# International Trade[1]

## Lecture Note 4: Successful Firms: Appeal vs Efficiency

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[1]The notes are still preliminary and in beta. Please, if you find any typo or mistake, send it to malfaro@ualberta.ca.

# Contents

# Notation

This is a derivation

This is some comment

This is a comment on advanced topics that are not part of the course (you can ignore it without loss of continuity regarding the text)

- The symbol ":=" means "by definition".
- Vectors are denoted by bold lowercase letters (for instance, $\mathbf{x}$) and matrices by bold capital letters (for instance, $\mathbf{X}$).
- The set of nonnegative real numbers is denoted by $\mathbb{R}_+ := [0, \infty)$
- The set of positive real numbers is denoted by $\mathbb{R}_{++} := (0, \infty)$
- The Cartesian product is denoted by $X_1 \times X_2 \times ... \times X_N$. If each set comprises the nonnegative real numbers, we use the notation $\mathbb{R}_+^N$.
- To differentiate between the verb "maximize" and the operator "maximum", I denote the former with "max" and the latter with "sup" (i.e., supremum). The same caveat applies to "minimize" and "minimum", where I use "min" and "inf", with the latter indicating infimum.
- "iff" means "if and only if"
- $\exp(x)$ is the function $e^x$.
- Random variables are denoted with a bar below. For instance, $\underline{x}$.

These notes contain hyperlinks in blue and red text. If you are using Adobe Acrobat Reader, click on the link to see the text, and navigate back by pressing Alt+Left Arrow.

# 1 Introduction

This note develops a formal framework to identifying features of successful firms, where we define **successful firms** as those having high profits. The goal is to provide a framework for understanding empirical studies that use microdata of companies.

The sources of success can be categorized into two broad areas: demand-side factors (termed **quality/appeal**), and supply-side factors (termed **efficiency**). In these types of models, the concept of quality is understood broadly, encompassing both objective and subjective characteristics. This explains why we also refer to quality as appeal.

The analysis examines the behavior of firms regarding prices, markups, and quantities, based on whether success stems from quality or efficiency. To focus on these decisions, we keep the model as simple as possible. In particular, we analyze an industry with only one firm operating. Despite the simplicity of the model, the insights presented are useful for more complex market structures, such as monopolistic competition and oligopolies with differentiated goods.

In economics, a single firm in an industry is referred to as a monopoly. However, it is important to distinguish between a monopoly and exercising monopoly power, defined as setting a price above marginal cost. The distinction is important, as **the number of firms does not necessarily determine the competitiveness of the market or the profits a firm can earn**.

First, a firm can exercise market power, even when competing against many rivals in its industry. Apple in the cell phone market is a typical example. Second, an industry with just a few firms does not necessarily mean those firms have market power. This is evident in a Bertrand competition model with two firms, a homogeneous good, and equal marginal costs, in which case a competitive outcome would emerge.

Finally, the presence of a single firm is not sufficient for exercising market power. This insight comes from Contestability Theory, which examines scenarios without actual competition but with non-existent barriers to entry and exit. The key takeaway is that the threat of potential entry can discipline a monopolist's behavior, leading to competitive outcomes. This arises because, if the single firm set prices above marginal cost, new entrants could easily enter and take away the entire demand.

## 1.1 Setup

We focus on an industry with a single firm supplying one good and no potential entrants. This firm operates with constant marginal costs $c$ and has no fixed costs. Formally, its cost function is $C(q) := cq$, where $q$ is the quantity produced by the firm.

The total demand for the good is described by a function $q(p;\alpha) \in \mathcal{C}^2$ (i.e., twice continuously differentiable) where $p \in [\underline{p}, \overline{p}]$ is the price of the good and $\alpha \in \mathbb{R}_+$ a parameter. We assume that $\frac{\partial q(p;\alpha)}{\partial p} < 0$. and suppose that the features of the good are exogenously given.

We refer to $\alpha$ as the **good's appeal**. This parameter captures that consumers make consumption decisions based not only on price, but also on non-price aspects of the good (e.g., quality, after-sale services). Below, we will make several assumptions consistent with this interpretation for $\alpha$ (for instance, that a greater $\alpha$ increases the demand).[1]

## 1.2 Basic Assumptions

We now state some basic assumptions about how the quantity demanded relates to $p$ and $\alpha$. These assumptions will be specified in terms of elasticities. After this, we will add some additional assumptions to get unambiguous comparative statics.

