i;\mathbb{A}\right)-1\right]\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}\ln z_i} > 0$ *or* $\lambda\left(\mathbf{x}_i;\mathbb{A}\right) < -\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln z_i}\left(\frac{\partial \ln h(\mathbf{x}_i)}{\partial \ln p_i}\right)^{-1}$ *for any* $(\mathbf{x}_i,\mathbb{A})$*.*

Assumption E.5. *For any* $(\mathbf{x}_i,\mathbb{A})$*, we suppose that*

$$\lambda\left(\mathbf{x}_i;\mathbb{A}\right) < \frac{\xi^z\left(\mathbf{x}_i,\mathbb{A}\right)}{\xi^p\left(\mathbf{x}_i,\mathbb{A}\right)-1}$$

when $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} - \left[\varepsilon^p\left(\mathbf{x}_i;\mathbb{A}\right)-1\right]\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}\ln z_i} < 0$.

Assumption E.6. *For any* $(\mathbf{x}_i,\mathbb{A})$*, we suppose that*

$$\lambda\left(\mathbf{x}_i;\mathbb{A}\right) < \frac{\xi^z\left(\mathbf{x}_i,\mathbb{A}\right)}{\xi^p\left(\mathbf{x}_i,\mathbb{A}\right)}$$

when $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} - \left[\varepsilon^p\left(\mathbf{x}_i;\mathbb{A}\right)-1\right]\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}\ln z_i} < 0$.

Making use of these assumptions, we determine the next proposition. The proof of each statement follows verbatim those for investments that do not affect marginal costs.

Proposition E.1. *Suppose a framework where investments additionally affect marginal costs. Then:*

- *Propositions 5.1 and 5.2 hold.*
- *Proposition 6.1 holds if we replace Assumption 6.1 by Assumption E.4, where leader* i *charges higher prices if* $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} - \left[\varepsilon^p\left(\mathbf{x}_i;\mathbb{A}\right)-1\right]\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}\ln z_i} < 0$ *and lower prices if* $\frac{\partial \ln \varepsilon^p(\mathbf{x}_i;\mathbb{A})}{\partial \ln z_i} - \left[\varepsilon^p\left(\mathbf{x}_i;\mathbb{A}\right)-1\right]\frac{\mathrm{d}\ln c_i(z_i)}{\mathrm{d}\ln z_i} > 0$.
- *Proposition 6.2 holds if we replace Assumption 6.2 by Assumption E.5.*
- *Proposition 6.3 holds if we replace Assumption 6.3 by Assumption E.6.*