Regarding price, we have already stated that $\frac{\partial q(p;\alpha)}{\partial p} < 0$, and so $\varepsilon_p(p;\alpha) > 0$ for all $(p;\alpha)$. We now add that demand is always elastic, formally represented by $\varepsilon_p(p;\alpha) > 1$ for all $(p;\alpha)$.[2] The magnitude of the price elasticity assesses the tradeoff faced by any firm: when prices are increased, revenue is directly increased, but also indirectly decreased through reducing the units sold. By assuming that demand is price elastic, we are basically ruling out the possibility of a corner solution, where the price is always set as high as possible.

As for $\alpha$, we now make two assumptions consistent with it being appeal. First, **greater appeal results in a greater quantity demanded**, captured by the assumption $\frac{\partial q(p;\alpha)}{\partial \alpha} > 0$, or $\varepsilon_\alpha := \frac{\partial \ln q(p;\alpha)}{\partial \ln \alpha} > 0$ in terms of elasticities. This assumption states that, holding price fixed, increases in $\alpha$ raise the quantity demanded. Note that we remain agnostic about the specific channel through which appeal boosts demand. It could occur by existing consumers purchasing more quantities or by the firm selling to new customers.

The second assumption refers to the relation between appeal with price. So far, we have assumed that quality increases the quantity sold of a product. But we could additionally imagine that a more attractive product reduces the sensitivity of consumers

[1] Note that we are implicitly defining $\alpha$ as a real number, rather than a vector. This implies that all non-price features of a good are encompassed in a single measure $\alpha$. Breaking down each tangible and intangible characteristic of the good into several parameters would unnecessarily complicate the model.

[2] This assumption is stronger than what we need, as we could allow for demands that are inelastic for some prices.

to higher prices. In other words, increases in price reduce the quantity sold, but to a lesser extended compared to a low-quality product.

This aspect is incorporated into the model by assuming that a higher $\alpha$ is associated with a lower $\varepsilon_p(p;\alpha)$, which formally means $\frac{\partial \varepsilon_p(p;\alpha)}{\partial \alpha} \leq 0$. Expressed in words, **when a good is more appealing, the demand becomes less price elastic**. Note that we allow for the possibility that appeal does not affect the willingness to pay for the good. This could occur if, for instance, greater appeal reflects improvements in distribution channels, and the new consumers have the same valuation for the product as the old ones. In that case, the quantities demanded would increase. However, the sensitivity of consumers to prices would remain unaffected, as the pool of new consumers would be equivalent to the original consumer base.

**Assumption 1.1.** *Summing up, we assume that*

- $\varepsilon_p > 1$
- $\varepsilon_\alpha(p;\alpha) > 0$ *and* $\frac{\partial \varepsilon_p(p;\alpha)}{\partial \alpha} \leq 0$.

We will actually require additional assumptions regarding $\frac{\partial \varepsilon_p(p;\alpha)}{\partial p}$. However, these assumptions will be added later in the text, as we first need to derive a few results.

## 1.3 The Optimization Problem

The firm's optimization problem is to maximize profits by choosing the price of its good. Once the prices are chosen, the quantity supplied is completely determined by the demand function $q(p;\alpha)$. This means that, once the firm selects the price, the market adjusts quantities until supply equals demand.

**Remark** *The firm could alternatively maximize profits by choosing the quantity supplied, letting the price be determined by the equality of supply and demand. Nevertheless,* both optimization problems yield the same result. *Differentiating between price and quantity as the choice variable is only needed when considering strategic interactions, as in the Cournot and Bertrand oligopoly models. In contrast, for mod-*

*els like the one considered or monopolistic competition, it does not matter whether we frame the problem as choosing price or quantity.*

Formally, the optimization problem is

$$\max_{p\in\left[\underline{p},\overline{p}\right]} \pi\left(p;\alpha,c\right):=q\left(p;\alpha\right)\left(p-c\right)$$

There are different conditions guaranteeing that the problem is well-defined. Nevertheless, we will proceed under the assumption that a solution exists, is unique, and interior. The goal is to make a clear distinction between assumptions we made to get unambiguous results in a comparative–static analysis, and those to have a well-behaved problem.

In recent decades, comparative statics analysis has seen a revival, particularly through the approach of monotone comparative statics. This approach has demonstrated that numerous assumptions previously deemed necessary can actually be relaxed. As a result, it has now become customary to focus on the minimal conditions that are necessary to obtain unambiguous comparative statics results.
For the model considered here, existence, uniqueness, and interior solutions can nevertheless be characterized in a simple manner. Firstly, an optimal price exists, as the price domain is compact and profits are continuous. Furthermore, by adding Inada conditions and strict quasiconcavity of profits, we can guarantee that the solution is interior and unique.
However, these sufficient conditions are stronger than needed. It is common that profits are not strictly quasiconcave, despite having a unique solution. Furthermore, we could be able predict how a parameter affects the solution, even without assuming uniqueness. As these remarks illustrate, many intricate details arise for well-behaved solutions, in a context where these considerations are tangential for our analysis.

We characterize the solution by the first-order condition:

$$p^{*}=\frac{\varepsilon_{p}\left(p^{*};\alpha\right)}{\varepsilon_{p}\left(p^{*};\alpha\right)-1}c. \tag{PRICE}$$

The first-order condition is
$\frac{\mathrm{d}\pi}{\mathrm{d}p}=\frac{\partial Q(p;\alpha)}{\partial p}\left(p-c\right)+Q\left(p,\alpha\right)=0\Rightarrow\frac{\partial Q(p;\alpha)}{\partial p}\frac{1}{Q(p,\alpha)}=-\frac{1}{p-c}$
By multiplying both sides by $p$, then $-\frac{\partial Q(p;\alpha)}{\partial p}\frac{p}{Q(p,\alpha)}=\frac{p}{p-c}$.
Since $\varepsilon\left(p;\alpha\right):=-\frac{\partial Q(p;\alpha)}{\partial p}\frac{p}{Q(p,\alpha)}$, then the first-order condition implies that $\varepsilon\left(p;\alpha\right)=\frac{p}{p-c}$, or simply $p=\frac{\varepsilon(p;\alpha)}{\varepsilon(p;\alpha)-1}c$.

Since we have assumed that $\varepsilon_{p}\left(p;\alpha\right)>1$ for all $\left(p;\alpha\right)$, (PRICE) determines that $p^{*}>c$. Notice that (PRICE) provides only an implicit characterization for optimal prices $p^{*}$, as the price elasticity still depends on prices. We denote the implicit value $p^{*}$ that satisfies (PRICE) by $p^{*}\left(\alpha,c\right)$.

Once that optimal prices are pinned down, **optimal profits** are

$$\pi^* (\alpha, c) := Q [p^* (\alpha, c), \alpha] [p^* (\alpha, c) - c].$$

Finally, we now introduce the concept of **markup** into the analysis. This is denoted by $\mu$ and defined by

$$\mu := \frac{p}{c},$$

providing information about the ratio of revenue over cost per unit sold.

In particular, when we replace $p^*$ into the expression, markup is defined by the following function:

$$\mu (p; \alpha) := \frac{\varepsilon_p (p; \alpha)}{\varepsilon_p (p; \alpha) - 1},$$

allowing us to rewrite equation (PRICE) by

$$p^* = \mu (p^*; \alpha) c. \tag{PRICE-1}$$

Markups are commonly employed as a measure of market power, taking into account that $\mu = \frac{p}{c} = 1$ in perfect competition.

# 2 Comparative Statics (CS)

CS identifies how a model's endogenous variables are affected by changes in its parameters. In our model, the endogenous variable is price, and there are two parameters, $c$ and $\alpha$. Once we determine the effect of these parameters on prices, we are also able to identify the effect of these parameters on other endogenous variables like quantities and markups.

Based on this, we start by determining how $\alpha$ and $c$ affect the optimal price $p^* (\alpha, c)$. We will perform a CS analysis by varying one parameter at a time, ultimately establishing the signs of $\frac{\partial p^*(\alpha,c)}{\partial c}$ and $\frac{\partial p^*(\alpha,c)}{\partial \alpha}$.

We treat $\alpha$ and $c$ as parameters, even though appeal and productivity are partly determined by firms. However, this simplification does not invalidate the conclusions of our model.
To see this, consider a model where a firm makes decisions on $\alpha$ and $c$. Additionally, suppose the firm has some given ability to differentiate products ($\varphi^\alpha$) and reduce costs ($\varphi^c$). For a given ($\varphi^c, \varphi^\alpha$), this firm would then make

choices $c^*(\varphi^c, \varphi^\alpha)$ and $\alpha^*(\varphi^c, \varphi^\alpha)$. In this context, we are essentially asking how firms with a greater $\varphi^c$ (lower $c$) or greater $\varphi^\alpha$ (higher $\alpha$) make their choices.

## 2.1 Some Additional Assumptions

Before conducting a CS analysis, we need to add some assumptions to obtain unambiguous results. These assumptions are related to the impact of prices on price elasticity and markups.

We begin by showing that the sign of the effect of prices on price elasticity coincides with the negative effect of prices on markups. To observe this, we know that $\mu(p;\alpha) := \frac{\varepsilon_p(p;\alpha)}{\varepsilon_p(p;\alpha)-1}$. Taking $\mu$ as a function of $\varepsilon_p$, the relation between both terms is:

$$\frac{\partial \mu(\varepsilon_p)}{\partial \varepsilon_p} = \frac{-1}{(\varepsilon_p - 1)^2}.$$

This indicates that markup increases when the price elasticity decreases. As a corollary, if any parameter or variable decreases the price elasticity, the markup will increase. Overall, markups are determined by whether the parameter or the variable makes the demand more inelastic (higher markup) or more elastic (lower markups). As a result, the assumptions we make about how $\varepsilon_p$ is impacted when $p$ or $\alpha$ varies determine completely how markups are affected.

So far, we have only supposed that $\frac{\partial \varepsilon_p(p;\alpha)}{\partial \alpha} \leq 0$. Therefore,

$$\frac{\partial \mu(p^*;\alpha)}{\partial \alpha} = \frac{-1}{(\varepsilon_p - 1)^2} \frac{\partial \varepsilon_p(p^*;\alpha)}{\partial \alpha} \geq 0.$$

Thus, **increases in appeal determine a higher markup,** as they reduce the price elasticity of demand.

Now, let's consider how variations in prices affect the price elasticity and hence markups. Formally,

$$\frac{\partial \mu(p^*;\alpha)}{\partial p} = \frac{-1}{(\varepsilon_p - 1)^2} \frac{\partial \varepsilon_p(p^*;\alpha)}{\partial p},$$

determining that there is a negative relation between $\frac{\partial \mu(p^*;\alpha)}{\partial p}$ and $\frac{\partial \varepsilon_p(p^*;\alpha)}{\partial p}$. It is not obvious what the sign $\frac{\partial \varepsilon_p(p^*;\alpha)}{\partial p}$ should be. Consistent with the results we want to get below, suppose that

$$\frac{\partial \varepsilon_p(p^*;\alpha)}{\partial p} < 0,$$

which implies that

$$\frac{\partial \mu\left(p^{*};\alpha\right)}{\partial p}>0.$$

This means that **firms charging a higher price set a higher markup**. Equivalently, firms charging a lower price set a lower markup. One way to justify this is to think about income-constrained consumers. Presumably, richer people are less sensitive to increases in prices. Following an increase in price, poor people might not afford the good, making rich customers be the only relevant portion of the demand. In this scenario, the price elasticity of the aggregate demand would be lower and determine that increases in price allow the firm to raise its markup.

The second assumption we make is that, even though $\frac{\partial \mu(p^{*};\alpha)}{\partial p}>0$, the effect is such that:

$$1-\frac{\partial \ln \mu\left(p;\alpha\right)}{\partial \ln p}>0 \tag{1}$$

What is the justification for Assumption (1)? It is necessary to get a specific characterization of firms we are interested in. However, it could also be justified in other grounds: Assumption (1) at the optimal price $p^{*}$ is necessary to ensure both the second-order condition and uniqueness of the equilibrium.

**Summary of the Asssumptions**

- $\varepsilon_{p}\left(p;\alpha\right)>1$ for any $\left(p;\alpha\right)$ (elastic demand at any point)
- $\alpha$ is appeal:
  - $\varepsilon_{\alpha}\left(p;\alpha\right)>0$ (increases of $\alpha$ boost demand)
  - $\frac{\partial \varepsilon_{p}(p;\alpha)}{\partial \alpha}\leq 0$ (greater $\alpha$ makes demand more inelastic/less elastic)
- $\frac{\partial \varepsilon_{p}(p^{*};\alpha)}{\partial p}<0$ which implies $\frac{\partial \ln \mu(p;\alpha)}{\partial \ln p}>0$ (for definite CS)
- $1-\frac{\partial \ln \mu(p;\alpha)}{\partial \ln p}>0$ (for definite CS)

## 2.2 Variations in $c$

Let's first consider the case where the parameter of interest is $c$. Formally, $\mathrm{d}c\neq 0$ and $\mathrm{d}\alpha=0$. Allowing for all the endogenous variable to react to this change, we can differentiate equation (PRICE-1) for $\mathrm{d}p^* \neq 0$ and $\mathrm{d}c \neq 0$ and obtain:

$$\frac{\partial p^*(\alpha,c)}{\partial c} = \frac{\mu(p^*;\alpha)}{1-\frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}} > 0 \tag{PRICE-$c$}$$

where we have used Assumption (1) to determine that $\frac{\partial p^*(\alpha,c)}{\partial c} > 0$. From this we conclude that **more efficient firms (lower $c$) charge lower prices.**

The FOC is $p^* = \mu(p^*;\alpha)\,c$ and differentiating it with $\mathrm{d}p^* \neq 0$ and $\mathrm{d}c \neq 0$:

$$\left[1-\frac{\partial \mu(p^*;\alpha)}{\partial p}c\right]\mathrm{d}p^* = \mu(p^*;\alpha)\,\mathrm{d}c$$

which implies that $\frac{\partial p^*(\alpha,c)}{\partial c} = \frac{\mu(p^*;\alpha)}{1-\frac{\partial \mu(p^*;\alpha)}{\partial p}c}$.

I want to show that $\frac{\partial \mu(p^*;\alpha)}{\partial p}c = \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}$. Starting from $\frac{\partial \mu(p^*;\alpha)}{\partial p}c$, we know by equation (PRICE-1) that $p^* = \mu(p^*;\alpha)\,c$ and so we can substitute $c$ for $\frac{p^*}{\mu(p^*;\alpha)}$, implying that $\frac{\partial \mu(p^*;\alpha)}{\partial p}c = \frac{\partial \mu(p^*;\alpha)}{\partial p}\frac{p^*}{\mu(p^*;\alpha)}$. Then, since $\frac{\partial \mu(p^*;\alpha)}{\partial p}\frac{p^*}{\mu(p^*;\alpha)} = \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}$, we have that $\frac{\partial p^*(\alpha,c)}{\partial c} = \frac{\mu(p^*;\alpha)}{1-\frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}}$. Notice this is positive since Assumption (1) is $1-\frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p} > 0$.

Once we have determined the effect of variations in $c$ on prices, we can determine the effect of $c$ on quantities and markups. Optimal quantities are given by $q^*\left[p(\alpha,c);\alpha\right]$. Thus,

$$\frac{\mathrm{d}q^*\left[p^*(\alpha,c);\alpha\right]}{\mathrm{d}c} = \underbrace{\frac{\partial q(p^*;\alpha)}{\partial p}}_{-}\underbrace{\frac{\partial p^*(\alpha,c)}{\partial c}}_{+} < 0 \tag{QUANT-$c$}$$

The result is intuitive. **Less efficient firms (firms with greater marginal costs) set a higher price, and sell less as a consequence**.

As far as markups go, the optimal value is given by $\mu\left[p^*(\alpha,c);\alpha\right]$ and so

$$\frac{\mathrm{d}\mu^*\left[p^*(\alpha,c);\alpha\right]}{\mathrm{d}c} = \underbrace{\frac{\partial \mu(p^*;\alpha)}{\partial p}}_{+}\underbrace{\frac{\partial p^*(\alpha,c)}{\partial c}}_{+} > 0 \tag{MK-$c$}$$

Thus, **more efficient firms charge a lower markup**.

Overall, we have determined **that less productive firms charge higher prices, sell less, and charge higher markups**. Equivalently, **more productive firms charge lower prices, sell more quantity, and charge lower markups**.

## 2.3 Variations in $\alpha$

Let's consider variations in $\alpha$. Differentiating equation (PRICE-1) for $\mathrm{d}p^* \neq 0$ and $\mathrm{d}\alpha \neq 0$, we obtain

$$\frac{\partial \ln p^*(\alpha, c)}{\partial \ln \alpha} = \frac{\frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln \alpha}}{1 - \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}} \geq 0 \qquad \text{(PRICE-}\alpha\text{)}$$

Since $p^*, \alpha > 0$, we have that $\frac{\partial \ln p^*(\alpha,c)}{\partial \ln \alpha} \geq 0$ iff $\frac{\partial p^*(\alpha,c)}{\partial \alpha} \geq 0$. Thus, a greater appeal makes the firm charge a higher (or the same) price .

The FOC is $p^* = \mu(p^*;\alpha)\, c$ and differentiating it where $\mathrm{d}p^* \neq 0$ and $\mathrm{d}\alpha \neq 0$:

$$\left[1 - \frac{\partial \mu(p^*;\alpha)}{\partial p} c\right] \mathrm{d}p^* = \frac{\partial \mu(p^*;\alpha)}{\partial \alpha} c\, \mathrm{d}\alpha$$

which implies that $\frac{\partial p^*(\alpha,c)}{\partial \alpha} = \frac{\frac{\partial \mu(p^*;\alpha)}{\partial \alpha} c}{1 - \frac{\partial \mu(p^*;\alpha)}{\partial p} c}$. Regarding the denominator, we have already shown in the derivation of (PRICE-$c$) that $\frac{\partial \mu(p^*;\alpha)}{\partial p} c = \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}$. Concerning the numerator, using that $c = \frac{p^*}{\mu(p^*;\alpha)}$, then $\frac{\partial \mu(p^*;\alpha)}{\partial \alpha} c = \frac{\partial \mu(p^*;\alpha)}{\partial \alpha} \frac{p^*}{\mu(p^*;\alpha)}$ which equals $\frac{\partial \ln \mu(p^*;\alpha)}{\partial \alpha} p^*$.
With all these results, we have shown that

$$\frac{\partial p^*(\alpha, c)}{\partial \alpha} = \frac{\frac{\partial \ln \mu(p^*;\alpha)}{\partial \alpha} p^*}{1 - \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}}$$

Multiplying both sides by $\alpha$, then $\frac{\partial p^*(\alpha,c)}{\partial \alpha} \alpha = \frac{\frac{\partial \ln \mu(p^*;\alpha)}{\partial \alpha} \alpha p^*}{1 - \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}}$. Dividing both sides by $p^*$ $\frac{\partial p^*(\alpha,c)}{\partial \alpha} \frac{\alpha}{p^*} = \frac{\frac{\partial \ln \mu(p^*;\alpha)}{\partial \alpha} \alpha}{1 - \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}}$.
Since $\frac{\partial \ln \mu(p^*;\alpha)}{\partial \alpha} \alpha = \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln \alpha}$ and $\frac{\partial p^*(\alpha,c)}{\partial \alpha} \frac{\alpha}{p^*} = \frac{\partial \ln p^*(\alpha,c)}{\partial \ln \alpha}$, the result follows.

To understand why prices are increasing in appeal, keep in mind that a firm faces a more inelastic demand when it sells a product with higher appeal. Hence, the firm might increase its price, without the quantities sold being heavily affected.

Regarding optimal quantities $q[p^*(\alpha, c);\alpha]$:

$$\frac{\mathrm{d}q[p^*(\alpha, c);\alpha]}{\mathrm{d}\alpha} = \underbrace{\frac{\partial q(p^*;\alpha)}{\partial \alpha}}_{+} + \underbrace{\frac{\partial q(p^*;\alpha)}{\partial p}}_{-} \underbrace{\frac{\partial p(\alpha, c)}{\partial \alpha}}_{+ \text{ or } 0} \gtreqless 0 \qquad \text{(QUANT-}\alpha\text{)}$$

and so the effect is ambiguous.

The intuition behind is the following. When there is increase in the appeal of the good, there are two effects working simultaneously. First, there is a direct effect, where the mere fact of selling a product with more appeal increases the demand for the good. However, there is also an indirect effect if appeal turns the demand more inelastic: the firm would have incentives to increase its price, thus reducing its demand. Overall, depending on which effect dominates, total demand can increase or decrease. Notice that if appeal does not affect price elasticity, then the quantity demanded would be necessarily greater. For future references, we define the two possibilities.

$$\text{Case I of (QUANT-}\alpha\text{): } \frac{\mathrm{d}q\left[p^*\left(\alpha,c\right);\alpha\right]}{\mathrm{d}\alpha}>0$$
$$\text{Case II of (QUANT-}\alpha\text{): } \frac{\mathrm{d}q\left[p^*\left(\alpha,c\right);\alpha\right]}{\mathrm{d}\alpha}<0$$

Concerning the effects on markups:

$$\frac{\mathrm{d}\mu\left[p^*\left(\alpha,c\right);\alpha\right]}{\mathrm{d}\alpha}=\underbrace{\frac{\partial\mu\left(p^*;\alpha\right)}{\partial\alpha}}_{+}+\underbrace{\frac{\partial\mu\left(p^*;\alpha\right)}{\partial p}}_{+}+\underbrace{\frac{\partial p^*\left(\alpha,c\right)}{\partial\alpha}}_{+}>0 \qquad (\mathrm{MK}-\alpha)$$

Intuitively, we have shown that there is a one-to-one relation between the sign of $\mu$ and of $\varepsilon_p$. Since a greater appeal turns the demand more inelastic directly through both $\alpha$ and indirectly through $p^*$ (increases in prices make the demand more inelastic), then the firm increases the markup.

# 3 What Makes A Firm Successful?

According to Michael Porter, a famous academic specialized in business, there are three strategies that firms can pursue to be successful. He refers to them as Generic Competitive Strategies, and comprise the following:

[1] **Overall cost leadership**

*Examples*: Walmart, Costco and Aldi (retailers), RyanAir and EasyJet (airlines), Ikea (furniture), H&M (apparel).

[2] **Differentiation**

*Examples*: Nike and Adidas (sport clothes), Coca Cola and Pepsi (carbonated beverages), Duracell and Energizer (batteries), Bayer (pharmaceutical products), Apple (computers).

[3] **Focus**

*Examples*: Ferrari, BMW, and Mercedes Benz (cars), Louis Vuitton and Gucci (apparel), Dom Pérignon (champagne), Rolex (clocks).

We now formalize the conceptualization of successful firms within the model. This requires identifying combinations of $(\alpha, c)$ that make a company earn high profits. The classification relies on that high appeal (high $\alpha$) and high efficiency (low $c$) result in high profits:

$$\frac{\partial \pi^* (\alpha, c)}{\partial \alpha} = \frac{\partial Q (p^*; \alpha)}{\partial \alpha} (p^* - c) > 0,$$
$$\frac{\partial \pi^* (\alpha, c)}{\partial c} = -Q (p^*; \alpha) < 0.$$

We know that optimal profits are $\pi^* (\alpha, c) := Q [p^* (\alpha, c), \alpha] [p^* (\alpha, c) - c]$.

For a change in $\alpha$

$$\begin{aligned}\frac{d\pi^* (\alpha, c)}{d\alpha} &= \frac{\partial \pi^* (\alpha, c)}{\partial \alpha} + \underbrace{\frac{\partial \pi^* (\alpha, c)}{\partial p}}_{=0 \text{ by the FOC}} \frac{\partial p^* (\alpha, c)}{\partial \alpha} \\ &= \frac{\partial \pi^* (\alpha, c)}{\partial \alpha} \\ &= \frac{\partial Q (p^*; \alpha)}{\partial \alpha} (p^* - c) > 0\end{aligned}$$

For a change in $c$:

$$\begin{aligned}\frac{d\pi^* (\alpha, c)}{dc} &= \frac{\partial \pi^* (\alpha, c)}{\partial c} + \underbrace{\frac{\partial \pi^* (\alpha, c)}{\partial c}}_{=0 \text{ by the FOC}} \frac{\partial p^* (\alpha, c)}{\partial c} \\ &= \frac{\partial \pi^* (\alpha, c)}{\partial c} \\ &= -Q (p^*; \alpha) < 0\end{aligned}$$

Next, we show how to construct these categories based on combinations of $\alpha$ and $c$.

## 3.1 What are the Strategies Followed by a Successful Firm?

We have shown that firms with a lower $c$ or greater $\alpha$ exhibit higher profits. Moreover, we have established the following results.

**Summary of the Results**

**Variations in $c$**

- $\frac{\partial \pi^*(\alpha,c)}{\partial c} < 0$
- $\frac{\partial p^*(\alpha,c)}{\partial c} > 0$
- $\frac{dq^*[p^*(\alpha,c);\alpha]}{dc} < 0$

- $\frac{\mathrm{d}\mu^*[p^*(\alpha,c);\alpha]}{\mathrm{d}c} > 0$

**Variations in $\alpha$**

- $\frac{\partial \pi^*(\alpha,c)}{\partial \alpha} > 0$
- $\frac{\partial p^*(\alpha,c)}{\partial \alpha} > 0$
- $\frac{\mathrm{d}q[p^*(\alpha,c);\alpha]}{\mathrm{d}\alpha} \gtreqless 0$
  - Case I: $\frac{\mathrm{d}q[p^*(\alpha,c);\alpha]}{\mathrm{d}\alpha} > 0$
  - Case II: $\frac{\mathrm{d}q[p^*(\alpha,c);\alpha]}{\mathrm{d}\alpha} < 0$
- $\frac{\mathrm{d}\mu[p^*(\alpha,c);\alpha]}{\mathrm{d}\alpha} > 0$

With the information provided through the comparative statics analysis, we can now establish Porter's taxonomy:

[1] **Overall Cost Leadership**: firms with lower $c$, such that they have high $q^*$, low $p^*$ and low $\mu^*$

[2] **Differentiation**: firms with high $\alpha$ and Case I of (QUANT-$\alpha$), such that they have high $q^*$, high $p^*$ and high $\mu^*$

[3] **Focus**: firms with high $\alpha$ and Case II of (QUANT-$\alpha$), such that they have low $q^*$, high $p^*$ and high $\mu^*$.

By using the definition of profits, we can also see how these strategies are reflected. There are two ways in which we can reexpress optimal profits. First,

$$\pi^*(\alpha, c) := \frac{R[p^*(\alpha,c),\alpha]}{\varepsilon_p[p^*(\alpha,c),\alpha]} \tag{PROFIT-1}$$

Let's indicate optimal variables without arguments and with a $*$ as a superscript. Optimal profits are $\pi^*(\alpha,c) := Q^*(p^* - c)$. By the FOC, $p^* = \frac{\varepsilon^*}{\varepsilon^*-1}c$ and so by subtracting $c = \frac{\varepsilon^*-1}{\varepsilon^*}p^*$. Thus, optimal profits are $\pi^*(\alpha,c) := Q^*\left(p^* - \frac{\varepsilon^*-1}{\varepsilon^*}p^*\right)$ or, just $\pi^*(\alpha,c) := Q^*p^*\left(1 - \frac{\varepsilon^*-1}{\varepsilon^*}\right)$ which determines the result.

Moreover, by the mere definition of a profit function, we can divide and multiply by $c$ and obtain:

$$\pi(p^*;\alpha,c) = \underbrace{cQ(p^*;\alpha)}_{=:(1)}\underbrace{[\mu(p^*;\alpha) - 1]}_{=:(2)} \tag{PROFIT-2}$$

where we have used the fact that $\frac{p^*}{c} = \mu(p^*;\alpha)$

Using (PROFIT-2), we can observe that one way to attain high profits is by setting lower markups (low $\mu$ and so a small term (2)) and increasing production scale (high $Q$ and so big term (1)). A firm that pursues this strategy would be characterized as massive and inexpensive. However, this requires the firm to be highly productive, enabling it to sell at a significantly low price. For this strategy to be feasible, consumers must be fairly sensitive to price, making demand highly responsive. In this way, the rise in quantities sold can compensate for the low prices. These conditions turn a cost leadership approach profitable.

At the other extreme, (PROFIT-2) indicates that high profits can also be achieved if the firm sells low quantities (small term (1)) and sets high markups (high $\mu$ and so big term (2)). These firms focus on a niche market, targeting a small number of high-purchasing power customers. The strategy is profitable as long as these customers have a high willing to pay for the distinctive features of the good. In terms of (PROFIT-1), its is implemented by enhancing the appeal of the good to such extent that $\varepsilon_p$ is reduced significantly, allowing the firm to charge a high price. Note that the strategy could still be profitable even if total revenues decrease, as the firm can save on production costs by selling low amounts.

Finally, successful firms could aim at maintaining some balance between sales and prices/markups charged. The scenario corresponds to the case of differentiation in Porter's taxonomy. Firms in this category set relatively high prices, but their target customer base is broader than a mere niche market. The strategy is implemented by increasing the price and appeal of the good, without doing it to such a degree that the good becomes unaffordable for a large segment of customers.